Class X Mathematics

CBSE Board, Set – 2

General Instructions:

(i) All questions re compulsory.

(ii) The question paper consists of 31 questions divided into four sections – A, B, C and D.

(iii) Section A contains 4 questions of 1 mark each, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.

(iv) Use of calculators is not permitted.

Section A

Q1. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Sol.1 There are 26 English alphabets in total, and so total out comes = 26, out of these 26, 5 are vowels .

⇒ Number of Consonants = favorable outcomes = 26 – 5 = 21.

Here the required probability i.e. prob (chosen is a consonant) $=\frac{21}{26}$.

Q2. In Fig. 1, PA and PB are tangents to the circle with centre O such that ∠ APB = 50°. Write the measure of ∠OAB.

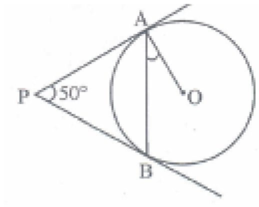
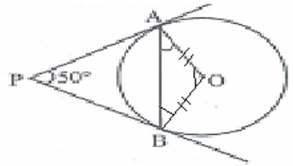


Figure 1

Sol.2 In Δ AOB, OA = OB (radius)



∴ ∠ OAB = ∠ OBA

= x (let)

In fig. , OAPB is quadrilateral, ∠ OAP and ∠ OBP is right angle (radius is r to the tangent at point of contact)

Using angle sum properly of quadrilateral,

$\therefore \angle AOB + \angle OBP + \angle BPA + \angle OAP = 360°$

$\angle AOB + 90° + 50° + 90° = 360°$

$\Rightarrow \angle AOB + 230° = 360°$

$\Rightarrow \angle AOB = 360° - 230°$

$= 130°$

In Δ OAB,

$\angle OAB + \angle AOB + \angle OBA = 180°$

$\Rightarrow x + 130° + x = 180°$

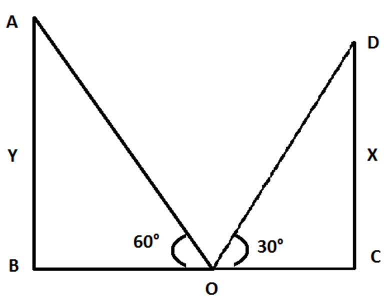
$\Rightarrow 2x + 130° = 180°$

$\Rightarrow 2x = 50°$

$\Rightarrow x = 25°$

Hence $\angle OAB = 25°$

Q3. The top of the two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.



Sol. 3 Let AB and CD be the two towers of height y and x respectively subtending angles of 60° & 30°.

We are required to find the ratio x:y.

Let O be the mid-point of the line joining their feet.

$\Longrightarrow OB = OC = \frac{1}{2}BC$ ... (i)

Now, in Δ AOB

$\tan 60° = \frac{P}{B} = \frac{AB}{OB}$

$\Longrightarrow \tan 60° = \frac{y}{(\frac{1}{2})BC}$ {from (i)}

$\Longrightarrow (\sqrt{3}/2)\, BC = y$ ... (ii)

And in Δ DOC

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$\tan 30° = \frac{P}{B} = \frac{DC}{OC}$

$\Rightarrow \tan 30° = \frac{x}{(1/2)BC}$ {from (i)}

$\Rightarrow \frac{BC}{2\sqrt{3}} = x$ ... (iii)

$x : y = \frac{BC}{2\sqrt{3}} : (\frac{\sqrt{3}}{2})BC$

$\Rightarrow x : y = \frac{2BC\sqrt{3}}{2\sqrt{3}BC} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$

$\Rightarrow x : y = 1 : 3$

Q4. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

Sol. 4 Since it is given that $x = -\frac{1}{2}$ is a solution of the given quadratic equation so it must satisfy the given equation.

So putting $x = -\frac{1}{2}$ is equation.

$3\left(-\frac{1}{2}\right)^2 + 2\,k\left(-\frac{1}{2}\right) - 3 = 0.$

$3 \times \frac{1}{4} + 2\,k\left(-\frac{1}{2}\right) - 3 = 0.$

$\frac{3}{4} - k - 3 = 0.$

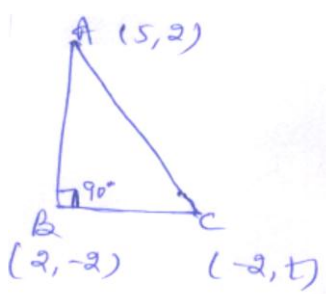
$\Rightarrow k = \frac{3}{4} - 3 = \frac{3-12}{4}$

$= -\frac{9}{4}$

Hence the value of k is $-\frac{9}{4}$.

Section B

Q5. If A (5, 2), B (2,-2) and C (-2, t) are the vertices of a right angled triangle with ∠B = 90°. Then find the value of t.



Sol.5 A (5,2), B (2,-2), C(-2, t) are verticals of right angles triangle

∠B = 90°

Using distance formula

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$AB = \sqrt{(5-2)^2 + (2+2)^2}$

$BC = \sqrt{(2+2)^2 + (-2-t)^2}$

$AC = \sqrt{(5+2)^2 + (2+t)^2}$

Now, using Pythagoras theorem in Δ ABC, we have

$AB^2 + BC^2 = AC^2$.

$(3^2 + 4^2) + (4^2 + (2 + t)^2) = T^2 + (2 - t)^2$

$\Rightarrow 9 + 16 + 16 + 4 + t^2 + 4t = 49 + 4 + t^2 - 4t$

$\Rightarrow 41 + 4 + 4t = 53 - 4t$

$\Rightarrow 8t = 53 - 4t$

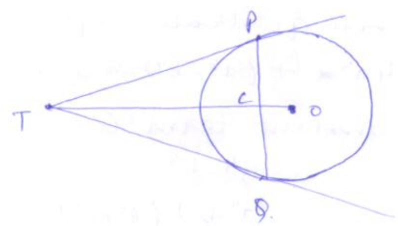
$\Rightarrow 8t = 8$

$\Rightarrow t = 1$

Hence the value & t is 1.

Q6. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Sol.6 Suppose OT intersects PQ at C.



Then, in Δ QTC, Δ PTC

TP = TQ (Tangents from an external point are equal)

∠PTC = ∠ QTC (TP & TQ are equally inclined to OT)

& TC = TC (common)

By SAS criterion of similarity, are have

Δ PTC ≅ ΔQTC.

⇒ PC = CQ and ∠ PCT = ∠ QCT

But ∠ PCT + ∠ QCT = 180°

⇒ PCT = ∠ QCT = 90°

⇒ OT ⊥ PQ. ⇒ Hence OT is the right bisector of line segment PQ.

Q7. In Fig.2, AB is the diameter of a circle with centre O and at is a tangent. If ∠AOQ = 58°, find ∠ATQ.

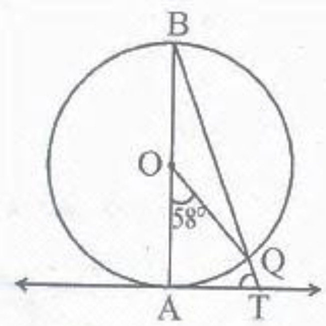
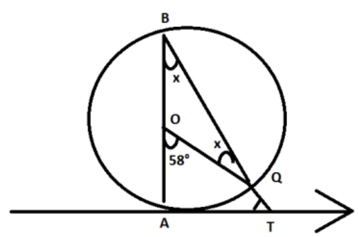


Figure 2

Sol. 7 Since A B is the diameter of the circle



So, ∠AOQ + ∠QOB = 180° (linear pair)

⟹ 58° + ∠QOB = 180°

⟹ ∠QOB = 180° – 58°

= 122°

In Δ OBQ,

OB = OQ (radius)

∠OBQ = ∠OQB = x

∠BOQ + ∠OBQ + ∠OQB = 180°

122° + x + x = 180°

⟹ 2x = 58°

⟹ x = 29°

∠OQT = 180° – ∠OQB

= 180° – 29°

∠OQT = 151°

AOQT is quadrilateral

∠D = 58°, ∠BAT = 90° ∠OQT = 151°

So, $\angle ATQ = 260° - (58° + 90° + 151°)$

$= 360° - 299°$

$\angle ATQ = 61°$

$\Rightarrow \angle ATQ = 61°$

Q8. Solve the following quadratic equation for x:

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$

Sol.8 The given quadratic equation is $4x^2 - 4a^2x + (a^4 - b^4)$

Hence, the constant term is $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

Coefficient of middle term is $= -4a^2$

$= - \{2(a^2 + b^2) + 2(a^2 - b^2)\}$

Now equation is

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$

$4x^2 - \{2\ (a^2+b^2) + 2\ (a^2 - b^2)\ x + (a^2 - b^2)\ (a^2 + b^2) = 0$

$4x^2 - 2\ (a^2 + b^2)\ x - 2\ (a^2 - b^2)\ x - (a^2 - b^2)\ (a^2 + b^2) = 0$

$\Rightarrow \{4x^2 - 2\ (a^2 + b^2)\ x\} - \{2(a^2 - b^2)\ x - (a^2 - b^2)\ (a^2 + b^2)\} = 0$

$\Rightarrow 2x\ \{2x - (a^2 + b^2)\} - (a^2 - b^2)\ \{2x - (a^2 + b^2)\} = 0$

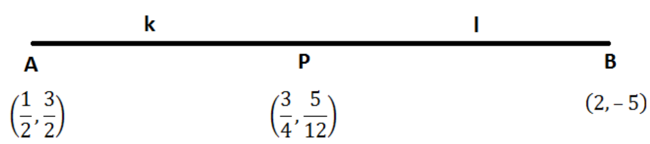
$\Rightarrow \{2x - (a^2 + b^2)\}\ \{\ 2x - (a^2 - b^2)\} = 0$

$\Rightarrow 2x - (a^2 + b^2) = 0$ or $2x - (a^2 - b^2) = 0.$

$\Rightarrow x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$

Q9. Find the ratio in which the point $P\left(\frac{1}{2}, \frac{3}{2}\right)$ and B (2,-5).

Sol. 9



Let us assume that the point P divides AB in the ratio k : 1.

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Then using section formula, we have

$P = \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$\Rightarrow \left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$\Rightarrow \frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$

and $\frac{5}{12} = \frac{-5\,k+\frac{3}{2}}{k+1}$

$\Rightarrow \frac{3}{4} = \frac{2k+\frac{1}{2}}{k+1}$

$\Rightarrow 3\ (k + 1) = \left(2k + \frac{1}{2}\right) 4.$

$\Rightarrow 3\ k + 3 = 8\ k + 2.$

$\Rightarrow 5\ k = 1. \Rightarrow k = \frac{1}{5}$

Hence the required ratio is k = 1 i.e. $\frac{1}{5} = 1$ or 1: 5.

Q10. Find the middle term of the A.P. 213, 205, 197, ---, 37.

Sol. 10 a = 213

l = 37

d = – 8

l = a + (n – 1) d

37 = 213 + (n – 1) (– 8)

$-\frac{176}{-8} = n - 1$

n = 23

Middle term a_{12} = 213 – 11 × 8

= 125

Section C

Q11. In Fig. 3, APB and AQO are semicircles, and AO = OB. If the perimeter of the figure is 40 cm, find the area of the shaded region. $\left[\text{User } \pi = \frac{22}{7}\right]$.

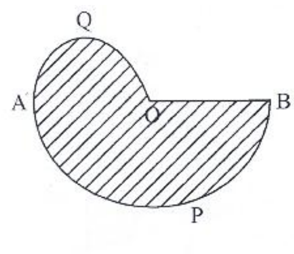
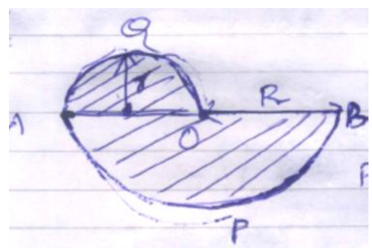


Figure 3

Sol. 11 AO = OB



Perimeter of figure

$= \pi r + \pi R + R$

$r = \frac{R}{2}$

$\text{Perimeter} = \frac{\pi R}{2} + \pi R + R = R\left(\frac{3\pi}{2} + 1\right)$

$40 = R\left(\frac{3\pi}{2} + 1\right)$

$R = \frac{40}{\frac{3\pi}{2}+1} = \frac{80}{3\pi+2} = \frac{80}{\frac{66+14}{7}} = 7$

$R = 7 \text{ cm}. \; r = \frac{R}{2} = 3.5 \text{ cm}$

$\text{Area of shaded region} = \frac{\pi R^2}{2} + \frac{\pi r^2}{2}$

$= \frac{\pi R^2}{2} + \frac{\pi r^2}{2}$

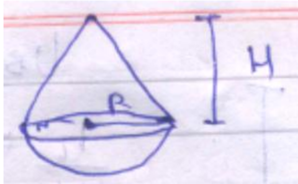
$= \frac{\pi R^2}{2} + \frac{\pi R^2}{8}$

$= \frac{22\times7\times7}{7\times2} + \frac{22\times7\times7}{7\times 8}$

$= \frac{5}{4} \times 77 = 1.25 \times 77 = 96.25 \text{ cm}^2$

Q12. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs. 10 per cm^2. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol. 12 R = 3.5 cm



$V = \frac{1001}{6}\text{cm}^3$

Height of cone = H

Volume $= \frac{2}{3}\pi R^3 + \frac{1}{3}\pi R^2 H$

$\frac{1001}{6} = \frac{\pi R^2}{3}\ [2R + H]$

$\frac{1001\times 7}{2\times 22\times 3.5\times 3.5} = 2 \times 3.5 + H$

6 = H

H = 6 cm

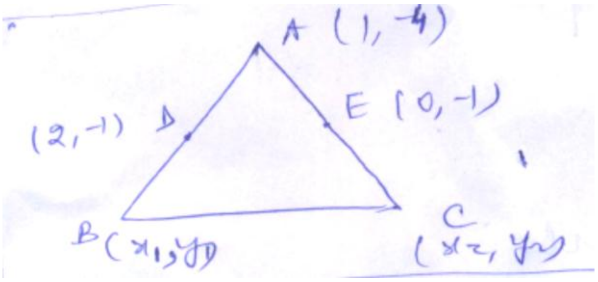
Surface area of hemispherical part to painted $= 2\pi R^2 = 2 \times 22/7 \times 3.5 \times 3.5$

$= 22 \times 3.5 = 77\ \text{cm}^2$

Total Cost = 10 × 77 = Rs. 770

Q13. Find the area of the triangle ABC with A (1, –4) and mid-points of sides through A being (2, – 1) and (0, –1).

Sol.13 Given:



ΔABC in which

A (1, -4)& mid – Pts of sides through

A being (2, -1) & (0, -1).

Let the co-ordinates of B (x_1, y_1) & C (x_2, y_2).

Co-ordinates of points B $= \left(\frac{1+x_1}{2}, \frac{-4+y_1}{2}\right)$

$(2, -1) = \left(\frac{x_1+1}{2}, \frac{y_1+4}{2}\right)$

$\Rightarrow \frac{x_1+1}{2} = 2\ \&\ \frac{y_1-4}{2} = -1$

$\Rightarrow x_1 = 3$ & $y_1 = 2$

So, Point B = (3,2).

Similarly co-ordinates of point C = $\left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$

(0, -1) = $\left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right)$

$\Rightarrow \frac{1+x_2}{2} = 0$ & $\frac{y_2-4}{2} = -1$.

$\Rightarrow x_2 = -1$ & $y_2 = 2$

Point C = (-1, 2)

Area of Δ ABC = where A (x_1, y_1), B(x_2,y_2) & C (x_3, y_3) is

$\frac{1}{2} \mid x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \mid$

$= \frac{1}{2} \mid 1 (2 - 2) + 3 (2 - (-4)) + (-1)(-4 - 2) \mid$

$= \frac{1}{2} \mid 0 + 3.6 + (-1)(-6) \mid$

$= \frac{1}{2} \mid 18 + 6 \mid$

$= \frac{1}{2} \times 24$

= 12 sq units.

Q14. In Fig. 4, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. $\left(\text{User } \pi = \frac{22}{7} \text{ and } \sqrt{5} = 2.236\right)$

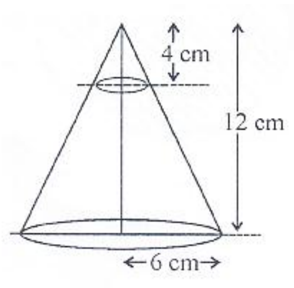


Figure 4

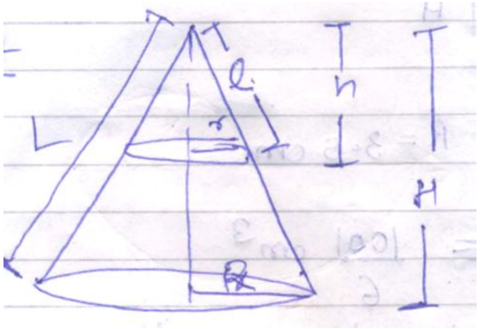
Sol. 14 Using similarity

$\frac{h}{H} = \frac{r}{R}$

$\frac{4}{12} = \frac{r}{6}$

r = 2 cm

r = 2 cm   (H – h) = 8 cm



R = 6 cm

$l = \sqrt{r^2 + h^2} = \sqrt{4 + 16} = 2\sqrt{5}$

$\frac{r}{R} = \frac{l}{L}$

$L = \frac{lR}{r} = \frac{2\sqrt{5}\times 6}{2} = 6\sqrt{5}$

TSA = $\pi r^2 + \pi R^2 + \pi(L - l)(r + R)$

$= \pi\,[4 + 36 + 4\sqrt{5} \times 8]$

$= 8\pi\,[5 + 4\sqrt{5}] = 350.592\ cm^2$

Q15. In Fig. 5, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\text{Use } \pi = \frac{22}{7}\right]$

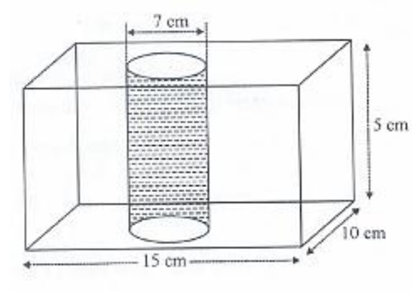


Figure 5

Sol. 15 Surface area of remaining block = Surface area of block - 2 area of circle + C·SA· of cylinder formed

$= 2\,(15 \times 10 + 10 \times 5 + 5 \times 15) - 2 \times \pi \times \frac{7}{2} \times \frac{7}{2} + 2 \times \pi \times \frac{7}{2} \times 5$

$= 2\,[275] - 77 + 110$

$= 583\ cm^2$

Q16. In Fig. 6, find the area of the shaded region [Use π = 3.14]

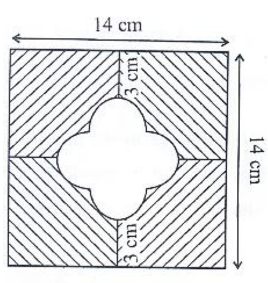
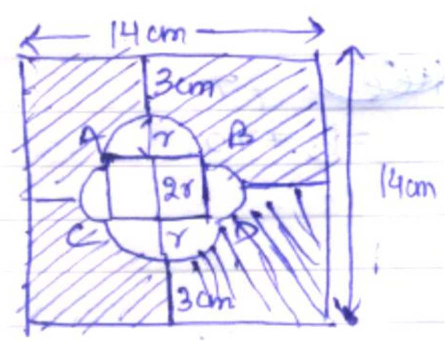


Figure 6



Sol. 16 ABCD will be square with side = diameter of semi-circle

Radius + side + radius = 14 – 3 – 3

r + 2r + r = 8

r = 2 cm

Area of unshaded figure = $4^2 + 4\,\frac{\pi r^2}{2}$

= 16 + 25·12

= 41·12 cm^2

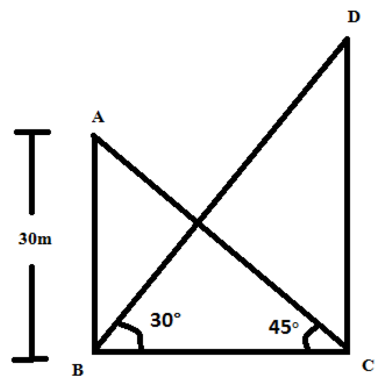
Area of shaded figure = 14^2 – 41·12

= 154·88 cm^2

Q17. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Sol.17 AB → Tower

CD → Building



In Δ ABC

$\tan 45 = \frac{AB}{BC}$

BC = AB

BC = 30

In Δ BDC

$\tan 30 = \frac{CD}{BC}$

CD $= \frac{30}{\sqrt{3}} = 10\sqrt{3}$m.

Q18. If the sum of the first n terms of an A.P. is $\frac{1}{2}(3n^2 + 7n)$, then find its n^{th} term.

Hence write its 20^{th} term.

Sol. 18 Sn $= \frac{1}{2}(3n^2 + 7n)$

n^{th} term = Sn = Sn – 1

$= \frac{1}{2}(3n^2 + 7n) - \frac{1}{2}\left(3\,(n-1)^2 + 7(n-1)\right)$

$= \frac{1}{2}(3n^2 + 7n) - \frac{1}{2}\,(3n^2 + n - 4)$

$= \frac{1}{2}\,[3n^2 + 7n - 3n^2 - n + 4]$

$= \frac{1}{2}\,[6n + 4]$

$a_n = 3n + 2$

$a_{20} = 3 \times 20 + 2 = 62$

Q19. Three distinct coins are tossed together. Find the probability of getting

(i) at least 2 heads

(ii) at most 2 heads

Sol. 19 Sample space = (HHH, HHT, HTH, THH, TTH, THT, HTT, TTT)

at least 2 Heads = (HHT, HTH, HH, HHH)

probability $= \frac{4}{8} = \frac{1}{2}$

at most 2 Heads = (TTT, THT, TTH, TTT, HHT, THH, HTH)

Probability $= \frac{7}{8}$

Q20. Find that value of p for which the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

Sol. 20 $(p + 1)\,x^2 - (p + 1)\,x + 3\,(p + 9)$

For equal roots

$D = 0$

$6^2(p + 1)^2 - 4 \times 3\,(p + q)\,(p + 1) = 0$

$3\,(p^2 + 1 + 2p) - (p^2 + 10p + q) = 0$

$2p^2 - 6 - 4p = 0$

$p^2 - 2p - 3 = 0$

$p^2 - 3p + p - 3 = 0$

$p(p - 3) + (-3) = 0$

$p = -1$(rejected) $p = 3$

Section D

Q21. In Fig.7, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

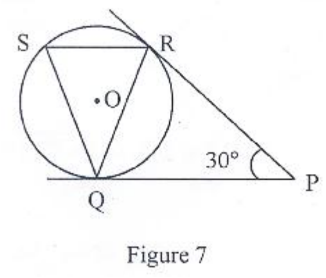
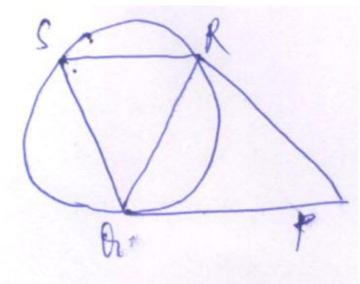


Figure 7

Sol. 21 Since, PQ = PR

$\angle PQR = \angle QPR = \frac{180-30}{2} = 75°$

Now, ∠RSQ = ∠RQP = 75° (because angle in equal segment)



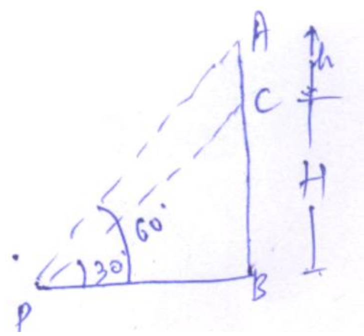
Also, ∠SQP = 180 – ∠RSQ (internal angles)

m ∠SQP = 105

∴ ∠RQS = 30°

Q22. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower.

Sol.22 In Δ CBP BC is tower → H



$\tan 30 = \frac{H}{BP}$ AC is flag → h

$\frac{1}{\sqrt{3}} = \frac{H}{BP}$

$BP = H\sqrt{3}$ .....(1)

$\tan 60 = \frac{AB}{BP} = \frac{H+h}{BP}$

$BP = \frac{H+h}{\sqrt{3}}$ .....(2)

From (1) and (2)

$\frac{H+h}{\sqrt{3}} = \sqrt{3}H \quad \Rightarrow H + h = 3H$

h = 2H

Length of flag staff h = 5m = 2H

Length of tower = 2.5m

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Q23. Ramkali required Rs. 2500 after 12 weeks to send her daughter to school. She saved Rs. 100 in the first week and increased her weekly saving by Rs. 20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

What value is generated in the above situation?

Sol.23 First weak = 100

Second weak = 120

Third weak = 140

So it will form A.P. with 12 terms

a = 100, d = 20

$S_{12} = \frac{12}{2}[2 \times 100 + (12 - 1)20]$

$= 6[200 + 220]$

$= 6 \times 440$

$=$ Rs. 2640 $>$ Rs. 2500

Yes she will be able to send money.

Q24. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Sol.24 (i) Divisible by 2 will be 10 numbers

Divisible by 3 will be 6 numbers

Divisible by both i.e. divisible by 6 will be counted twice which will be 6, 12, 18

So, numbers divisible by 2 or 3 = 10+6 -3

= 13

Probability $= \frac{13}{20}$

(ii) Prime numbers = 2, 3, 5, 7, 11,17,17,19 $\Longrightarrow$ Probability $= \frac{8}{20} = \frac{2}{5}$

Q25. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Sol. 25 Rate of flowing = 2.52 km/hr

$= 2.52 \times \frac{5}{18}\frac{m}{s}$

$= 0.7\ m/s$

Let r be radius of pipe

Volume flowing through pipe $= \pi r^2\ 0.7\ \frac{m^3}{s}$

$= 0.7 \times \frac{22}{7} r^2$

$= 2.2r^2\ \frac{m^3}{s}$

Time = 30 min = 1800s

Volume flown $= 2.2 \times r^2 \times 1800\ m^3$

In tank

Height = 3.15 m

Radius = 0.4 m

Volume flown from pipe = volume rised in tank

$2.2 \times r^2 \times 1800 = \pi \times 0.4 \times 0.4 \times 3.15$

$r^2 = \frac{22\times0.4\times0.4\times0.45}{7\times2.2\times1800}$

$r^2 = \frac{0.4\times0.4\times0.45}{180\times100}$

$r = \frac{0.4}{2\times10} = \frac{0.2}{10} = 2$ cm.

Diameter = 2r = 4 cm.

Q26. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread event all around the well to form a 40 cm high embankment. Find the width of the embankment.

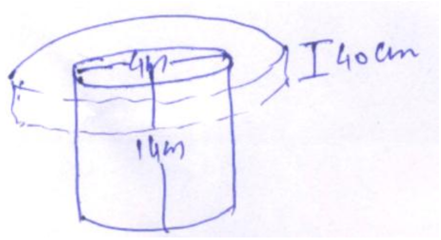
Sol. 26 Diameter = 4

Radius = 2

Height (h) = 14m

Let w be width of embankment

Height (H) = 40m



Volume of earth taken out $= \pi r^2 h$

$= \pi \times 2 \times 2 \times 14$

$= 176 m^2$

Radius of embankment = 2 + w

Volume of earth spread out = Volume of embankment

$176 = \pi (r + w)^2 H - \pi r^2 H$ (No earth being filled here)

$\pi H \left((r + w)^2 - r^2\right) = 176$

$\frac{22}{7} \times 0.4 \, [(r + w + r)(r + w - r)] = 176$

$(2r + w)(w) = \frac{176 \times 7 \times 10}{22 \times 0.4} = 140$

$(4 + w)(w) = 140$

$= (10 \times 14)$

$w = 10$ m

Q27. Solve for x:

$$\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2$$

Sol.27 $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}$

$\Rightarrow \quad \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x}$

$\Rightarrow \quad \frac{7x-5}{2(x+1)(x-2)} = \frac{23}{5x}$

$\Rightarrow \quad 5x(7x - 5) = 46(x + 1)(x - 2)$

$\Rightarrow \quad 35x^2 - 25x = 46x^2 - 46x - 92$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$\Rightarrow \quad 11x^2 - 21x - 92 = 10$

$\Rightarrow \quad 11x^2 - 44x + 23x - 92 = 0$

$\Rightarrow \quad 11x\,(x - 4) + 23\,(x - 4) = 0$

$x = 4 / x = -\frac{23}{11}$

Q28. To fill a swimming pool two pipes are to be used. If the pipe of large diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of large diameter to fill the poo.

Sol. 28 Let V be volume of swimming pool.

t_1 be time taken by smaller pipe to fill

t_2 be time taken by layer pipe

$\frac{V}{t_1} \times 9 + \frac{V}{t_2} \times 4 = \frac{V}{2}$

$\frac{9}{t_1} + \frac{4}{t_2} = \frac{1}{2}$

Given $t_1 = t_2 = 10$

$\frac{9}{t_2+10} + \frac{4}{t_2} = \frac{1}{2}$

$2\,(9t_2 + 4t_2 + 40) = t_2^2 + 10\,t_2$

$26t_2 + 80 = t_2^2 + 10t_2$

$t_2^2 - 16t_2 - 80 = 0$

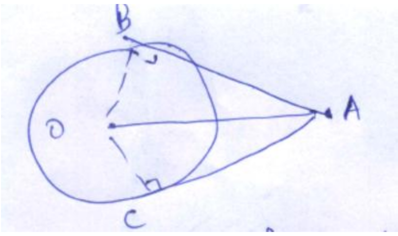
$t_2 = 20$ hrs.

$t_2 = -4 \rightarrow$ rejected

$t_1 = t_2 + 10 = 30$ hrs

Q29. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. 29 Let AB and AC be tangents drawn from external point A



To prove – AB = AC.

Proof:- In Δ AOB and Δ AOC

AO = AO (common)

OB = OB (radi of same circle)

$\angle ABO = \angle ACO$ (90° each tangent ⊥r to radius)

Using RHS ≅

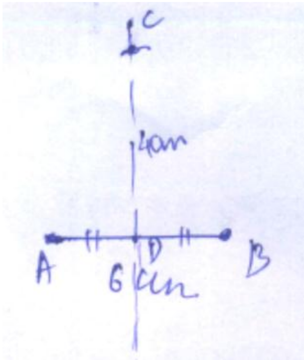
Δ AOB ≅ Δ AOC

AB = AC Hence Proved.

Q30. Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the isosceles triangle.

Sol. 30 Steps

1. Draw AB of 6 cm



2. Draw perpendicular bisector of AB.

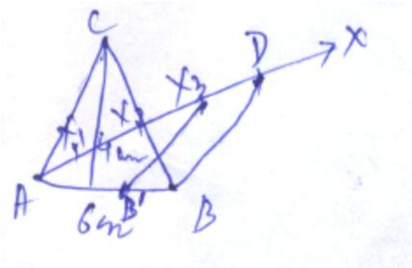
3. from 'D' cut on arc of 4 cm Radius ray this point is C

4. Join $\overline{AC}$ and $\overline{BC}$.

Proof II

S.1) Draw any on ray

$\overrightarrow{AX}$ at any angle



S.2) Cut on arc of same radius at equal intervals on $\overrightarrow{AX}$ starting from A.

S.3) Let the point of interaction of 4th arc the D.

Draw parallel to BD from X_3 and ray point of intersection be B'.

S.4) Now from B' draw a line parallel to $\overline{BC}$ and ray the point where it intersects AC as C'. Now A C' B' is new triangle.

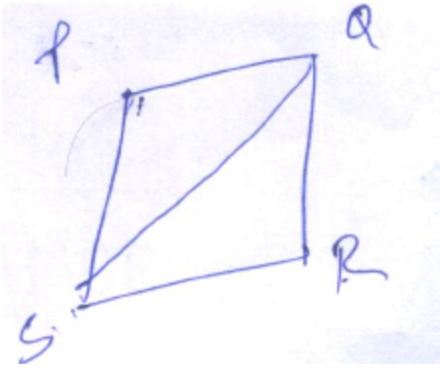
Q31. If P(−5, −3), Q(−4, −6), R(2, −3) and S(1, 2) are the vertices of a quadrilateral PQRS, find it area.

Sol. 31 P (−5, −3)

Q (− 4, − 6)

R (2, – 3)

S (1, 2)



Area of PQRS = Ar PQS + Ar SRQ

Ar PQS $= \frac{1}{2}\left|-5\,(-6-2) + (-4)\,(2+3) + 1\,(-3+6)\right|$

$= \frac{1}{2}\,[40 - 20 + 3]$

$= \frac{23}{2}$ sq units

Ar SRQ $= \frac{1}{2}\,[1\,(-3+6) + 2\,(-6-2) + (-4)(2+3)\,]$

$= \left|\frac{1}{2}[-3-16-20]\right|$

$= \frac{33}{2}$ sq units $\Longrightarrow$ Area of PQRS $= \frac{33}{2} + \frac{23}{2} = \frac{56}{2} = 28$ sq units