

# MODEL TEST PAPER

FIRST TERM (SA-I)

MATHEMATICS

(With Answers)

CLASS X

Time Allowed : 3 to 3½ Hours

Maximum Marks : 80

## General Instructions :

- (i) All questions are compulsory.
- (ii) The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
- (iii) Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
- (iv) There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

### Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If  $\cot \theta = \frac{5}{2}$ , then the value of  $\frac{(2 + 2 \operatorname{cosec} \theta)(1 - \operatorname{cosec} \theta)}{(2 + 2 \sec \theta)(1 - \sec \theta)}$  is

(a)  $\frac{625}{16}$

(b)  $\frac{615}{16}$

(c)  $\frac{635}{16}$

(d)  $\frac{605}{16}$

2. If the pair of linear equations  $10x + 5y - (k - 5) = 0$  and  $20x + 10y - k = 0$  has infinitely many solutions, then the value of  $k$  is

(a) 5

(b) 10

(c) 2

(d) 8

3. Which of the following numbers has non-terminating repeating decimal expansion ?

(a)  $\frac{17}{320}$

(b)  $\frac{7}{80}$

(c)  $\frac{84}{400}$

(d)  $\frac{93}{420}$

4.  $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$  is equal to

(a) 0

(b) 1

(c) -1

(d) none of these

5. If  $\sec 2\theta = \operatorname{cosec}(\theta - 45^\circ)$  where  $2\theta$  is an acute angle, then  $\theta$  is equal to

(a)  $42^\circ$

(b)  $43^\circ$

(c)  $44^\circ$

(d)  $45^\circ$

6. If  $x \cos(90^\circ - \theta) \tan(90^\circ - \theta) = \sin(90^\circ - \theta)$ , then  $x$  is equal to

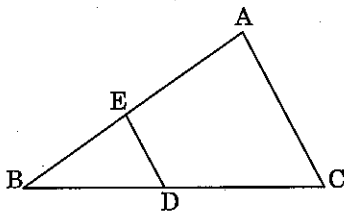
(a)  $\frac{1}{2}$

(b)  $\frac{1}{\sqrt{2}}$

(c) 1

(d)  $\sqrt{3}$

7. In figure,  $DE \parallel CA$  and  $D$  is a point on  $BC$  such that  $BD : DC = 2 : 3$ . The ratio of area of  $\triangle ABC$  to area of  $\triangle BDE$  is



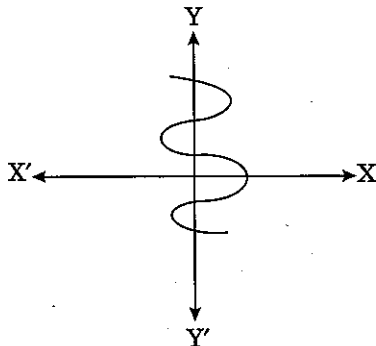
(a) 25 : 4

(b) 16 : 25

(c) 25 : 16

(d) 3 : 2

8. In figure, the graph of a polynomial  $p(x)$  is shown. The number of zeroes of  $p(x)$  is



(a) 0

(b) 1

(c) 2

(d) 3

9. For a given data with 70 observations the 'less than ogive' and the 'more than ogive' intersect at (20.5, 35). The median of the data is

(a) 20

(b) 35

(c) 70

(d) 20.5

10. If  $\text{HCF}(84, 630) = 42$  then  $\text{LCM}(84, 630)$  is

(a) 1240

(b) 1260

(c) 1280

(d) 1200

### Section B

Question numbers 11 to 18 carry 2 marks each.

11. Determine the values of  $a$  and  $b$  for which the following pair of linear equations has infinite many solutions :

$$3x - (a + 1)y = 2b - 1$$

$$5x + (1 - 2a)y = 3b$$

12. Show that every positive even integer is of the form  $2q$  and every odd positive integer is of the form  $2q + 1$  where  $q$  is some whole number.

13. On dividing the polynomial  $3x^3 - 4x^2 - 3x + 25$  by a polynomial  $g(x)$ , the quotient and remainder were  $3x + 5$  and  $5$  respectively, find  $g(x)$ .

14. Prove that :

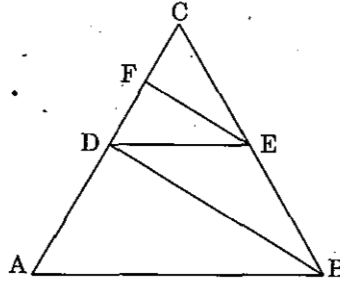
$$\frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot^2 A} = 1$$

Or

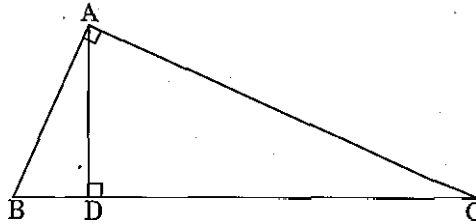
Prove that :

$$\frac{\cot A - 1}{2 - \sec^2 A} = \frac{\cot A}{1 + \tan A}$$

15. In figure,  $AB \parallel DE$  and  $BD \parallel EF$ . Prove that  $CD^2 = AC \times FC$ .



16. In  $\triangle ABC$ ,  $AD \perp BC$  and  $AD^2 = BD \times CD$ . Prove that  $\angle BAC$  is a right angle.



17. A life insurance agent found the following data for distribution of ages of 100 policy holders, when the policies are given only to persons having age 18 years but less than 60 years.

| Age (in years)        | 0 - 20 | 20 - 25 | 25 - 30 | 30 - 35 | 35 - 40 | 40 - 45 | 45 - 50 | 50 - 55 | 55 - 60 |
|-----------------------|--------|---------|---------|---------|---------|---------|---------|---------|---------|
| No. of policy holders | 2      | 4       | 18      | 21      | 33      | 11      | 3       | 6       | 2       |

Write the above distribution as less than type cumulative frequency distribution.

18. Find the mode of the following distribution of house-hold expenditure (in ₹) of manual workers in a city :

| Expenditure (in ₹) | Frequency | Expenditure (in ₹) | Frequency |
|--------------------|-----------|--------------------|-----------|
| 1000 - 2000        | 24        | 5000 - 6000        | 30        |
| 2000 - 3000        | 40        | 6000 - 7000        | 22        |
| 3000 - 4000        | 33        | 7000 - 8000        | 16        |
| 4000 - 5000        | 28        | 8000 - 9000        | 7         |

### Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Show that  $21^n$  cannot end with the digits 0, 2, 4, 6 and 8 for any natural number  $n$ .

20. Prove that  $\frac{13}{7}\sqrt{5}$  is irrational.

Or

Prove that  $\sqrt{3} + \sqrt{5}$  is irrational.

21. Student of a class are made to stand in (complete) rows. If 4 students are extra in a row there would be two rows less. If four students are less in a row there would be four more rows. Find the number of students in the class.

Or

A and B have some money with them. A said to B, 'If you give me ₹ 100, my money will become 75% of the money left with you'. B said to A, 'Instead if you give me ₹ 100, your money will become 40% of my money'. How much money did A and B have originally?

22. If  $\alpha, \beta$  are zeroes of the polynomial  $x^2 - 2x - 15$ , then form a quadratic polynomial whose zeroes are  $(2\alpha)$  and  $(2\beta)$ .

23. Prove that :

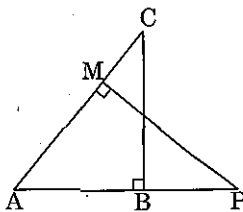
$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$$

24. Prove that :

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A \cos^2 A} - 2$$

25. A ladder reaches a window which is 12 m above the ground on one side of the street. Keeping its foot at the same point is turned to the other side of the street to reach a window 9 m high. Find the width of the street if the length of the ladder is 15 m.

26. In figure,  $\triangle ABC$  and  $\triangle AMP$  are right angled at  $B$  and  $M$  respectively, prove that  $CA \times MP = PA \times BC$



27. Find the median of the following data :

|                |         |         |         |         |         |         |         |         |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|
| Age (in years) | 20 - 25 | 25 - 30 | 30 - 35 | 35 - 40 | 40 - 45 | 45 - 50 | 50 - 55 | 55 - 60 |
| No. of Persons | 50      | 70      | 100     | 180     | 150     | 120     | 70      | 59      |

28. Find the mean of the following distribution using step-deviation method.

|               |        |          |           |           |           |           |
|---------------|--------|----------|-----------|-----------|-----------|-----------|
| Income (in ₹) | 0 - 50 | 50 - 100 | 100 - 150 | 150 - 200 | 200 - 250 | 250 - 300 |
| Frequency     | 90     | 150      | 100       | 80        | 70        | 10        |

Or

The mean of 10 numbers is 12.5. The mean of first six numbers is 15 and the mean of last five numbers is 10. Find the sixth number.

**Section 'D'**

Question numbers 29 to 34 carry 4 marks each.

29. Prove that :

$$\frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$$

Or

Evaluate :

$$\frac{\sin(90^\circ - \theta) \cdot \operatorname{cosec}(90^\circ - \theta) \cot \theta}{\sec(90^\circ - \theta) \cdot \cos(90^\circ - \theta) \tan(90^\circ - \theta)} + \frac{\sin^2 65^\circ + \sin^2 25^\circ}{\tan 10^\circ \tan 35^\circ \tan 55^\circ \tan 80^\circ} + 2 \cos^2 32^\circ \operatorname{cosec}^2 58^\circ - \cos^2 45^\circ$$

30. Find all the zeroes of the polynomial  $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ , if two of its zeroes are  $\sqrt{2}$  and  $-\sqrt{2}$ .

31. Prove that :

$$\sin^8 \theta - \cos^8 \theta = (\sin^2 \theta - \cos^2 \theta)(1 - 2 \sin^2 \theta \cos^2 \theta)$$

32. Solve the following system of linear equations graphically :

$$2x + y = 8$$

$$3x - 2y = 12$$

Also find the coordinates of the points where these lines meet the  $x$ -axis.

33. The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution :

| Profit (in lakhs ₹)      | Number of shops (frequency) |
|--------------------------|-----------------------------|
| More than or equal to 5  | 30                          |
| More than or equal to 10 | 28                          |
| More than or equal to 15 | 16                          |
| More than or equal to 20 | 14                          |
| More than or equal to 25 | 10                          |
| More than or equal to 30 | 7                           |
| More than or equal to 35 | 3                           |

Draw both ogives for the data above.

34. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Or

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**ANSWERS**

**Section 'A'**

1. (a)

2. (b)

3. (d)

4. (b)  
7. (c)  
10. (b)

5. (d)  
8. (b)

6. (c)  
9. (d)

**Section 'B'**

11.  $a = 8, b = 5$   
13.  $g(x) = x^2 - 3x + 4$

17.

| Age (in years) | No. of policy holders (cf) |
|----------------|----------------------------|
| Less than 20   | 2                          |
| Less than 25   | 6                          |
| Less than 30   | 24                         |
| Less than 35   | 45                         |
| Less than 40   | 78                         |
| Less than 45   | 89                         |
| Less than 50   | 92                         |
| Less than 55   | 98                         |
| Less than 60   | 100                        |

18. Mode = 2695.65

**Section 'C'**

19. **Hint :**  $21^n = (7 \times 3)^n = 7^n \times 3^n$

If  $21^n$  ends with any one of the digits 0, 2, 4, 6 or 8, then it must be divisible by 2  
 $\Rightarrow 2$  divides  $21^n$  but 2 is prime  $\Rightarrow 2$  divides 21, which is wrong.

So,  $21^n$  cannot end with the digits 0, 2, 4, 6 or 8 for any natural number  $n$ .

21. Number of students in the class = 96 Or Originally A have money = ₹ 500, Originally B have money = ₹ 900  
22.  $x^2 - 4x - 60$   
25. Width of the street = 21 m  
27. Median = 39.9 years  
28. Mean = 117 Or Sixth number is 15.

**Section 'D'**

29. Or  $\frac{7}{2}$

30. The zeroes of the polynomial  $f(x)$  are  $\sqrt{2}, -\sqrt{2}, 1$  and  $\frac{1}{2}$ .

32.  $x = 4, y = 0; (0, 8)$  and  $(0, -6)$