CBSE 2011
CCE QUESTION PAPER

FIRST TERM (SA-I)
MATHEMATICS
CODE NO. 1040105-A1

(With Solutions)

CLASS X

Time Allowed : 3 to 3½ Hours] *[Maximum Marks : 80*

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(iii) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted.*

(vi) *An additional 15 minutes time has been allotted to read this question paper only.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. Which of the following numbers has terminating decimal expansion ?

(a) $\frac{37}{45}$ (b) $\frac{21}{2^3 5^6}$

(c) $\frac{17}{49}$ (d) $\frac{89}{2^2 3^2}$

Solution. Choice (b) is correct.

The rational number $\mathbf{\frac{21}{2^3 5^6}}$ has terminating decimal expansion because the prime factorisation of $q = 2^3 \cdot 5^6$ is of the form $2^m \cdot 5^n$, where m and n are non-negative integers.

2. The value of p for which the polynomial $x^3 + 4x^2 - px + 8$ is exactly divisible by $(x - 2)$ is

(a) 0 (b) 3

(c) 5 (d) 16

Solution. Choice (d) is correct.

Since the polynomial $f(x) = x^3 + 4x^2 - px + 8$ is exactly divisible by $(x - 2)$, therefore 2 is a zero of polynomial $f(x)$

$\Rightarrow \qquad f(2) = 0$

$\Rightarrow \quad (2)^3 + 4(2)^2 - p(2) + 8 = 0$

$\Rightarrow \quad 8 + 16 - 2p + 8 = 0$

$\Rightarrow \quad 2p = 32$

$\Rightarrow \quad \mathbf{p = 16.}$

3. ΔABC and ΔPQR are similar triangles such that $\angle A = 32°$ and $\angle R = 65°$, then $\angle B$ is

(*a*) 83° (*b*) 32°

(*c*) 65° (*d*) 97°

Solution. Choice (*a*) is correct.

Since ΔABC and ΔPQR are similar triangles, therefore

$$\angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R$$

But $\angle A = 32°$ and $\angle R = 65°$ (given)

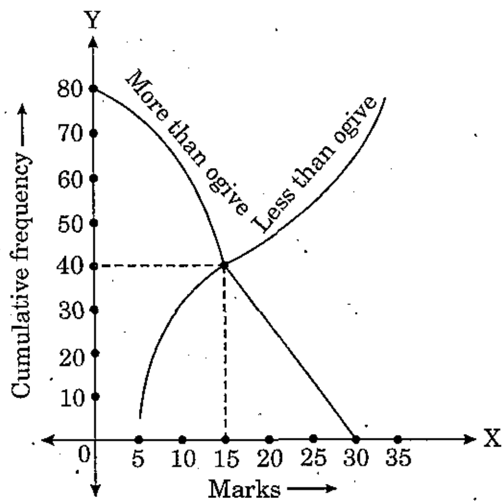
$$\therefore \quad \angle B = 180° - \angle A - \angle C$$

$$= 180° - 32° - 65° \qquad [\because \angle C = \angle R = 65° \text{ (given)}]$$

$$= 180° - 97°$$

$$= \mathbf{83°}.$$

4. In figure, the value of the median of the data using the graph of less than ogive and more than ogive is



(*a*) 5 (*b*) 40

(*c*) 80 (*d*) 15

Solution. Choice (*d*) is correct.

The median of the given data is given by the *x*-coordinate of the point of intersection of 'more than ogive' and 'less than ogive'.

Here, the *x*-coordinate of the point of intersection of the given graph (see figure) of less than and more than ogives is **15**.

5. If $\theta = 45°$, the value of $\text{cosec}^2\, \theta$ is

(*a*) $\dfrac{1}{\sqrt{2}}$ (*b*) 1

(*c*) $\dfrac{1}{2}$ (*d*) 2

Solution. Choice (*d*) is correct.

$\text{cosec}^2 45° = (\text{cosec } 45°)^2 = (\sqrt{2})^2 = 2.$ $\left[\because \text{cosec } 45° = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2}\right]$

6. sin (60° + θ) – cos (30° – θ) is equal to

(*a*) 2 cos θ (*b*) 2 sin θ

(*c*) 0 (*d*) 1

Solution. Choice (*c*) is correct.

$\sin (60° + \theta) - \cos (30° - \theta)$

$= \cos [90° - (60° + \theta)] - \cos (30° - \theta)$ $[\because \cos (90° - A) = \sin A]$

$= \cos (30° - \theta) - \cos (30° - \theta)$

$= \mathbf{0}$

7. The [HCF × LCM] for the numbers 50 and 20 is

(*a*) 10 (*b*) 100

(*c*) 1000 (*d*) 50

Solution. Choice (*c*) is correct.

We know that

HCF × LCM = Product of two positive numbers.

∴ HCF × LCM = 50 × 20

= **1000.**

8. The value of *k* for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is

(*a*) $k = 3$ (*b*) $k = 2$

(*c*) $k = 4$ (*d*) $k = -2$

Solution. Choice (*a*) is correct.

Since the lines represented by the given pair of linear equations are parallel, therefore

$$\frac{4}{2} = \frac{6}{k} \neq \frac{-1}{-7}$$

$\Rightarrow$ $2 = \frac{6}{k}$

$\Rightarrow$ $k = 6 \div 2$

$\Rightarrow$ $\mathbf{k = 3.}$

9. If $\sin A + \sin^2 A = 1$, then the value of $\cos^2 A + \cos^4 A$ is

(*a*) 2 (*b*) 1

(*c*) –2 (*d*) 0

Solution. Choice (*b*) is correct.

Given, $\sin A + \sin^2 A = 1$

$\Rightarrow$ $\sin A = 1 - \sin^2 A$

$\Rightarrow$ $\sin A = \cos^2 A$ $[\because 1 - \sin^2 \theta = \cos^2 \theta]$

$\Rightarrow$ $\sin^2 A = \cos^4 A$ [Squaring both sides]

$\Rightarrow$ $1 - \cos^2 A = \cos^4 A$

$\Rightarrow$ $\mathbf{\cos^4 A + \cos^2 A = 1}$

10. The value of [(sec *A* + tan *A*)(1 – sin *A*)] is equal to

(*a*) $\tan^2 A$ (*b*) $\sin^2 A$

(*c*) $\cos A$ (*d*) $\sin A$

Solution. Choice (*c*) is correct.

$(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \mathbf{\cos A}.$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find a quadratic polynomial with zeroes $3 + \sqrt{2}$ and $3 - \sqrt{2}$.

Solution. Let S and P denote the sum and product of a required quadratic polynomial $p(x)$, then

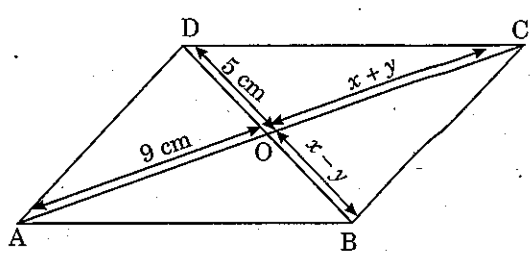
$$S = (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$$

and $$P = (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$$

$\therefore$ $p(x) = k[x^2 - Sx + P]$, where k is non-zero real number

or $p(x) = \mathbf{k[x^2 - 6x + 7]}$, where k is non-zero real number.

12. In figure, *ABCD* is a parallelogram. Find the values of *x* and *y*.



Solution. Since *ABCD* is a parallelogram, therefore

$$x + y = 9 \quad \ldots(1)$$

and $$x - y = 5 \quad \ldots(2)$$

[Diagonals of a parallelogram bisect each other.
$\Rightarrow$ $OC = AO$ and $OB = DO$
where O is the point of intersection of diagonals AC and BD]

Adding (1) and (2), we get

$$(x + y) + (x - y) = 9 + 5$$

$\Rightarrow$ $$2x = 14$$

$\Rightarrow$ $$x = \mathbf{7}$$

Subtracting (2) from (1), we get

$$(x+y)-(x-y)=9-5$$

$$\Rightarrow \quad 2y=4$$

$$\Rightarrow \quad y=2$$

13. If sec 4A = cosec (A – 20°) where 4A is an acute angle, find the value of A.

Solution. We have

$$\sec 4A = \operatorname{cosec}(A-20^\circ)$$

$$\Rightarrow \quad \operatorname{cosec}(90^\circ-4A)=\operatorname{cosec}(A-20^\circ) \qquad [\because \operatorname{cosec}(90^\circ-\theta)=\sec\theta]$$

$$\Rightarrow \quad 90^\circ-4A=A-20^\circ$$

$$\Rightarrow \quad 4A+A=90^\circ+20^\circ$$

$$\Rightarrow \quad 5A=110^\circ$$

$$\Rightarrow \quad \mathbf{A=22^\circ}$$

Or

If 5 tan θ = 4, find the value of $\dfrac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$.

Solution. We have

$$5\tan\theta=4 \quad\Rightarrow\quad \tan\theta=\frac{4}{5} \qquad \ldots(1)$$

$$\frac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}=\frac{(5\sin\theta-3\cos\theta)/\cos\theta}{(5\sin\theta+2\cos\theta)/\cos\theta}$$

[Dividing numerator and denominator by cos θ]

$$=\frac{\dfrac{5\sin\theta}{\cos\theta}-\dfrac{3\cos\theta}{\cos\theta}}{\dfrac{5\sin\theta}{\cos\theta}+\dfrac{2\cos\theta}{\cos\theta}}$$

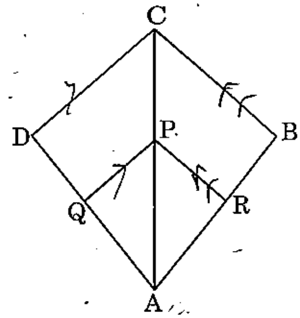
$$=\frac{5\tan\theta-3}{5\tan\theta+2}$$

$$=\frac{5\left(\dfrac{4}{5}\right)-3}{5\left(\dfrac{4}{5}\right)+2} \qquad \text{[using (1)]}$$

$$=\frac{4-3}{4+2}$$

$$=\mathbf{\frac{1}{6}}$$

14. In figure, $PQ \parallel CD$ and $PR \parallel CB$. Prove that $\frac{AQ}{QD} = \frac{AR}{RB}$.



Solution. We have

In ΔACD, since $PQ \parallel CD$, then by BPT,

$$\frac{AQ}{QD} = \frac{AP}{PC} \quad \text{...(1)}$$

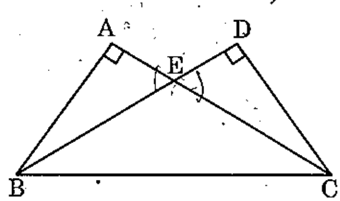
Again, in ΔABC, since $PR \parallel CB$, then by BPT,

$$\frac{AP}{PC} = \frac{AR}{RB} \quad \text{...(2)}$$

From (1) and (2), we have

$$\boldsymbol{\frac{AQ}{QD} = \frac{AR}{RB}}$$

15. In figure, two triangles ABC and DBC are on the same base BC in which $\angle A = \angle D = 90°$. If CA and BD meet each other at E, show that $AE \times CE = BE \times ED$.



Solution. In ΔAEB and ΔDEC

$\angle A = \angle D = 90°$ [given]

and $\angle AEB = \angle DEC$ [Vertically opposite $\angle s$]

Therefore, by AA-criterion of similarity, we have

$\Delta AEB \sim \Delta DEC$

$\Rightarrow \quad \frac{AE}{DE} = \frac{BE}{CE}$

$\Rightarrow \quad \boldsymbol{AE \times CE = BE \times ED}$ [$\because DE = ED$]

16. Check whether 6^n can end with the digit 0 for any natural number n.

Solution. We know that any positive integer ending with the digit 0 is divisible by 5 and so its prime factorisation must contain the prime 5.

We have

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

⇒ There are two prime in the factorisation of $6^n = 2^n \times 3^n$

⇒ *5 does not occur in the prime factorisation of 6^n for any n.*

[By uniqueness of the Fundamental Theorem of Arithmetic]

Hence, 6^n can never end with the digit **0** for any natural number.

17. Find the mean of the following frequency distribution :

Class	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**8**	**12**	**10**	**11**	**9**

Solution. Let the assumed mean be $a = 25$ and $h = 10$

Class	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 – 10	8	5	– 2	– 16
10 – 20	12	15	– 1	– 12
20 – 30	10	25	0	0
30 – 40	11	35	1	11
40 – 50	9	45	2	18
Total	$n = \Sigma f_i = 50$			$\Sigma f_i u_i = 1$

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 25 + \frac{1}{50} \times 10$$

$$= 25 + \frac{1}{5}$$

$$= 25 + 0.2$$

$$= 25.2$$

Hence the mean is **25.2**.

18. Find the mode of the following data :

Class	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**
Frequency	**15**	**6**	**18**	**10**

Solution. Since the class 40 – 60 has the maximum frequency 18, therefore 40 – 60 is the modal class.

$\therefore l = 40, h = 20, f_1 = 18, f_0 = 6, f_2 = 10$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 40 + \frac{18 - 6}{2 \times 18 - 6 - 10} \times 20$$

$$= 40 + \frac{12}{36 - 16} \times 20$$

$$= 40 + \frac{12}{20} \times 20$$

$$= 40 + 12$$

$$= 52$$

Hence the mode is **52.**

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $\sqrt{7}$ is irrational.

Solution. Let us assume, to the contrary, that $\sqrt{7}$ is rational. Then

$$\sqrt{7} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0.$$

Suppose p and q have a common factor other than 1. Then we can divide by the common factor, we get

$$\sqrt{7} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are co-prime}$$

So, $\sqrt{7}\, b = a$

Squaring both sides and rearranging, we get $7b^2 = a^2$

$\Rightarrow$ a^2 is divisible by 7

$\Rightarrow$ a is also divisible by 7 [If r (prime) divides a^2, then r divides a]

Let $a = 7m$, where m is an integer

Substituting $a = 7m$ in $7b^2 = a^2$, we get

$$7b^2 = 49m^2$$

$$\Rightarrow \quad b^2 = 7m^2$$

This means that b^2 is divisible by 7, and so b is also divisible by 7. Therefore, a and b have at least 7 as a common factor. But this contradicts the fact that a and b are co-prime. This contradiction has arisen because of our incorrect assumption that $\sqrt{7}$ is rational.

So, we conclude that $\sqrt{7}$ is **irrational.**

Or

Prove that $3 + \sqrt{5}$ is an irrational number.

Solution. Let us assume, to the contrary, that $3 + \sqrt{5}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that

$$3 + \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{a}{b} - 3 = \sqrt{5}$$

Rearranging the equation, we have

$$\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

Since a and b are integers, we get $\frac{a-3b}{b}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3+\sqrt{5}$ is rational.

So, we conclude that $3+\sqrt{5}$ is **irrational.**

20. Use Euclid's division algorithm to find the HCF of 10224 and 9648.

Solution. Given integers are 10224 and 9648.

Applying Euclid division algorithm to 9648 and 10224, we get

$$10224 = 9648 \times 1 + 576 \quad \text{...(1)}$$
$$9648 = 576 \times 16 + 432 \quad \text{...(2)}$$
$$576 = 432 \times 1 + 144 \quad \text{...(3)}$$
$$432 = 144 \times 3 + 0 \quad \text{...(4)}$$

In equation (4), the remainder is zero. So, the last divisor or the non-zero remainder at the earliest stage, *i.e.*, in equation (3) is 144.

Therefore, HCF of 10224 and 9648 is **144.**

21. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$; find the value of 'a' if $3\alpha + 2\beta = 20$.

Solution. Since α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 6x + a$

$$\therefore \quad \alpha + \beta = \frac{-(-6)}{1} = 6 \quad \text{...(1)}$$

$$\text{and} \quad \alpha\beta = \frac{a}{1} = a \quad \text{...(2)}$$

Given : $3\alpha + 2\beta = 20$

$\Rightarrow \quad \alpha + (2\alpha + 2\beta) = 20$

$\Rightarrow \quad \alpha + 2(\alpha + \beta) = 20$

$\Rightarrow \quad \alpha + 2(6) = 20$ [using (1)]

$\Rightarrow \quad \alpha + 12 = 20$

$\Rightarrow \quad \alpha = 20 - 12$

$\Rightarrow \quad \alpha = 8$

Substituting $\alpha = 8$ in (1), we get

$8 + \beta = 6$

$\Rightarrow \quad \beta = 6 - 8$

$\Rightarrow \quad \beta = -2$

Further, substituting $\alpha = 8$ and $\beta = -2$ in (2), we obtain

$(8)(-2) = a$

$\Rightarrow \quad \boldsymbol{a = -16.}$

22. Solve for x and y.

$$4x + \frac{y}{3} = \frac{8}{3}$$

$$\frac{x}{2} + \frac{3y}{4} = -\frac{5}{2}$$

Solution. We have

$$4x + \frac{y}{3} = \frac{8}{3} \qquad ...(1)$$

and
$$\frac{x}{2} + \frac{3y}{4} = -\frac{5}{2} \qquad ...(2)$$

Multiplying (2) by 8, we get

$$8\left(\frac{x}{2} + \frac{3y}{4}\right) = 8 \times \left(-\frac{5}{2}\right)$$

$$\Rightarrow \quad 4x + 6y = -20 \qquad ...(3)$$

Subtracting (1) from (3), we get

$$(4x + 6y) - \left(4x + \frac{y}{3}\right) = -20 - \frac{8}{3}$$

$$\Rightarrow \quad 6y - \frac{y}{3} = \frac{-60-8}{3}$$

$$\Rightarrow \quad \frac{18y - y}{3} = \frac{-68}{3}$$

$$\Rightarrow \quad 17y = -68$$

$$\Rightarrow \quad y = -4$$

Substituting $y = -4$ in (2), we get

$$\frac{x}{2} + \frac{3}{4}(-4) = -\frac{5}{2}$$

$$\Rightarrow \quad \frac{x}{2} - 3 = -\frac{5}{2}$$

$$\Rightarrow \quad \frac{x}{2} = -\frac{5}{2} + 3$$

$$\Rightarrow \quad \frac{x}{2} = \frac{-5+6}{2}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{1}{2}$$

$$\Rightarrow \quad x = 1$$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = -4}$.

Or

The sum of the numerator and the denominator of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

Solution. Let the fraction be $\frac{x}{y}$.

It is given that : **the sum of the numerator and the denominator of a fraction is 8.**

$$\therefore \quad x + y = 8 \qquad ...(1)$$

Also, it is given that : **if 3 is added to both the numerator and the denominator, the fraction becomes** $\frac{3}{4}$.

$$\therefore \quad \frac{x+3}{y+3} = \frac{3}{4}$$

$$\Rightarrow \quad 4x + 12 = 3y + 9$$

$$\Rightarrow \quad 4x - 3y = -3 \qquad \ldots(2)$$

Multiplying (1) by 3, we get

$$3x + 3y = 24 \qquad \ldots(3)$$

Adding (2) and (3), we get

$$(4x - 3y) + (3x + 3y) = -3 + 24$$

$$\Rightarrow \quad 4x + 3x = 21$$

$$\Rightarrow \quad 7x = 21$$

$$\Rightarrow \quad x = 3$$

Substituting $x = 3$ in (1), we get

$$3 + y = 8$$

$$\Rightarrow \quad y = 8 - 3 = 5$$

Hence, the fraction is $\mathbf{\frac{3}{5}}$.

23. Prove that $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \tan^2\theta - \cot^2\theta$.

Solution. We have

$$\text{L.H.S.} = \frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta}$$

$$= \frac{(\tan\theta - \cot\theta)}{\sin\theta\cos\theta} \times \frac{(\tan\theta + \cot\theta)}{(\tan\theta + \cot\theta)} \quad \text{[Multiplying and dividing by } \tan\theta + \cot\theta\text{]}$$

$$= \frac{\tan^2\theta - \cot^2\theta}{\sin\theta\cos\theta(\tan\theta + \cot\theta)}$$

$$= \frac{\tan^2\theta - \cot^2\theta}{\sin\theta\cos\theta\tan\theta + \sin\theta\cos\theta\cot\theta}$$

$$= \frac{\tan^2\theta - \cot^2\theta}{\sin\theta\cdot\cos\theta\cdot\frac{\sin\theta}{\cos\theta} + \sin\theta\cdot\cos\theta\cdot\frac{\cos\theta}{\sin\theta}}$$

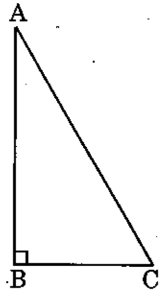
$$= \frac{\tan^2\theta - \cot^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \frac{\tan^2\theta - \cot^2\theta}{1} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \tan^2\theta - \cot^2\theta$$

$$= \text{R.H.S.}$$

24. In figure, ΔABC is right-angled at B, $BC = 7$ cm and $AC - AB = 1$ cm. Find the value of $\cos A - \sin A$.



Solution. Given :

$$BC = 7 \text{ cm} \quad \ldots(1)$$

and $$AC - AB = 1 \text{ cm} \quad \ldots(2)$$

In right-angled ΔABC, we have

$$AC^2 = AB^2 + BC^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow \quad AC^2 - AB^2 = BC^2$$

$$\Rightarrow \quad (AC - AB)(AC + AB) = BC^2 \quad [\because x^2 - y^2 = (x - y)(x + y)]$$

$$\Rightarrow \quad (1)(AC + AB) = (7)^2 \quad \text{[using (1) and (2)]}$$

$$\Rightarrow \quad AC + AB = 49 \quad \ldots(3)$$

Adding (2) and (3), we get

$$(AC - AB) + (AC + AB) = 1 + 49$$

$$\Rightarrow \quad 2AC = 50$$

$$\Rightarrow \quad AC = 25 \text{ cm}$$

Substituting $AC = 25$ cm in (3), we obtain

$$25 + AB = 49$$

$$\Rightarrow \quad AB = 49 - 25$$

$$\Rightarrow \quad AB = 24 \text{ cm}$$

Thus, $AB = 24$ cm, $BC = 7$ cm and $AC = 25$ cm.

In ΔCAB, $$\sin A = \frac{\text{Perpendicular } (BC)}{\text{Hypotenuse } (AC)}$$

$$\Rightarrow \quad \sin A = \frac{7}{25}$$

and $$\cos A = \frac{\text{Base } (AB)}{\text{Hypotenuse } (AC)}$$

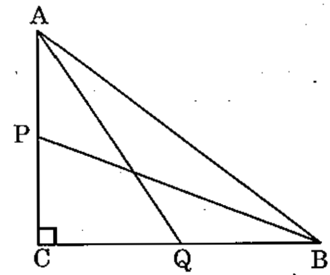
$$\Rightarrow \quad \cos A = \frac{24}{25}$$

Now, $$\cos A - \sin A = \frac{24}{25} - \frac{7}{25}$$

$$= \frac{24 - 7}{25}$$

$$= \mathbf{\frac{17}{25}}.$$

25. In figure, P and Q are the mid-points of the sides CA and CB respectively of ΔABC right-angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$.



Solution. Since ΔACB is a right triangle, right-angled at C, therefore

$$AB^2 = AC^2 + BC^2 \qquad \text{...(1)}$$

Since ΔACQ is a right triangle, right-angled at C, therefore

$$AQ^2 = AC^2 + CQ^2 \qquad \text{...(2)}$$

Again, ΔPCB is a right triangle, right-angled at C, therefore

$$BP^2 = BC^2 + PC^2 \qquad \text{...(3)}$$

Adding (2) and (3), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (BC^2 + PC^2)$$

$$\Rightarrow \quad AQ^2 + BP^2 = (AC^2 + BC^2) + (CQ^2 + PC^2)$$

$$\Rightarrow \quad AQ^2 + BP^2 = AB^2 + (CQ^2 + PC^2) \qquad \text{[using (1)]}$$

$$\Rightarrow \quad AQ^2 + BP^2 = AB^2 + \left[\left(\frac{1}{2}BC\right)^2 + \left(\frac{1}{2}AC\right)^2\right]$$

[$\because$ P and Q are the mid-points of the sides CA and CB.

$\therefore \quad PC = AP = \frac{1}{2}AC$

$CQ = BQ = \frac{1}{2}BC$]

$$\Rightarrow \quad AQ^2 + BP^2 = AB^2 + \left(\frac{1}{4}BC^2 + \frac{1}{4}AC^2\right)$$

$$\Rightarrow \quad AQ^2 + BP^2 = AB^2 + \frac{1}{4}(BC^2 + AC^2)$$

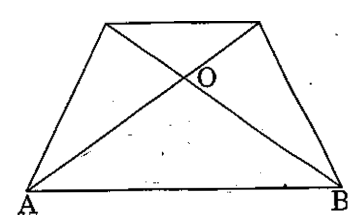
$$AQ^2 + BP^2 = AB^2 + \frac{1}{4}AB^2 \qquad \text{[using (1)]}$$

$$\Rightarrow \quad 4(AQ^2 + BP^2) = 4AB^2 + AB^2$$

$$\Rightarrow \quad \mathbf{4(AQ^2 + BP^2) = 5AB^2}$$

26. The diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at point O. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

Solution. $ABCD$ is a trapezium in which O is the point of intersection of the diagonals AC and BD and $AB \parallel CD$.



In triangles AOB and COD, we have

$\angle AOB = \angle COD$ [Vertically opposite $\angle s$]

and $\angle OAB = \angle OCD$ [Alternate $\angle s$]

So, by AA-criterion of similarity of triangles, we have

$$\Delta AOB \sim \Delta COD$$

$$\Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$$ [∵ The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.]

$$\Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{(2CD)^2}{CD^2}$$ [∵ $AB = 2CD$ (given)]

$$\Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{4CD^2}{CD^2}$$

$$\Rightarrow \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{4}{1}$$

Thus, the ratio of the areas of triangles AOB and COD is **4 : 1.**

27. The mean of the following frequency distribution is 50. Find the value of *p*.

Classes	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**	**80 – 100**
Frequency	**17**	**28**	**32**	***p***	**19**

Solution. **Calculation of Mean**

Classes	*Class-mark* (x_i)	*Frequency* (f_i)	$f_i x_i$
0 – 20	10	17	170
20 – 40	30	28	840
40 – 60	50	32	1600
60 – 80	70	p	$70p$
80 – 100	90	19	1710
Total		$n = \Sigma f_i = 96 + p$	$\Sigma f_i x_i = 4320 + 70p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$(\text{given})\ 50 = \frac{4320 + 70p}{96 + p}$$

$$\Rightarrow \quad 4800 + 50p = 4320 + 70p$$

$$\Rightarrow \quad 4800 - 4320 = 70p - 50p$$

$$\Rightarrow \quad 20p = 480$$

$$\Rightarrow \quad \boldsymbol{p = 24.}$$

28. Compute the median for the following cumulative frequency distribution :

Weight in (kg)	***Less than* 38**	***Less than* 40**	***Less than* 42**	***Less than* 44**	***Less than* 46**	***Less than* 48**	***Less than* 50**	***Less than* 52**
Number of students	**0**	**3**	**5**	**9**	**14**	**28**	**32**	**35**

Solution. **Calculation of Median**

Weight in (kg)	*No. of students (f)*	*Cumulative frequency (cf)*
Less than 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	14
46 – 48	14	28
48 – 50	4	32
50 – 52	3	35

Here, $\frac{n}{2} = \frac{35}{2} = 17.5$. Now, 46 – 48 is the class whose cumulative frequency is 28 is greater than $\frac{n}{2}$, *i.e.*, 17.5.

$\therefore$ 46 – 48 is the median class.

From the table, $f = 14$, $cf = 14$, $h = 2$

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{14} \times 2$$

$$= 46 + \frac{1}{2}$$

$$= 46 + 0.5$$

$$= \mathbf{46.5}$$

Or

Find the missing frequencies in the following frequency distribution table, if $N = 100$ and median is 32.

Marks obtained	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	***Total***
No. of students	**10**	**?**	**25**	**30**	**?**	**10**	**100**

Solution. Let x and y be the missing frequencies of classes 10 – 20 and 40 – 50 respectively.

Calculation of Median

Marks obtained	*No. of students*	*Cumulative Frequency*
0 – 10	10	10
10 – 20	x	$10 + x$
20 – 30	25	$35 + x$
30 – 40	30	$65 + x$
40 – 50	y	$65 + x + y$
50 – 60	10	$75 + x + y$
Total	100	

It is given that, $n = 100$ = Total Frequency

$\therefore \quad 75 + x + y = 100$

$\Rightarrow \quad x + y = 100 - 75$

$\Rightarrow \quad x + y = 25 \qquad \ldots(1)$

The median is 32 (given), which lies in the class 30 – 40

So, l = lower limit of median class = 30

f = frequency of median class = 30

cf = cumulative frequency of class preceding the median class = $35 + x$

h = class size = 10

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 32 = 30 + \left(\frac{50 - (35 + x)}{30}\right) \times 10$$

$$\Rightarrow \quad 32 - 30 = \frac{15 - x}{3}$$

$\Rightarrow \quad 2 \times 3 = 15 - x$

$\Rightarrow \quad 6 = 15 - x$

$\Rightarrow \quad x = 15 - 6$

$\Rightarrow \quad x = 9$

Substituting $x = 9$ in (1), we get

$9 + y = 25$

$\Rightarrow \quad y = 25 - 9$

$\Rightarrow \quad y = 16$

Hence, the missing frequencies of the classes 10 – 20 and 40 – 50 are **9** and **16** respectively.

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Divide $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $(3x^2 + 2x - 4)$ and verify the result by division algorithm.

Solution. We have $p(x) = 30x^4 + 11x^3 - 82x^2 - 12x + 48$ and $g(x) = 3x^2 + 2x - 4$
Now we divide $p(x)$ by $g(x)$ to get $q(x)$ and $r(x)$.

$$\begin{array}{r|l} & 10x^2 - 3x - 12 \\ \hline 3x^2 + 2x - 4 & 30x^4 + 11x^3 - 82x^2 - 12x + 48 \\ & \underset{-}{30x^4} \underset{-}{+ 20x^3} \underset{+}{- 40x^2} \\ \hline & \quad -9x^3 - 42x^2 - 12x + 48 \\ & \quad \underset{+}{-9x^3} \underset{+}{- 6x^2} \underset{-}{+ 12x} \\ \hline & \qquad -36x^2 - 24x + 48 \\ & \qquad \underset{+}{-36x^2} \underset{+}{- 24x} \underset{-}{+ 48} \\ \hline & \qquad\qquad 0 \end{array}$$

$\left[\text{First term of the quotient is } \dfrac{30x^4}{3x^2} = 10x^2\right]$

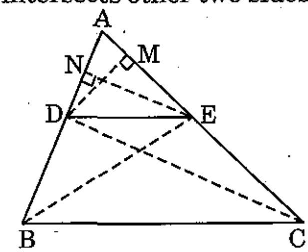
$\left[\text{Second term of the quotient is } \dfrac{-9x^3}{3x^2} = -3x\right]$

$\left[\text{Third term of the quotient is } \dfrac{-36x^2}{3x^2} = -12\right]$

Now, $p(x) = g(x) \cdot q(x) + r(x) = (3x^2 + 2x - 4) \times (10x^2 - 3x - 12) + 0$
$= 30x^4 - 9x^3 - 36x^2 + 20x^3 - 6x^2 - 24x - 40x^2 + 12x + 48$
$= \mathbf{30x^4 + 11x^3 - 82x^2 - 12x + 48}$

30. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Solution. Given : A triangle ABC in which a line parallel to BC intersects other two sides AB and AC at D and E respectively.



To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$.

Construction : Join BE, CD and draw $DM \perp AC$ and $EN \perp AB$.

Proof : Since EN is perpendicular to AB, therefore, EN is the height of triangles ADE and BDE.

$\therefore \quad \text{ar}(\Delta ADE) = \dfrac{1}{2}(\text{base} \times \text{height})$

$= \dfrac{1}{2}(AD \times EN)$ ...(1)

and $\quad \text{ar}(\Delta BDE) = \dfrac{1}{2}(\text{base} \times \text{height})$

$= \dfrac{1}{2}(DB \times EN)$ ...(2)

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \dfrac{\frac{1}{2}(AD \times EN)}{\frac{1}{2}(DB \times EN)}$ [using (1) and (2)]

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \dfrac{AD}{DB}$ ...(3)

Similarly, $\dfrac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta DEC)} = \dfrac{\frac{1}{2}(AE \times DM)}{\frac{1}{2}(EC \times DM)} = \dfrac{AE}{EC}$ ...(4)

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

$\therefore \quad ar(\Delta BDE) = ar(\Delta DEC)$ ...(5)

From (4) and (5), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

Again from (3) and (6), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, $\mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$.

Or

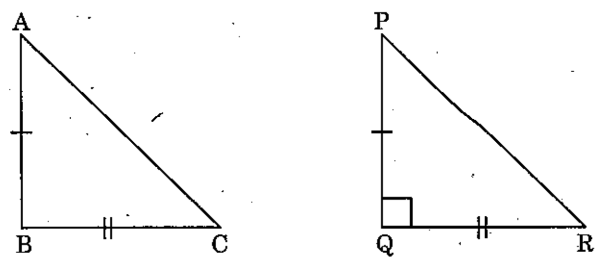
Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Solution. Given : A triangle ABC such that :

$$AC^2 = AB^2 + BC^2$$

To prove : ΔABC is a right-angled at B, *i.e.*, $\angle B = 90°$.

Construction : Construct a ΔPQR such that $\angle Q = 90°$ and $PQ = AB$ and $QR = BC$.



Proof : In ΔPQR, as $\angle Q = 90°$, we have

$PR^2 = PQ^2 + QR^2$ [By Pythagoras Theorem]

$\Rightarrow \quad PR^2 = AB^2 + BC^2$ ...(1) [As $PQ = AB$ and $QR = BC$]

But $\quad AC^2 = AB^2 + BC^2$ ...(2)

From (1) and (2), we have

$PR^2 = AC^2$

$\Rightarrow \quad PR = AC$ ...(3)

Now in ΔABC and ΔPQR, we have

$AB = PQ$

$BC = QR$

and $\quad AC = PR$ [using (3)]

$\therefore \quad \Delta ABC \cong \Delta PQR$ [SSS congruency]

$\Rightarrow \quad \angle B = \angle Q = 90°$ [CPCT]

Hence, $\quad \mathbf{\angle B = 90°}$.

31. Without using trigonometric tables, evaluate the following :

$$\frac{\sec 37^\circ}{\operatorname{cosec} 53^\circ} + 2 \cot 15^\circ \cot 25^\circ \cot 45^\circ \cot 75^\circ \cot 65^\circ - 3(\sin^2 18^\circ + \sin^2 72^\circ)$$

Solution. We have

$$\frac{\sec 37^\circ}{\operatorname{cosec} 53^\circ} + 2 \cot 15^\circ \cot 25^\circ \cot 45^\circ \cot 75^\circ \cot 65^\circ - 3(\sin^2 18^\circ + \sin^2 72^\circ)$$

$$= \frac{\sec 37^\circ}{\operatorname{cosec}(90^\circ - 37^\circ)} + 2 \cot 15^\circ \cot 25^\circ \cot 45^\circ \cot(90^\circ - 15^\circ) \cot(90^\circ - 25^\circ) - 3[\sin^2 18^\circ + \sin^2(90^\circ - 18^\circ)]$$

$$= \frac{\sec 37^\circ}{\sec 37^\circ} + 2 \cot 15^\circ \cot 25^\circ \cot 45^\circ \tan 15^\circ \tan 25^\circ - 3(\sin^2 18^\circ + \cos^2 18^\circ)$$

$[\because \operatorname{cosec}(90^\circ - \theta) = \sec\theta,\ \cot(90^\circ - \theta) = \tan\theta \text{ and } \sin(90^\circ - \theta) = \cos\theta]$

$= 1 + 2(\cot 15^\circ \cdot \tan 15^\circ)(\cot 25^\circ \cdot \tan 25^\circ) \cot 45^\circ - 3(1)$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + 2(1)(1)(1) - 3$ $\quad [\because \cot\theta . \tan\theta = 1 \text{ and } \cot 45^\circ = 1]$

$= 1 + 2 - 3$

$= \mathbf{0}.$

Or

Prove that : $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{ʿ}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

Solution. We have

$$\text{L.H.S.} = \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta}$$

$$= \frac{\dfrac{\sin\theta}{\cos\theta}}{1 - \dfrac{\cos\theta}{\sin\theta}} + \frac{\dfrac{\cos\theta}{\sin\theta}}{1 - \dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{(\sin\theta/\cos\theta)}{(\sin\theta - \cos\theta)/\sin\theta} + \frac{(\cos\theta/\sin\theta)}{(\cos\theta - \sin\theta)/\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{(\sin\theta - \cos\theta)(\sin\theta\cos\theta)} \qquad [\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{(\sin\theta\cos\theta)} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= \sec\theta \operatorname{cosec}\theta + 1$

= R.H.S.

32. If $2\cos\theta - \sin\theta = x$ and $\cos\theta - 3\sin\theta = y$. Prove that $2x^2 + y^2 - 2xy = 5$.

Solution. Given

$2\cos\theta - \sin\theta = x$ ...(1) and $\cos\theta - 3\sin\theta = y$ ...(2)

L.H.S. $= 2x^2 + y^2 - 2xy$

$= x^2 + (x^2 + y^2 - 2xy)$

$= x^2 + (x - y)^2$

$= (2\cos\theta - \sin\theta)^2 + [(2\cos\theta - \sin\theta) - (\cos\theta - 3\sin\theta)]^2$ [using (1) and (2)]

$= (2\cos\theta - \sin\theta)^2 + [(2\cos\theta - \cos\theta) + (-\sin\theta + 3\sin\theta)]^2$

$= (2\cos\theta - \sin\theta)^2 + (\cos\theta + 2\sin\theta)^2$

$= (4\cos^2\theta + \sin^2\theta - 4\cos\theta\sin\theta) + (\cos^2\theta + 4\sin^2\theta + 4\cos\theta\sin\theta)$

$= (4\cos^2\theta + \cos^2\theta) + (\sin^2\theta + 4\sin^2\theta) + (-4\cos\theta\sin\theta + 4\cos\theta\sin\theta)$

$= 5\cos^2\theta + 5\sin^2\theta + 0$

$= 5(\cos^2\theta + \sin^2\theta)$

$= 5(1)$ $[\because \cos^2\theta + \sin^2\theta = 1]$

$= 5$

= R.H.S.

33. Check graphically whether the pair of linear equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$ is consistent. Also, find the vertices of the triangle formed by these lines with the x-axis.

Solution. We have

$4x - y - 8 = 0$

$\Rightarrow y = 4x - 8$

and $2x - 3y + 6 = 0$

$\Rightarrow 3y = 2x + 6$

$\Rightarrow y = \frac{2x+6}{3}$

Table of $y = 4x - 8$

x	0	2	3
y	– 8	0	4
	A	B	C

Table of $y = \frac{2x+6}{3}$

x	0	– 3	3
y	2	0	4
	D	E	C

Take XOX' and YOY' as the axes of co-ordinates. Plotting the points $A(0, -8)$, $B(2, 0)$, $C(3, 4)$ and joining them by a line, we get a line 'l' which is the graph of $4x - y - 8 = 0$.

Further, plotting the point $D(0, 2)$, $E(-3, 0)$, $C(3, 4)$ and joining them by a line, we get a line 'm' which is the graph of $2x - 3y + 6 = 0$.

From the graph of the two equations, we find that the two lines l and m intersect each other at the point $C(3, 4)$.

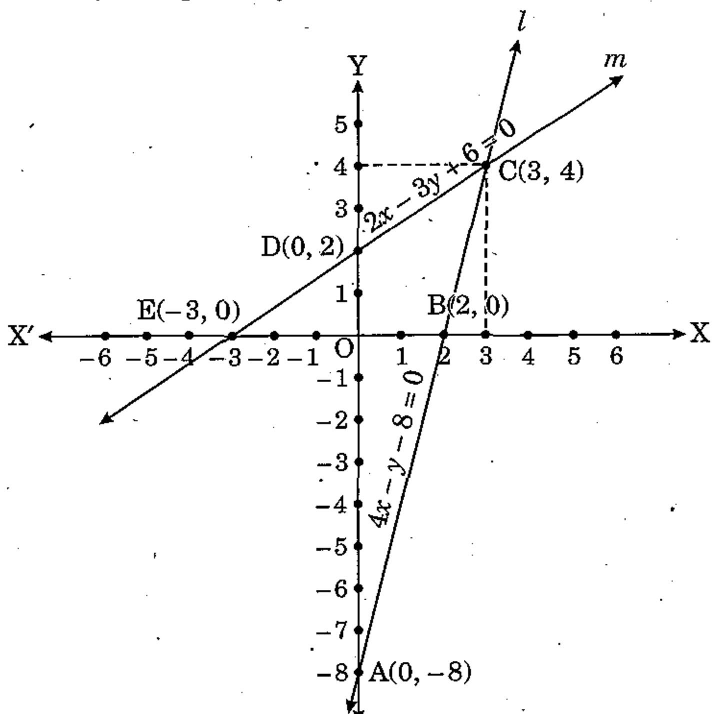
Yes, the pair of linear equations $4x - y - 8 = 0$ and $2x - 3y + 6 = 0$ is **consistent.**

$\therefore$ $\quad$ $\boldsymbol{x = 3, y = 4}$ is the solution.

The first line $4x - y - 8 = 0$ meets the x-axis at the points $B(2, 0)$.

The second line $2x - 3y + 6 = 0$ meets the x-axis at the point $E(-3, 0)$.

Hence, the vertices of the triangle ECB formed by the given lines with the x-axis are $\boldsymbol{E(-3, 0)}$, $\boldsymbol{C(3, 4)}$ and $\boldsymbol{B(2, 0)}$ respectively.



34. The following table shows the ages of 100 persons of a locality.

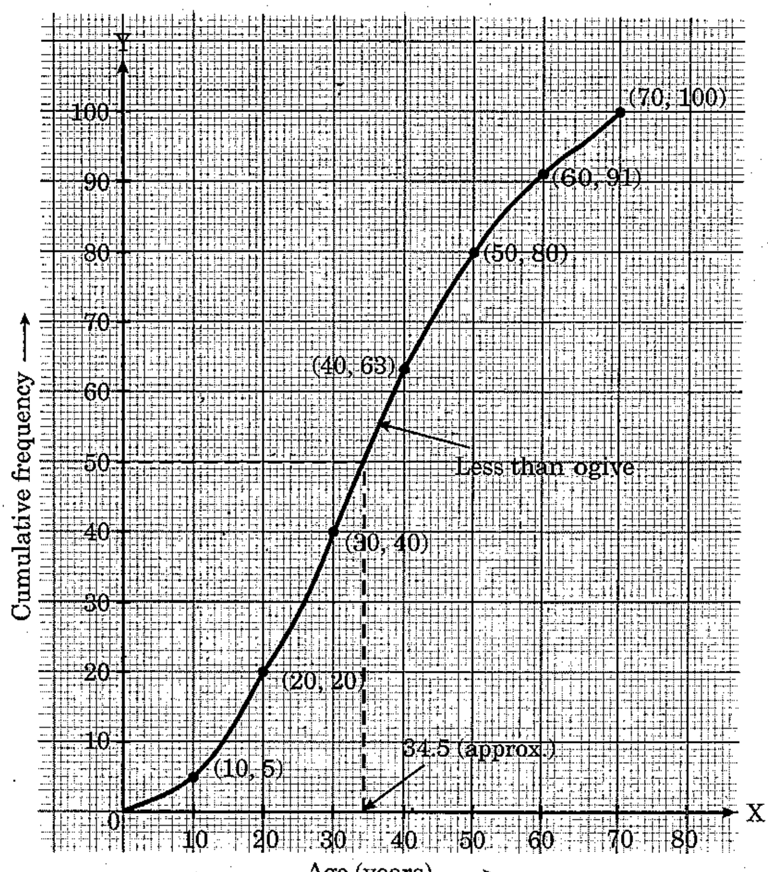
Age (years)	*Number of persons*
0 – 10	**5**
10 – 20	**15**
20 – 30	**20**
30 – 40	**23**
40 – 50	**17**
50 – 60	**11**
60 – 70	**9**

Draw the less than ogive and find the median.

Solution. We prepare the cumulative frequency table by less than type method as given below :

Age (years)	*Number of persons (Frequency)*	*Age (years) less than*	*Cumulative frequency*
0 – 10	5	10	5
10 – 20	15	20	20
20 – 30	20	30	40
30 – 40	23	40	63
40 – 50	17	50	80
50 – 60	11	60	91
60 – 70	9	70	100

Here 10, 20, 30, 40, 50, 60, 70 are the upper limits of the respective class-intervals less than 0 – 10, 10 – 20, 20 – 30, 30 – 40, 40 – 50, 50 – 60, 60 – 70. To represent the data in the table graphically, we mark the upper limits of the class-intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis), choosing a convenient scale other than the class intervals, we assume a class interval – 10 – 0 prior to the first class interval 0 – 10 with zero frequency.



Now, we plot the points (0, 0), (10, 5), (20, 20), (30, 40) (40, 63), (50, 80), (60, 91) and (70, 100) on a graph paper and join them by a free hand smooth curve to get the "less than ogive." (see figure)

Locate $\frac{n}{2} = \frac{100}{2} = 50$ on y-axis.

From this point, draw a line parrallel to x-axis cutting the curve at a point. From this point, draw a perpendicular to x-axis. The point of intersection of this perpendicular with x-axis determine the median age (see figure) *i.e.*, median age is **34.5 years** (approx).