CBSE
CCE SAMPLE QUESTION PAPER

FIRST TERM (SA-I)
MATHEMATICS
(*With Solutions*)
CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

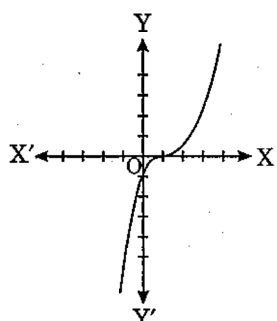
1. Euclid's Division Lemma states that for any two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy.

(*a*) $1 < r < b$ (*b*) $0 < r < b$

(*c*) $0 \leq r < b$ (*d*) $0 < r \leq b$

Solution. Choice (*c*) is correct.

2. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



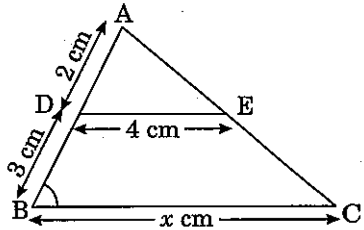
(*a*) 4 (*b*) 1

(*c*) 2 (*d*) 3

Solution. Choice (*b*) is correct.

The number of zeroes is 1 as the graph intersects the x-axis at one point in the given figure.

3. In figure, if $DE \parallel BC$, then x equals



(*a*) 6 cm (*b*) 8 cm
(*c*) 10 cm (*d*) 12.5 cm

Solution. Choice (*c*) is correct.

In figure, since $DE \parallel BC$, therefore

$\angle D = \angle B$ [Corresponding $\angle$s]

In ΔADE and ΔABC, we have

$\angle D = \angle B$ [Proved above]

$\angle A = \angle A$ [Common]

So, by AA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$

$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$ [Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{2}{5} = \frac{4}{x}$ [$\because AB = AD + DB = (2 + 3)$ cm $= 5$ cm]

$\Rightarrow \quad x = \frac{5 \times 4}{2}$

$\Rightarrow \quad x =$ **10 cm.**

4. If $\sin 3\theta = \cos(\theta - 6°)$, where (3θ) and $(\theta - 6°)$ are both acute angles, then the value of θ is

(*a*) 18° (*b*) 24°
(*c*) 36° (*d*) 30°

Solution. Choice (*b*) is correct.

Given, $\sin 3\theta = \cos(\theta - 6°)$

$\Rightarrow \cos(90° - 3\theta) = \cos(\theta - 6°)$ [$\because \cos(90° - A) = \sin A$]

$\Rightarrow \quad 90° - 3\theta = \theta - 6°$

$\Rightarrow \quad 96° = 4\theta$

$\Rightarrow \quad \theta = 96° \div 4$

$\Rightarrow \quad \theta = 24°$

5. Given that $\tan\theta = \frac{1}{\sqrt{3}}$, the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is

(*a*) -1 (*b*) 1
(*c*) $\frac{1}{2}$ (*d*) $-\frac{1}{2}$

Solution. Choice (*c*) is correct.

Given, $\tan\theta = \frac{1}{\sqrt{3}}$, then

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$= \frac{(\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{2+(\sqrt{3})^2+\left(\frac{1}{\sqrt{3}}\right)^2} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta} = \frac{1}{1/\sqrt{3}} = \sqrt{3}\right]$$

$$= \frac{3-\frac{1}{3}}{2+3+\frac{1}{3}}$$

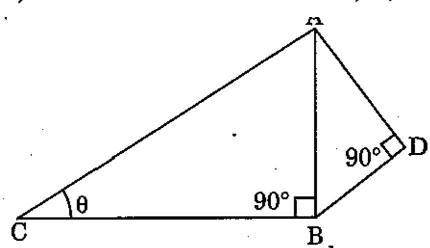
$$= \frac{(9-1)/3}{(6+9+1)/3}$$

$$= \frac{8/3}{16/3}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

6. In figure, *AD* = 4 cm, *BD* = 3 cm and *CB* = 12 cm, then cot θ equals



(*a*) $\frac{3}{4}$ (*b*) $\frac{5}{12}$

(*c*) $\frac{4}{3}$ (*d*) $\frac{12}{5}$

Solution. Choice (*d*) is correct.

In right ΔADB, $AB^2 = AD^2 + BD^2$

$\Rightarrow \quad AB^2 = (4)^2 + (3)^2$

$\Rightarrow \quad AB^2 = 16 + 9 = 25 = (5)^2$

$$\Rightarrow \quad AB = 5 \text{ cm}$$

In ΔACB, $\cot\theta = \frac{CB}{AB}$

$$= \frac{12}{5} \qquad [\because CB = 12 \text{ cm (given)}]$$

7. The decimal expansion of $\frac{147}{120}$ will terminate after how many places of decimal?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) will not terminate

Solution. Choice (*c*) is correct.

$$\frac{147}{120} = \frac{3 \times 49}{3 \times 40}$$
$$= \frac{49}{40}$$
$$= \frac{49 \times 25}{40 \times 25}$$
$$= \frac{1225}{1000}$$
$$= 1.275$$

It shows that after **3** places of decimal, decimal expansion $\frac{147}{120}$ will terminate.

8. The pair of linear equations $3x + 2y = 5$; $2x - 3y = 7$ have

(*a*) one solution (*b*) two solutions

(*c*) many solutions (*d*) no solution

Solution. Choice (*a*) is correct.

Here, $a_1 = 3, b_1 = 2, c_1 = -5$

$a_2 = 2, b_2 = -3, c_2 = -7$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}, \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The given pair of lines have **one solution.**

9. If $\sec A = \operatorname{cosec} B$, then $A + B$ is equal to

(*a*) Zero (*b*) 90°

(*c*) < 90° (*d*) > 90°

Solution. Choice (*b*) is correct.

Given, $\sec A = \operatorname{cosec} B$

$\Rightarrow \quad \sec A = \operatorname{cosec} B$

$\Rightarrow \quad \operatorname{cosec}(90° - A) = \operatorname{cosec} B \qquad [\because \operatorname{cosec}(90° - \theta) = \sec\theta]$

$\Rightarrow \quad 90° - A = B$

$\Rightarrow \quad A + B = \mathbf{90°}$

10. For a given data with 70 observations the 'less than ogive' and the 'more than ogive' intersect at (20.5, 35). The median of the data is

(*a*) 20 (*b*) 35

(*c*) 70 (*d*) 20.5

Solution. Choice (*d*) is correct.

The x-coordinate of the point of intersection of the 'less than ogive' and the 'more than ogive' is the median. Since the given intersection point is (20.5, 35), therefore the median is **20.5.**

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Is $7 \times 5 \times 3 \times 2 + 3$ a composite number ? Justify your answer.

Solution. We have

$7 \times 5 \times 3 \times 2 + 3$

$= 7 \times 5 \times 2 \times 3 + 1 \times 3$ [Writing 3 as 1×3]

$= (7 \times 5 \times 2 + 1) \times 3$ [Taking 3 as common]

$= (70 + 1) \times 3$

$= 71 \times 3$

$= 213$

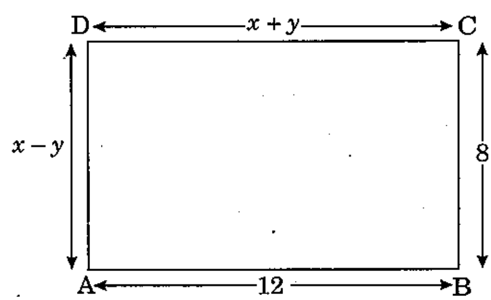
Fundamental Theorem of Arithmetic states that "Every composite number can be expressed (factorised) as a product of primes and their factorisation is unique, apart from the order in which the prime factors occur".

So, $7 \times 5 \times 3 \times 2 + 3 = 71 \times 3$ is a **composite number.**

12. Can $(x - 2)$ be the remainder on division of a polynomial $p(x)$ by $(2x + 3)$? Justify your answer.

Solution. No, $(x - 2)$ cannot be the remainder on division of given polynomial $p(x)$ by $(2x + 3)$ as the degree of remainder polynomial and quotient polynomial is same.

13. In figure, *ABCD* is a rectangle. Find the values of x and y.



Solution. Since *ABCD* is a rectangle, therefore

$AB = DC$ (opposite sides of a rectangle) and $AD = BC$ (opposite sides of a rectangle)

$\Rightarrow \quad 12 = x + y \quad ...(1)$ and $x - y = 8 \quad ...(2)$

Solving these equations by adding and subtracting the given equations, *i.e.*, we have

$(x + y) + (x - y) = 12 + 8$ $\qquad$ $(x + y) - (x - y) = 12 - 8$

$\Rightarrow \quad 2x = 20$ $\qquad$ $\Rightarrow \quad 2y = 4$

$\Rightarrow \quad \boldsymbol{x = 10}$ $\qquad$ $\Rightarrow \quad \boldsymbol{y = 2}$

14. If $7\sin^2\theta + 3\cos^2\theta = 4$, show that $\tan\theta = \dfrac{1}{\sqrt{3}}$.

Solution. Given :

$$7\sin^2\theta + 3\cos^2\theta = 4$$

$$\Rightarrow \quad \frac{7\sin^2\theta + 3\cos^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta} \qquad \text{[Dividing both sides by } \cos^2\theta\text{]}$$

$$\Rightarrow \quad 7\frac{\sin^2\theta}{\cos^2\theta} + 3\frac{\cos^2\theta}{\cos^2\theta} = \frac{4}{\cos^2\theta}$$

$$\Rightarrow \quad 7\tan^2\theta + 3 = 4\sec^2\theta$$

$$\Rightarrow \quad 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$$

$$\Rightarrow \quad 7\tan^2\theta + 3 = 4 + 4\tan^2\theta$$

$$\Rightarrow \quad 7\tan^2\theta - 4\tan^2\theta = 4 - 3$$

$$\Rightarrow \quad 3\tan^2\theta = 1$$

$$\Rightarrow \quad \tan^2\theta = \frac{1}{3}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \qquad [\because \theta \text{ lies in 1st quadrant}]$$

Or

If $\cot\theta = \dfrac{15}{8}$, evaluate $\dfrac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$.

Solution. We have

$$\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$$

$$= \frac{[2(1 + \sin\theta)](1 - \sin\theta)}{(1 + \cos\theta)[2(1 - \cos\theta)]}$$

$$= \frac{2(1 - \sin^2\theta)}{2(1 - \cos^2\theta)}$$

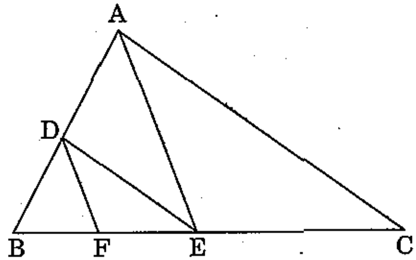
$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \left(\frac{15}{8}\right)^2 \qquad \left[\cot\theta = \frac{15}{8} \text{ (given)}\right]$$

$$= \frac{225}{64}.$$

15. In figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{FE}{BF} = \frac{EC}{BE}$.



Solution. In ΔBCA, $DE \parallel AC$

$\therefore \quad \frac{BE}{EC} = \frac{BD}{DA}$ ...(1) [By Thales Theorem]

In ΔBEA, $DF \parallel AE$

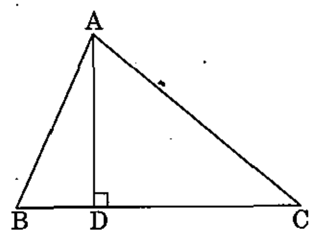
$\therefore \quad \frac{BF}{FE} = \frac{BD}{DA}$ ...(2) [By Thales Theorem]

From (1) and (2), we conclude that

$$\frac{BE}{EC} = \frac{BF}{FE}$$

$$\Rightarrow \quad \frac{FE}{BF} = \frac{EC}{BE}$$

16. In figure, $AD \perp BC$ and $BD = \frac{1}{3}CD$. Prove that $2CA^2 = 2AB^2 + BC^2$.



Solution. We have

$$BD = \frac{1}{3}CD$$

$\Rightarrow \quad 3BD = CD$

$\Rightarrow \quad 3BD + BD = CD + BD$ [Adding BD to both sides]

$\Rightarrow \quad 4BD = BC$

In right ΔABD, right-angled at D, we have

$AB^2 = AD^2 + BD^2$ [Pythagoras Theorem]

$\Rightarrow \quad 2AB^2 = 2AD^2 + 2BD^2$ ...(1)

In right ΔACD, right-angled at D, we have

$CA^2 = AD^2 + CD^2$ [Pythagoras Theorem]

$\Rightarrow \quad 2CA^2 = 2AD^2 + 2CD^2$ ...(2)

Subtracting (1) from (2), we get

$$2CA^2 - 2AB^2 = (2AD^2 + 2CD^2) - (2AD^2 + 2BD^2)$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = 2(CD^2 - BD^2)$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = 2[(3BD)^2 - BD^2] \quad \left[\because BC = \frac{1}{3}CD \Rightarrow CD = 3BD\right]$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = 2[9BD^2 - BD^2]$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = 2(8BD^2)$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = 16BD^2$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = (4BD)^2$$

$\Rightarrow$ $$2CA^2 - 2AB^2 = BC^2 \quad [\because BC = 4BD \text{ (proved above)}]$$

$\Rightarrow$ $$\mathbf{2CA^2 = 2AB^2 + BC^2.}$$

17. The following distribution gives the daily income of 50 workers of a factory :

Daily income (in ₹)	**100 – 120**	**120 – 140**	**140 – 160**	**160 – 180**	**180 – 200**
Number of Workers	**12**	**14**	**8**	**6**	**10**

Write the above distribution as less than type cumulative frequency distribution.

Solution. We calculate "less than type" cumulative frequency distribution by adding frequencies from top to bottom.

Less Than Type Cumulative Frequency Distribution

Daily income (in ₹)	*Number of Workers*	*Daily income Less than*	*Cumulative Frequency*
100 – 120	12	120	12
120 – 140	14	140	26
140 – 160	8	160	34
160 – 180	6	180	40
180 – 200	10	200	50

18. Find the mode of the following distribution of marks obtained by 80 students :

Marks obtained	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Number of students	**6**	**10**	**12**	**32**	**20**

Solution. Since the maximum number of students is 32, therefore, the modal class is 30 – 40. Thus, the lower limit (l) of the modal class = 30.

$\therefore$ $$f_1 = 32, f_0 = 12, f_2 = 20, h = 10.$$

Using the formula :

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \frac{32 - 12}{2 \times 32 - 12 - 20} \times 10$$

$$= 30 + \frac{20}{64 - 32} \times 10$$

$$= 30 + \frac{200}{32}$$

$$= 30 + \frac{25}{4}$$
$$= 30 + 6.25$$
$$= 36.25$$

So, the maximum number of students have **36.25 marks.**

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is a positive integer.

Solution. Let a be any positive integer, when a is divided by 4. Then, by Euclid's division lemna, we get

$$a = 4q + r,\ 0 \le r < 4, \text{ where } q \text{ and } r \text{ are whole numbers.}$$

So, $r = 0, 1, 2, 3$.

When $r = 0$ or 2, then a becomes an even integer, *i.e.*, $a = 4q$ or $4q + 2$.

When $r = 1$ or 3, then a becomes positive odd integer, *i.e.*, $a = 4q + 1$ or $4q + 3$.

Hence any positive odd integer is of the form $\mathbf{4q + 1}$ or $\mathbf{4q + 3}$, where q is a positive integer.

20. Prove that $\frac{2\sqrt{3}}{5}$ is irrational.

Solution. Let us assume, to the contrary, that $\frac{2\sqrt{3}}{5}$ is rational.

$$\frac{2\sqrt{3}}{5} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are coprime, } i.e., \text{ their HCF is 1}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5p}{2q}$$

Since $\frac{p}{q}$ is a rational number, therefore, $\frac{5}{2}\frac{p}{q}$ is also rational number.

$\Rightarrow \sqrt{3}$ is rational number

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{2\sqrt{3}}{5}$ is rational.

So, we conclude that $\frac{2\sqrt{3}}{5}$ is **irrational.**

Or

Prove that $(5 - \sqrt{2})$ is irrational.

Solution. Let us assume, to the contrary, that $(5 - \sqrt{2})$ is rational, *i.e.*, we can find coprime a and b $(b \ne 0)$ such that

$$5 - \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 5 - \frac{a}{b} = \sqrt{2}$$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational.

$\Rightarrow \sqrt{2}$ is rational

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{2}$ is rational.

So, we conclude that $(5 - \sqrt{2})$ is **irrational.**

21. A person rowing a boat at the rate of 5 km/hour in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

Solution. Let the speed of the stream be x km/h and the speed of the boat in still water be 5 km/h

$\therefore$ Relative speed of the boat downstream = $(5 + x)$ km/h

and relative speed of the boat upstream = $(5 - x)$ km/h

$\therefore$ Time taken by the boat downstream = $\frac{40}{5+x}$ h

and time taken by the boat upstream = $\frac{40}{5-x}$ h

According to the given condition, a boat takes thrice as much time in going 40 km upstream as in going 40 km downstream, we have

$$\frac{40}{5-x} = 3\left(\frac{40}{5+x}\right)$$

$$\Rightarrow \quad \frac{1}{5-x} = \frac{3}{5+x}$$

$$\Rightarrow \quad 5 + x = 15 - 3x$$

$$\Rightarrow \quad 3x + x = 15 - 5$$

$$\Rightarrow \quad 4x = 10$$

$$\Rightarrow \quad x = 2.5$$

Hence the speed of the stream is **2.5 km/h.**

Or

In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for each wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly ?

Solution. Let the number of correctly answered questions be x and the number of wrongly answered questions be y.

It is given that "the total number questions answered is 120".

$\therefore \quad x + y = 120$ ...(1)

Also, it is given that "one mark is awarded for each correct answer and $\frac{1}{2}$ mark is deducted for each wrong answer and Jayanti has scored 90 marks".

$$\therefore \quad 1 \cdot x - \frac{1}{2} \cdot y = 90$$

$$\Rightarrow \quad 2x - y = 180 \quad \ldots(2)$$

Adding (1) and (2), we get

$$(x + y) + (2x - y) = 120 + 180$$

$$\Rightarrow \quad 3x = 300$$

$$\Rightarrow \quad x = 100$$

Substituting $x = 100$ in (1), we get : $y = 120 - x = 120 - 100 = 20$.

Hence, the number of correctly answered questions by Jayanti are **100.**

Alternative Method :

Total number of questions = 120.

Let the number of correctly answered questions be x, then the number of wrongly answered questions = $120 - x$.

It is given that "one mark is awarded for each correct answer and $\frac{1}{2}$ mark is deducted for each wrong answer".

Thus, marks obtained for correctly answered question = $1 \cdot x = x$.

and, marks obtained for wrongly answered question = $\frac{1}{2}(120 - x)$

$$\therefore \quad \text{Total marks obtained} = x - \frac{1}{2}(120 - x)$$

But the total marks obtained by Jayanti = 90 (given)

$$\therefore \quad x - \frac{1}{2}(120 - x) = 90$$

$$\Rightarrow \quad x - 60 + \frac{1}{2}x = 90$$

$$\Rightarrow \quad \frac{3}{2}x = 90 + 60$$

$$\Rightarrow \quad \frac{3}{2}x = 150$$

$$\Rightarrow \quad x = \frac{150 \times 2}{3} = 100$$

Hence, the number of correctly answered questions by Jayanti are **100.**

22. If α, β are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are (2α) and (2β).

Solution. Since α and β are zeroes of the polynomial, $x^2 - 2x - 15$, therefore

$$\alpha + \beta = -\left(\frac{-2}{1}\right) = 2$$

$$\alpha\beta = \left(-\frac{15}{1}\right) = -15$$

Let S and P denote respectively the sum and product of zeroes of the required polynomial, then

$$S = 2\alpha + 2\beta = 2(\alpha + \beta) = 2(2) = 4$$

and $\qquad P = (2\alpha)(2\beta) = 4(\alpha\beta) = 4(-15) = -60$

Hence, the required polynomial $p(x)$ is given by

$$p(x) = (x^2 - Sx + P)$$

$$\Rightarrow \qquad p(x) = \boldsymbol{x^2 - 4x - 60}.$$

23. Prove that $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta) = \dfrac{1}{\tan\theta + \cot\theta}$.

Solution. We have

$$\begin{aligned}
\text{L.H.S.} &= (\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)\\
&= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\\
&= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\\
&= \frac{\cos^2\theta}{\sin\theta}\cdot\frac{\sin^2\theta}{\cos\theta}\\
&= \frac{\sin\theta\cos\theta}{1}\\
&= \frac{\sin\theta\cos\theta}{\sin^2\theta + \cos^2\theta} && [\because 1 = \sin^2\theta + \cos^2\theta]\\
&= \frac{\sin\theta\cos\theta/\sin\theta\cos\theta}{\dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}} && [\text{Dividing numerator and denominator by } \sin\theta\cos\theta]\\
&= \frac{1}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}\\
&= \frac{1}{\tan\theta + \cot\theta}\\
&= \text{R.H.S.}
\end{aligned}$$

24. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Solution. Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$\Rightarrow \qquad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \qquad \sin\theta = (\sqrt{2} - 1)\cos\theta$$

$$\Rightarrow \qquad \frac{\sin\theta}{\sqrt{2}-1} = \cos\theta$$

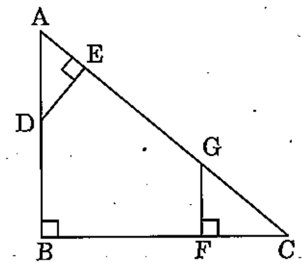
$$\Rightarrow \qquad \frac{(\sqrt{2}+1)\sin\theta}{(\sqrt{2}-1)(\sqrt{2}+1)} = \cos\theta$$

$$\Rightarrow \qquad \frac{\sqrt{2}\sin\theta + \sin\theta}{2-1} = \cos\theta$$

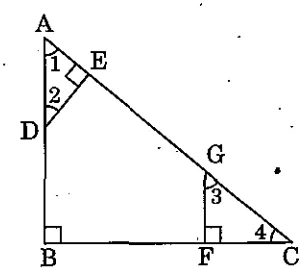
$\Rightarrow \qquad \sqrt{2} \sin \theta + \sin \theta = \cos \theta$

$\Rightarrow \qquad \cos \theta - \sin \theta = \sqrt{2} \sin \theta$

25. In figure, $AB \perp BC$, $FG \perp BC$ and $DE \perp AC$. Prove that $\Delta ADE \sim \Delta GCF$.



Solution. Since $AB \perp BC$ and $GF \perp BC$, therefore $AB \parallel GF$.



$\therefore \qquad \angle 1 = \angle 3$ [Corresponding $\angle s$] ...(1)

In ΔADE, we have

$\angle 1 + \angle 2 = 90°$

$\Rightarrow \qquad \angle 3 + \angle 2 = 90°$ ...(2) [using (1)]

In ΔGCF, we have

$\angle 3 + \angle 4 = 90°$ ...(3)

From (2) and (3), we have

$\angle 3 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \qquad \angle 2 = \angle 4$ ...(4)

In ΔADE and ΔGCF, we have

$\angle 1 = \angle 3$ [Proved in (1)]

and $\quad \angle 2 = \angle 4$ [Proved in (4)]

So, by AA similarity of criterion, we have

$\Delta ADE \sim \Delta GCF$

Alternative Method :

Since $AB \perp BC$ and $GF \perp BC$, therefore $AB \parallel GF$

$\therefore \qquad \angle 1 = \angle 3$ ...(1) [Corresponding $\angle s$]

In ΔADE and ΔGCF, we have

$\angle 1 = \angle 3$ [Proved in (1)]

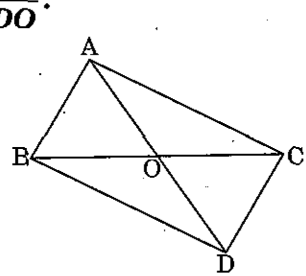
and $\quad \angle AED = \angle GFC$ [Each = 90°]

So, by AA similarity criterion, we have

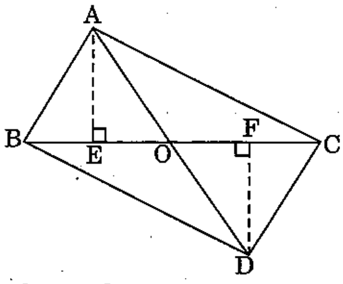
$\Delta ADE \sim \Delta GCF$

26. ΔABC and ΔDBC are on the same base BC and on opposite sides of BC and O is the point of intersection of AD and BC.

Prove that $\dfrac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \dfrac{AO}{DO}$.



Solution. In the figure ΔABC and ΔDBC are on the same base BC, AD and BC intersect at O. Draw $AE \perp BC$ and $DF \perp BC$.



In ΔAEO and ΔDFO, we have

$\angle AEO = \angle DFO$ [Each = 90°]

and $\angle AOE = \angle DOF$ [Vertically opposite $\angle s$]

So, by AA similarity criterion, we have

$$\Delta AEO \sim \Delta DFO$$

$$\frac{AE}{DF} = \frac{AO}{DO} \quad \text{...(1) [In similar triangles corresponding sides are proportional]}$$

Now, $$\frac{\text{area }(\Delta ABC)}{\text{area }(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$$

$$= \frac{AE}{DF}$$

$$= \frac{AO}{DO} \quad \text{[using (1)]}$$

Hence proved.

27. Find mean of the following frequency distribution, using step-deviation method :

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**7**	**12**	**13**	**10**	**8**

Solution. Let the assumed mean be $a = 25$ and $h = 10$.

Calculation of Mean

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 – 10	7	5	– 20	– 2	– 14
10 – 20	12	15	– 10	– 1	– 12
20 – 30	13	25	0	0	0
30 – 40	10	35	10	1	10
40 – 50	8	45	20	2	16
Total	$n = \Sigma f_i = 50$				$\Sigma f_i u_i = 0$

From the table, $n = \Sigma f_i = 50$, $\Sigma f_i u_i = 0$.

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 25 + \frac{0}{50} \times 10$$

$$= 25 + 0$$

$$= \mathbf{25}$$

Or

The mean of the following frequency distribution is 25. Find the value of p.

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**2**	**3**	**5**	**3**	***p***

Solution. **Calculation of Mean**

Classes	Frequency (f_i)	Class-mark (x_i)	$f_i x_i$
0 – 10	2	5	10
10 – 20	3	15	45
20 – 30	5	25	125
30 – 40	3	35	105
40 – 50	p	45	$45p$
Total	$n = \Sigma f_i = 13 + p$		$\Sigma f_i x_i = 285 + 45p$

From the table, $n = \Sigma f_i = 13 + p$, $\Sigma f_i x_i = 285 + 45p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 25 = \frac{285 + 45p}{13 + p}$$

$$\Rightarrow \quad 25(13 + p) = 285 + 45p$$

$$\Rightarrow \quad 325 + 25p = 285 + 45p$$

$$\Rightarrow \quad 45p - 25p = 325 - 285$$

$$\Rightarrow \quad 20p = 40$$

$$\Rightarrow \quad \mathbf{p = 2}$$

28. Find the median of the following data :

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**	**80 – 90**	**90 – 100**
Frequency	**5**	**3**	**4**	**3**	**3**	**4**	**7**	**9**	**7**	**8**

Solution. **Calculation of Median**

Classes	Frequency (f)	Cumulative Freqeuncy (cf)
0 – 10	5	5
10 – 20	3	8
20 – 30	4	12
30 – 40	3	15
40 – 50	3	18
50 – 60	4	22 (cf)
60 – 70	7 (f)	29
70 – 80	9	38
80 – 90	7	45
90 – 100	8	53
Total	$n = \Sigma f = 53$	

Here, $n = 53$ and $\frac{n}{2} = \frac{53}{2} = 26.5$.

Now, 60 – 70 is the class whose cumulative frequency is 29 is greater than $\frac{n}{2} = 26.5$.

$\therefore$ 60 – 70 is the median class.
From the table, $f = 7$, $cf = 22$, $h = 10$
Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 60 + \frac{26.5 - 22}{7} \times 10$$

$$= 60 + \frac{4.5}{7} \times 10$$

$$= 60 + \frac{45}{7}$$

$$= 60 + 6.43$$

$$= \mathbf{66.43}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Find other zeroes of the polynomial $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$ if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution. Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$, therefore $(x - \sqrt{2})(x + \sqrt{2}) = (x^2 - 2)$ is a factor of the given polynomial.

Now, we divide the given polynomial by $(x^2 - 2)$.

$$\begin{array}{r|l} & 2x^2 + 7x - 15 \\ \hline x^2 - 2 & 2x^4 + 7x^3 - 19x^2 - 14x + 30 \\ & \underline{-2x^4 \qquad\quad \mp 4x^2} \\ & \qquad 7x^3 - 15x^2 - 14x + 30 \\ & \qquad \underline{-7x^3 \qquad\quad \mp 14x} \\ & \qquad\qquad -15x^2 + 30 \\ & \qquad\qquad \underline{\mp 15x^2 \pm 30} \\ & \qquad\qquad\qquad 0 \end{array}$$

[First term of quotient is $\frac{2x^4}{x^2} = 2x^2$]

[Second term of quotient is $\frac{7x^3}{x^2} = 7x$]

[Third term of quotient is $\frac{-15x^2}{x^2} = -15$]

So, $2x^4 + 7x^3 - 19x^2 - 14x + 30 = (x^2 - 2)(2x^2 + 7x - 15)$

$$= (x - \sqrt{2})(x + \sqrt{2})[2x^2 + 10x - 3x - 15]$$

$$= (x - \sqrt{2})(x + \sqrt{2})[2x(x + 5) - 3(x + 5)]$$

$$= (x - \sqrt{2})(x + \sqrt{2})(x + 5)(2x - 3)$$

So, the zeroes of $2x^2 + 7x - 15 = (x + 5)(2x - 3)$ are given by -5 and $\frac{3}{2}$.

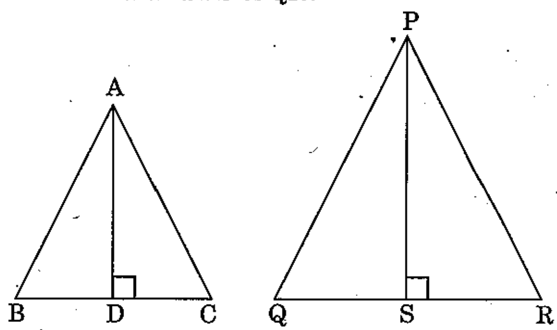
Hence all the zeroes of the given polynomial are $\sqrt{2}, -\sqrt{2}, -5$ and $\frac{3}{2}$.

30. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.



Proof : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\frac{AB}{PQ} = \frac{BC}{QR}$ [$\because \Delta ABC \sim \Delta PQR$]

$\Rightarrow \frac{AD}{PS} = \frac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$$

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$ ...(4)

As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad ...(5)$$

Hence, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$ [From (4) and (5)]

Or

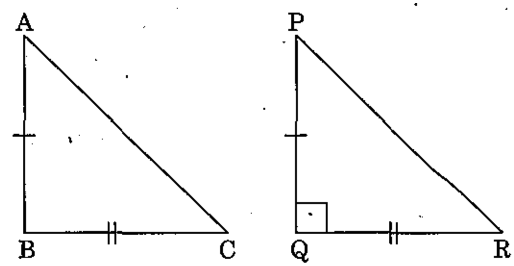
Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

Solution. Given : A triangle ABC such that :

$$AC^2 = AB^2 + BC^2$$

To prove : ΔABC is a right-angled at B, *i.e.*, $\angle B = 90°$.

Construction : Construct a ΔPQR such that $\angle Q = 90°$ and $PQ = AB$ and $QR = BC$.



Proof : In ΔPQR, as $\angle Q = 90°$, we have

$PR^2 = PQ^2 + QR^2$ [By Pythagoras Theorem]

$\Rightarrow \quad PR^2 = AB^2 + BC^2$ ...(1) [As $PQ = AB$ and $QR = BC$]

But $\quad AC^2 = AB^2 + BC^2$ ...(2)

From (1) and (2), we have

$PR^2 = AC^2$

$\Rightarrow \quad PR = AC$ ...(3)

Now in ΔABC and ΔPQR, we have

$AB = PQ$

$BC = QR$

and $\quad AC = PR$ [using (3)]

$\therefore \quad \Delta ABC \cong \Delta PQR$ [SSS congruency]

$\Rightarrow \quad \angle B = \angle Q = 90°$ [CPCT]

Hence, $\quad \angle \boldsymbol{B} = \mathbf{90°}$.

31. Prove that $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1 - \sin\theta}$

Solution. We have

$$\text{L.H.S.} = \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + (\sec^2\theta - \tan^2\theta)} \qquad [\because 1 = \sec^2\theta - \tan^2\theta]$$

$$= \frac{\sec\theta + \tan\theta - 1}{(\tan\theta - \sec\theta) + (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta - 1}{-(\sec\theta - \tan\theta) + (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$$

$$= \frac{\sec\theta + \tan\theta - 1}{(\sec\theta - \tan\theta)[-1 + \sec\theta + \tan\theta]}$$

$$= \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1 - \sin\theta}$$

= R.H.S.

Or

Evaluate : $\mathbf{\dfrac{\sec\theta \operatorname{cosec}(90^\circ - \theta) - \tan\theta \cot(90^\circ - \theta) + \sin^2 55^\circ + \sin^2 35^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}}$

Solution. We have

$$\frac{\sec\theta \operatorname{cosec}(90^\circ - \theta) - \tan\theta \cot(90^\circ - \theta) + \sin^2 55^\circ + \sin^2 35^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}$$

$$= \frac{\sec\theta \sec\theta - \tan\theta \tan\theta + \sin^2(90^\circ - 35^\circ) + \sin^2 35^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan(90^\circ - 20^\circ) \tan(90^\circ - 10^\circ)}$$

$[\because \operatorname{cosec}(90^\circ - \theta) = \sec\theta, \cot(90^\circ - \theta) = \tan\theta]$

$$= \frac{\sec^2\theta - \tan^2\theta + \cos^2 35^\circ + \sin^2 35^\circ}{\tan 10^\circ \tan 20^\circ (\sqrt{3}) \cot 20^\circ \cot 10^\circ} \qquad [\because \sin(90^\circ - \theta) = \cos\theta, \tan(90^\circ - \theta) = \cot\theta]$$

$$= \frac{1 + 1}{(\tan 10^\circ . \cot 10^\circ)(\tan 20^\circ . \cot 20^\circ)(\sqrt{3})}$$

$[\because \sec^2\theta - \tan^2\theta = 1, \sin^2\theta + \cos^2\theta = 1, \tan(90^\circ - \theta) = \cot\theta]$

$$= \frac{2}{(1)(1)(\sqrt{3})} \qquad [\because \tan\theta \cdot \cot\theta = 1]$$

$$= \mathbf{\frac{2}{\sqrt{3}}}$$

32. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \dfrac{p^2-1}{p^2+1}$.

Solution. We have

$$\frac{p^2-1}{p^2+1} = \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{(\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta)-1}{(\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta)+1}$$

$$= \frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(1+\tan^2\theta)+2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta} \quad [\because \sec^2\theta-1=\tan^2\theta \text{ and } 1+\tan^2\theta=\sec^2\theta]$$

$$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)}$$

$$= \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\sin\theta}{\cos\theta\cdot\sec\theta}$$

$$= \sin\theta.$$

33. Draw the graphs of following equations :

$2x - y = 1,\ x + 2y = 13$

(*i*) Find the solution of the equation from the graph.

(*ii*) Shade the triangular region formed by the lines and the *y*-axis.

Solution. We have

$2x - y = 1$ ...(1) and $x + 2y = 13$ ...(2)

$\Rightarrow \quad y = 2x - 1 \qquad \Rightarrow \quad y = \dfrac{13-x}{2}$

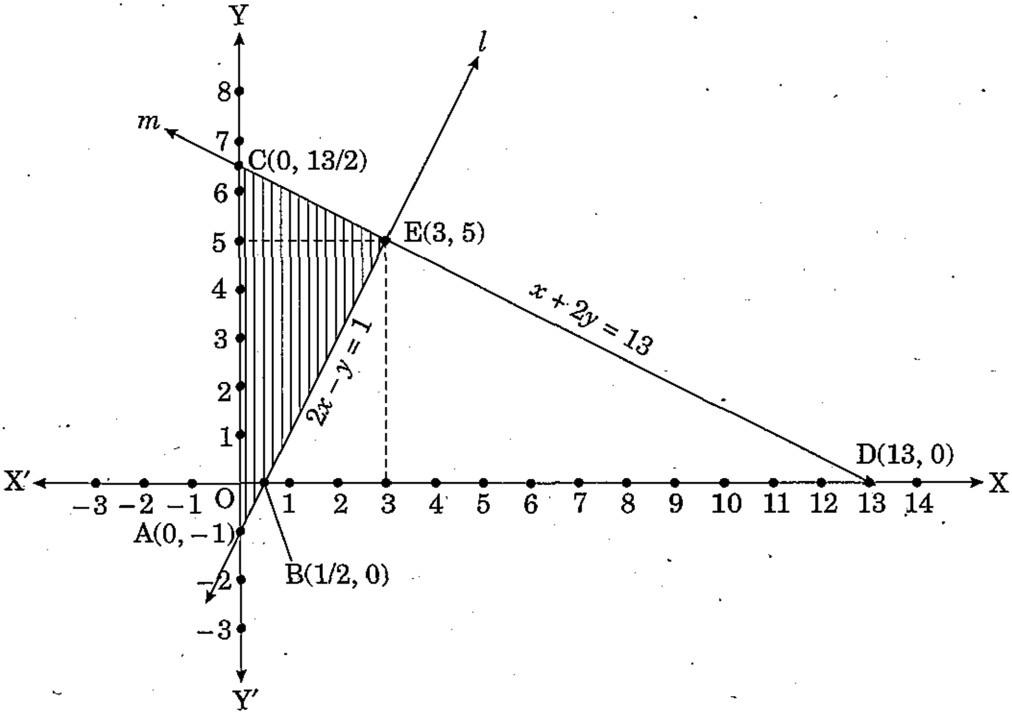
Table of $y = 2x - 1$

x	0	1/2	3
y	-1	0	5
	A	B	E

Table of $y = \dfrac{13-x}{2}$

x	0	13	3
y	13/2	0	5
	C	D	E

Plot the points $A(0,-1)$, $B\left(\frac{1}{2},0\right)$, $C\left(0,\frac{13}{2}\right)$, $D(13,0)$ and $E(3,5)$ on graph paper and join the points to form the lines l and m as shown in figure.



(*i*) From the graph, clearly lines (1) and (2) intersect each other at the point $E(3, 5)$.

So, $x = 3, y = 5$ is the required solution of the given equations.

(*ii*) Shaded triangular region *ACE* is shown in the figure.

34. The following table gives the production yield per hectare of wheat of 100 farms of a village :

Production yield in kg/hectare	**50 – 55**	**55 – 60**	**60 – 65**	**65 – 70**	**70 – 75**	**75 – 80**
Number of farms	**2**	**8**	**12**	**24**	**38**	**16**

Change the above distribution to more than type distribution and draw its ogive.

Solution. First we change the distribution to a more than type distribution and draw its ogive.

Here 50, 55, 60, 65, 70, 75 are the lower limits of the respective class intervals more than 50 – 55, 55 – 60, 60 – 65, 65 – 70, 70 – 75 and 75 – 80.

To represent the data in the table graphically, we mark the lower limits of class intervals on the horizontal axis (x-axis) and their corresponding cumulative frequencies on the vertical axis (y-axis), choosing convenient scale.

Now plot the point corresponding to the ordered pairs given by (lower limit, corresponding cumulative frequency)

i.e., (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16) on a graph paper and join them by a free hand smooth curve. The curve we get is called a cumulative frequency curve or an ogive (of more than type) (see figure).

Classes	*Production field (in kg/hectare)*	*Number of farms (Frequency)*	*Cumulative Frequency (more than type)*
50 – 55	more than or equal to 50	2	100
55 – 60	more than or equal to 55	8	98
60 – 65	more than or equal to 60	12	90
65 – 70	more than or equal to 65	24	78
70 – 75	more than or equal to 70	38	54
75 – 80	more than or equal to 75	16	16