CCE SAMPLE QUESTION PAPER

FIRST TERM (SA-I)
MATHEMATICS
(*With Solutions*)
CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The decimal expansion of the rational number $\frac{23}{2^3 5^2}$, will terminate after how many places of decimal ?

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) 5

Solution. Choice (*b*) is correct.

$$\frac{23}{2^3 5^2} = \frac{23}{2(2^2)(5^2)} = \frac{23}{2(2 \times 5)^2} = \frac{23}{2(10)^2} = \frac{115}{100} = \mathbf{0.115}$$

Thus, shows that decimal expansion of the rational number $\frac{23}{2^3 5^2}$, will terminate after **three** places of decimal.

2. If HCF (105, 120) = 15, then LCM (105, 120) is

(*a*) 210 (*b*) 420

(*c*) 840 (*d*) 1680

Solution. Choice (*c*) is correct.

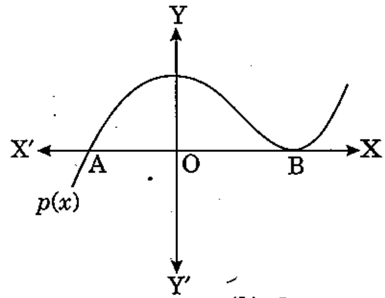
HCF × LCM = Product of two numbers

$15 \times \text{LCM} = 105 \times 120$

$\Rightarrow \quad \text{LCM} = \frac{105 \times 120}{15}$

$\Rightarrow \quad \text{LCM} = 105 \times 8 = \mathbf{840}.$

3. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



(a) 1 (b) 2

(c) 3 (d) 4

Solution. Choice (b) is correct.

The number of zeroes of $p(x)$ is **2** as the graph intersects the x-axis at two points A and B in figure.

4. The value of k for which $6x - 3y + 10 = 0$ and $2x + ky + 9 = 0$ has no solution is

(a) -1 (b) -2

(c) 1 (d) 3

Solution. Choice (a) is correct.

Here $a_1 = 6, b_1 = -3, c_1 = 10$ and $a_2 = 2, b_2 = k, c_2 = 9$

The given system of equations will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{6}{2} = \frac{-3}{k}$$

$$\Rightarrow \quad k = \frac{-3 \times 2}{6}$$

$$\Rightarrow \quad k = -1$$

5. Let $\Delta ABC \sim \Delta PQR$, area of $\Delta ABC = 81$ cm^2, area of $\Delta PQR = 144$ cm^2 and $QR = 6$ cm, then length of BC is

(a) 4 cm (b) 4.5 cm

(c) 9 cm (d) 12 cm

Solution. Choice (b) is correct.

Since $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{81}{144} = \frac{BC^2}{QR^2} \quad \left[\because \begin{array}{l} \text{area}(\Delta ABC) = 81\,\text{cm}^2 \text{ (given) and} \\ \text{area}(\Delta PQR) = 144\,\text{cm}^2 \text{ (given)} \end{array}\right]$$

$$\Rightarrow \quad \left(\frac{9}{12}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

$$\Rightarrow \quad \frac{9}{12} = \frac{BC}{QR}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{BC}{6} \qquad [\because QR = 6 \text{ cm (given)}]$$

$$\Rightarrow \quad BC = \frac{3 \times 6}{4}$$

$$\Rightarrow \quad BC = \frac{3 \times 3}{2}$$

$$\Rightarrow \quad BC = \mathbf{4.5\ cm}$$

6. The value of $\frac{\cos(90° - \theta)\cos\theta}{\tan\theta} - 1$ is

(*a*) $-\sin^2\theta$ (*b*) $-\text{cosec}^2\theta$
(*c*) $-\cos^2\theta$ (*d*) $-\cot\theta$

Solution. Choice (*a*) is correct.

$$\frac{\cos(90° - \theta)\cos\theta}{\tan\theta} - 1 = \frac{\sin\theta\cos\theta}{\tan\theta} - 1$$

$$= \frac{\sin\theta\cos\theta}{\sin\theta/\cos\theta} - 1$$

$$= \cos^2\theta - 1$$

$$= -(1 - \cos^2\theta)$$

$$= \mathbf{-\sin^2\theta}$$

7. If $\tan\theta + \frac{1}{\tan\theta} = 2$, then the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ is

(*a*) 3 (*b*) 4
(*c*) 2 (*d*) -4

Solution. Choice (*c*) is correct.

Given, $$\tan\theta + \frac{1}{\tan\theta} = 2$$

$$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2\tan\theta \cdot \frac{1}{\tan\theta} = 4 \qquad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$$

$$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} = 4 - 2$$

$$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} = \mathbf{2}$$

8. If $5\tan\alpha = 4$, then the value of $\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha}$ is

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{2}{7}$

Solution. Choice (c) is correct.

$$5 \tan \alpha = 4 \Rightarrow \tan \alpha = \frac{4}{5}$$

$$\frac{5 \sin \alpha - 3 \cos \alpha}{5 \sin \alpha + 2 \cos \alpha}$$

$$= \frac{(5 \sin \alpha - 3 \cos \alpha)/\cos \alpha}{(5 \sin \alpha + 2 \cos \alpha)/\cos \alpha}$$ [Dividing numerator and denominator by $\cos \alpha$]

$$= \frac{5 \tan \alpha - 3}{5 \tan \alpha + 2} = \frac{5(4/5) - 3}{5(4/5) + 2}$$

$$= \frac{4-3}{4+2} = \frac{1}{6}$$

9. If $\sec 4A = \operatorname{cosec}(A - 20°)$, where $4A$ is an acute angle, then the value of A is

(a) 21° (b) 22°

(c) 23° (d) 24°

Solution. Choice (b) is correct.

Given, $\sec 4A = \operatorname{cosec}(A - 20°)$

$\Rightarrow \operatorname{cosec}(90° - 4A) = \operatorname{cosec}(A - 20°)$ [$\because \operatorname{cosec}(90° - \theta) = \sec \theta$]

$\Rightarrow 90° - 4A = A - 20°$

$\Rightarrow 4A + A = 90° + 20°$

$\Rightarrow 5A = 110°$

$\Rightarrow \mathbf{A = 22°}$.

10. Which measure of central tendency is given by the x-coordinate of the point of intersection of 'more than ogive' and 'less than ogive' ?

(a) Mean (b) Mode

(c) Median (d) Mean and Median both.

Solution. Choice (c) is correct.

The median of a grouped data of central tendency is given by the x-coordinate of the point of intersection of the 'more than ogive' and 'less than ogive'.

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Write 21975 as a product of its prime factors.

Solution. We have

$$\begin{aligned} 21975 &= 5 \times 4395 \\ &= 5 \times 5 \times 879 \\ &= 5 \times 5 \times 3 \times 293 \\ &= \mathbf{3 \times 5^2 \times 293}. \end{aligned}$$

12. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q.

Solution. We divide the given polynomial by $x^2 + 5$.

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x^2+5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\ & \underline{\overset{-}{}x^4 \qquad \overset{-}{+}5x^2} \\ & 2x^3 + 3x^2 + 12x + 18 \\ & \underline{\overset{-}{}2x^3 \qquad \overset{-}{+}10x} \\ & 3x^2 + 2x + 18 \\ & \underline{\overset{-}{}3x^2 \qquad \overset{-}{+}15} \\ & 2x + 3 \end{array}$$

$\left[\text{First term of quotient is } \frac{x^4}{x^2} = x^2\right]$

$\left[\text{Second term of quotient is } \frac{2x^3}{x^2} = 2x\right]$

$\left[\text{Third term of quotient is } \frac{3x^2}{x^2} = 3\right]$

So, $x^4 + 2x^3 + 8x^2 + 12x + 18 = (x^2 + 5)\underbrace{(x^2 + 2x + 3)}_{\text{Quotient}} + \underbrace{(2x + 3)}_{\text{Remainder}}$

But the given remainder is $px + q$. Therefore,

$2x + 3 = px + q$

Comparing both sides, we get : $p = 2$ and $q = 3$.

Hence $\boldsymbol{p = 2}$ and $\boldsymbol{q = 3}$.

13. Solve the following system of equations :

$$\frac{1}{2x} - \frac{1}{y} = -1$$

$$\frac{1}{x} + \frac{1}{2y} = 8, \text{ where } x \neq 0, y \neq 0$$

Solution. The given system of equations can be written as :

$$\frac{1}{x} - \frac{2}{y} = -2 \qquad \ldots(1)$$

[Multiplying both sides of both the given equations by 2]

and $$\frac{2}{x} + \frac{1}{y} = 16 \qquad \ldots(2)$$

Now, multiplying (2) by 2 and adding in (1), we get

$$\left(\frac{1}{x} - \frac{2}{y}\right) + 2\left(\frac{2}{x} + \frac{1}{y}\right) = -2 + 32$$

$$\Rightarrow \quad \frac{1}{x} + \frac{4}{x} = 30$$

$$\Rightarrow \quad \frac{5}{x} = 30$$

$$\Rightarrow \quad x = \frac{5}{30} = \frac{1}{6}$$

Substituting $x = \frac{1}{6}$ in (1), we get

$$6 - \frac{2}{y} = -2$$

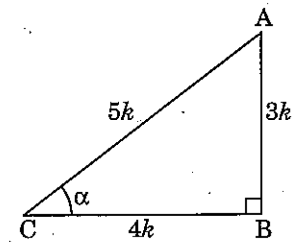
$$\Rightarrow \quad \frac{2}{y} = 6 + 2$$

$$\Rightarrow \quad y = \frac{2}{8} = \frac{1}{4}$$

Thus, the solution of the given system of equations is $\boldsymbol{x} = \frac{\mathbf{1}}{\mathbf{6}}$ and $\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{4}}$.

14. If $\sec \alpha = \frac{5}{4}$, evaluate $\frac{1 - \tan \alpha}{1 + \tan \alpha}$.

Solution. Let us draw a right ΔABC (see figure).



We have : $\sec \alpha = \frac{\text{Hypotenuse}}{\text{Base}}$

$$\Rightarrow \quad \frac{5}{4} = \frac{AC}{BC}$$

$\therefore AC = 5k, BC = 4k$, where k is a positive number
Using the Pythagoras theorem, we have

$$AB^2 = AC^2 - BC^2$$

$$\Rightarrow \quad AB^2 = (5k)^2 - (4k)^2$$

$$\Rightarrow \quad AB^2 = 25k^2 - 16k^2 = 9k^2$$

$$\Rightarrow \quad AB = 3k$$

$$\Rightarrow \quad \tan \alpha = \frac{AB}{BC} = \frac{3k}{4k} = \frac{3}{4}$$

Now, $\frac{1 - \tan \alpha}{1 + \tan \alpha} = \frac{1 - (3/4)}{1 + (3/4)} = \frac{(4 - 3)/4}{(4 + 3)/4} = \frac{4 - 3}{4 + 3} = \frac{\mathbf{1}}{\mathbf{7}}$

Alternative Method :

Given, $\sec \alpha = \frac{5}{4}$

$$\Rightarrow \quad \sec^2 \alpha = \frac{25}{16}$$

$$\Rightarrow \quad 1 + \tan^2 \alpha = \frac{25}{16}$$

$$\Rightarrow \quad \tan^2 \alpha = \frac{25}{16} - 1$$

$$\Rightarrow \quad \tan^2 \alpha = \frac{25 - 16}{16}$$

$$\Rightarrow \quad \tan^2 \alpha = \frac{9}{16} = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \quad \tan \alpha = \frac{3}{4}$$

Now, $$\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1-(3/4)}{1+(3/4)} = \frac{(4-3)/4}{(4+3)/4}$$

$$= \frac{4-3}{4+3} = \frac{\mathbf{1}}{\mathbf{7}}$$

Or

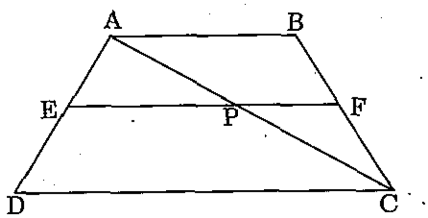
Prove that :

$$\mathbf{(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta.}$$

Solution. We have

$$\begin{aligned} \text{L.H.S.} &= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \\ &= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\cdot\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\cdot\sec\theta) \\ &= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + 2\sin\theta\cdot\frac{1}{\sin\theta} + 2\cos\theta\cdot\frac{1}{\cos\theta} \\ &= 1 + (1+\cot^2\theta) + (1+\tan^2\theta) + 2 + 2 \quad [\because \sec^2\theta = 1+\tan^2\theta;\ \operatorname{cosec}^2\theta = 1+\cot^2\theta] \\ &= (1+1+1+2+2) + \tan^2\theta + \cot^2\theta \\ &= 7 + \tan^2\theta + \cot^2\theta \\ &= \text{R.H.S.} \end{aligned}$$

15. Prove that in the figure if *ABCD* is a trapezium with *AB* || *DC* || *EF*, then $\boldsymbol{\frac{AE}{ED} = \frac{BF}{FC}}$.



Solution. Given : A trapezium *ABCD* in which *AB* || *DC* || *EF*.

To prove : $\frac{AE}{ED} = \frac{BF}{FC}$.

Proof : In ΔADC, we have

$$EP \parallel DC \qquad [\because EF \parallel DC \text{ (given)}]$$

$$\Rightarrow \quad \frac{AE}{ED} = \frac{AP}{PC} \qquad \ldots(1) \text{ [using BPT]}$$

Also, in ΔABC, we have

$$PF \parallel AB \qquad [\because EF \parallel AB \text{ (given)}]$$

$$\Rightarrow \quad \frac{AP}{PC} = \frac{BF}{FC} \qquad \ldots(2) \text{ [using BPT]}$$

From (1) and (2), we have

$$\boldsymbol{\frac{AE}{ED} = \frac{BF}{FC}}$$

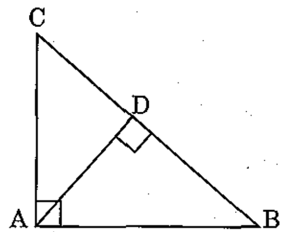
Hence, proved.

16. In ΔABC, $\angle A = 90°$ and $AD \perp BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

Solution. Let ABC be a triangle such that $\angle A = 90°$ and $AD \perp BC$.

We have to prove that

$$AB^2 + CD^2 = BD^2 + AC^2$$



In right ΔABD, we have

$$AB^2 = AD^2 + BD^2 \quad \ldots(1) \text{ [using Pythagoras Theorem]}$$

In right ΔACD, we have

$$AC^2 = AD^2 + CD^2 \quad \ldots(2) \text{ [using Pythagoras Theorem]}$$

Now, $AB^2 - AC^2 = (AD^2 + BD^2) - (AD^2 + CD^2)$ [using (1) and (2)]

$$= BD^2 - CD^2$$

$\Rightarrow \quad AB^2 + CD^2 = BD^2 + AC^2$.

17. The following distribution gives the scores of 230 students of a school :

Scores	***No. of students***
400 – 450	**20**
450 – 500	**35**
500 – 550	**40**
550 – 600	**32**
600 – 650	**24**
650 – 700	**27**
700 – 750	**18**
750 – 800	**34**
Total	**230**

Write the above distribution as more than type cumulative frequency distribution.

Solution. The cumulative frequency table by more than type method :

Scores	*No. of students*	*Score more than*	*Cumulative frequency*
400 – 450	20	400	230
450 – 500	35	450	210
500 – 550	40	500	175
550 – 600	32	550	135
600 – 650	24	600	103
650 – 700	27	650	79
700 – 750	18	700	52
750 – 800	34	750	34

We calculate more than cumulative frequencies by adding successive classes from bottom to top.

18. For the following grouped frequency distribution find the mode :

Class	**3 – 6**	**6 – 9**	**9 – 12**	**12 – 15**	**15 – 18**	**18 – 21**	**21 – 24**
Frequency	**2**	**5**	**10**	**23**	**21**	**12**	**3**

Solution. Here the maximum frequency is 23 and the class corresponding to this frequency is 12 – 15.

So, the modal class is 12 – 15.
Lower limit (l) of the modal class = 12.
Class size (h) = 15 – 12 = 3
Frequency (f_1) of the modal class = 23
Frequency (f_0) of the class preceeding the modal class = 10
Frequency (f_2) of the class following the modal class = 21
Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{23 - 10}{2 \times 23 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{46 - 31} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + \frac{13}{5}$$

$$= 12 + 2.6 = \mathbf{14.6}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Show that any positive odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$, where q is a positive integer.

Solution. Let a be any positive integer, when a is divided by 6. Then, by Euclid's division lemna, we get

$$a = 6q + r,\ 0 \le r < 6, \text{ where } q \text{ and } r \text{ are whole numbers.}$$

So, $r = 0, 1, 2, 3, 4, 5$.

When $r = 0$ or 2 or 4, then a becomes an even integer, *i.e.*, $a = 6q$ or $6q + 2$ or $6q + 4$.

When $r = 1$ or 3 or 5, then a becomes a positive odd integer, *i.e.*, $a = 6q + 1$ or $6q + 3$ or $6q + 5$.

Hence any positive odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$, where q is a positive integer.

20. Prove that $\sqrt{2} + \sqrt{5}$ is irrational.

Solution. Let us assume, to the contrary, that $\sqrt{2} + \sqrt{5}$ is a rational number.

That is we can find coprime p and q ($q \neq 0$) such that

$$\sqrt{2} + \sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{p}{q} - \sqrt{2} = \sqrt{5}$$

$$\Rightarrow \quad \left(\frac{p}{q} - \sqrt{2}\right)^2 = \left(\sqrt{5}\right)^2 \qquad \text{[Squaring both sides]}$$

$$\Rightarrow \quad \frac{p^2}{q^2} - 2\frac{p}{q}\cdot\sqrt{2} + 2 = 5$$

$$\Rightarrow \quad \frac{p^2}{q^2} - 3 = \frac{2p}{q}\cdot\sqrt{2}$$

$$\Rightarrow \quad \frac{p^2 - 3q^2}{q^2} = \frac{2p}{q}\cdot\sqrt{2}$$

$$\Rightarrow \quad \frac{p^2 - 3q^2}{2pq} = \sqrt{2}$$

Since, p and q are integers, $\frac{p^2 - 3q^2}{2pq}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $\sqrt{2} + \sqrt{5}$ is **irrational**.

Or

Prove that $\frac{5\sqrt{2}}{7}$ is irrational.

Solution. Let us assume to the contrary, that $\frac{5\sqrt{2}}{7}$ is a rational number.

$\therefore \quad \frac{5\sqrt{2}}{7} = \frac{p}{q}$, where p and q are coprime, *i.e.*, their HCF is 1.

$$\Rightarrow \quad \sqrt{2} = \frac{7p}{5q}$$

Since $\frac{p}{q}$ is a rational number, therefore, $\frac{7}{5} = \frac{p}{q}$ is also rational number.

$\Rightarrow \quad \sqrt{2}$ is a rational number.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arises because of our incorrect assumption that $\frac{5\sqrt{2}}{7}$ is rational.

So, we conclude that $\frac{5\sqrt{2}}{7}$ **is irrational.**

21. A two digit number is four times the sum of its digits and twice the product of the digits. Find the number.

Solution. Let the unit place digit be x and ten's place digit be y.

Then, Original number $= 10y + x$

It is given that a two digit number is four times the sum of its digits.

$\therefore \quad 10y + x = 4(y + x)$

$\Rightarrow \quad 10y - 4y = 4x - x$

$\Rightarrow \quad 6y = 3x$

$\Rightarrow \quad x = 2y$ ...(1)

Further, it is also given that a two digit number is twice the product of the digits.

$10y + x = 2(xy)$

$\Rightarrow \quad 10y + 2y = 2[(2y)y]$ [using (1)]

$\Rightarrow \quad 12y = 4y^2$

$\Rightarrow \quad 3 = y$ [$\because y \neq 0$]

Substituting $y = 3$ in (1), we get

$x = 2(3) = 6$

Hence, the original number is **36.**

Or

Solve for x and y :

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$ax - by = a^2 - b^2$$

Solution. The given system of equations can be written as

$$\frac{x}{a} + \frac{y}{b} = 2 \Rightarrow \frac{bx + ay}{ab} = 2 \Rightarrow bx + ay = 2ab \quad ...(1)$$

and $\quad ax - by = a^2 - b^2$ ...(2)

Multiplying (1) by b and (2) by a, we have

$b^2x + aby = 2ab^2$ ...(3)

and $\quad a^2x - aby = a^3 - ab^2$ ...(4)

Adding (3) and (4), we get

$b^2x + a^2x = 2ab^2 + a^3 - ab^2$

$\Rightarrow \quad (b^2 + a^2)x = ab^2 + a^3$

$\Rightarrow \quad (b^2 + a^2)x = a(b^2 + a^2)$

$$\Rightarrow \quad x = \frac{a(b^2 + a^2)}{(b^2 + a^2)}$$

$\Rightarrow \quad x = a$

Substituting $x = a$ in (2), we get

$a^2 - by = a^2 - b^2$

$\Rightarrow \quad by = a^2 - a^2 + b^2$

$\Rightarrow \quad by = b^2$

$\Rightarrow \quad y = b$

Hence, $\boldsymbol{x = a, y = b}$ is the solution of the given system of equations.

22. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find the quadratic polynomial whose zeroes are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Solution. Since α and β are zeroes of the polynomial $2x^2 - 5x + 7$, therefore

$$\alpha + \beta = -\left(\frac{-5}{2}\right) = \frac{5}{2}$$

and $$\alpha\beta = \frac{7}{2}$$

Let S and P denote respectively the sum and product of zeroes of the required polynomial, then

$$S = (2\alpha + 3\beta) + (3\alpha + 2\beta)$$

$$\Rightarrow \quad S = 5\alpha + 5\beta = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$$

and $\quad P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$$\Rightarrow \quad P = 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2$$

$$\Rightarrow \quad P = 6\alpha^2 + 6\beta^2 + 13\alpha\beta$$

$$\Rightarrow \quad P = 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta$$

$$\Rightarrow \quad P = 6(\alpha + \beta)^2 + \alpha\beta$$

$$\Rightarrow \quad P = 6\left(\frac{5}{2}\right)^2 + \frac{7}{2}$$

$$\Rightarrow \quad P = \frac{75}{2} + \frac{7}{2} = \frac{82}{2} = 41$$

Hence, the required polynomial $p(x)$ is given by :

$$p(x) = k(x^2 - Sx + P)$$

or $\quad p(x) = k\left[x^2 - \frac{25}{2}x + 41\right]$

or $\quad p(x) = \mathbf{k(2x^2 - 25x + 82)}$, where k is any non-zero real number.

23. Prove that : $\dfrac{\mathbf{1 + \cos A}}{\mathbf{\sin A}} + \dfrac{\mathbf{\sin A}}{\mathbf{1 + \cos A}} = \mathbf{2 \operatorname{cosec} A}$

Solution. We have

$$\text{L.H.S.} = \frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A}$$

$$= \frac{(1 + \cos A)^2 + \sin^2 A}{\sin A\,(1 + \cos A)}$$

$$= \frac{1 + 2\cos A + \cos^2 A + \sin^2 A}{\sin A\,(1 + \cos A)}$$

$$= \frac{1 + 2\cos A + 1}{\sin A\,(1 + \cos A)} \qquad [\because \cos^2 A + \sin^2 A = 1]$$

$$= \frac{2 + 2\cos A}{\sin A\,(1 + \cos A)}$$

$$= \frac{2\,(1 + \cos A)}{\sin A\,(1 + \cos A)}$$

$$= \frac{2}{\sin A}$$

$$= 2 \operatorname{cosec} A$$

$$= \text{R.H.S.}$$

24. Prove that

$$\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

Solution. We have

$$\text{R.H.S.} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

$$= 2 + \frac{\sin\theta(\cot\theta + \operatorname{cosec}\theta)}{(\cot\theta - \operatorname{cosec}\theta)(\cot\theta + \operatorname{cosec}\theta)}$$

$$= 2 + \frac{\sin\theta(\cot\theta + \operatorname{cosec}\theta)}{\cot^2\theta - \operatorname{cosec}^2\theta}$$

$$= 2 + \frac{\sin\theta\cot\theta + \sin\theta\operatorname{cosec}\theta}{\cot^2\theta - (1 + \cot^2\theta)} \qquad [\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta]$$

$$= 2 + \frac{\sin\theta\cot\theta + \sin\theta\operatorname{cosec}\theta}{-1}$$

$$= \frac{-2 + \sin\theta\cot\theta + 1}{-1} \qquad [\because \sin\theta.\operatorname{cosec}\theta = 1]$$

$$= \frac{-1 + \sin\theta\cot\theta}{-1}$$

$$= \frac{1 - \sin\theta\cot\theta}{1}$$

$$= \frac{1 - \sin\theta\cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{\sin\theta\operatorname{cosec}\theta - \sin\theta\cot\theta}{(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}$$

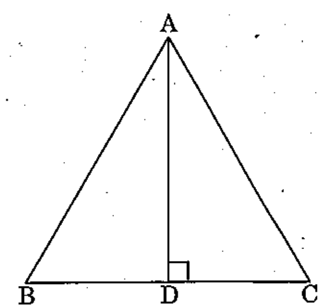
$$= \frac{\sin\theta(\operatorname{cosec}\theta - \cot\theta)}{(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}$$

$$= \frac{\sin\theta}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \text{L.H.S.}$$

25. If ABC is an equilateral triangle with $AD \perp BC$, then prove that $AD^2 = 3DC^2$.

Solution. Let ABC be an equilateral triangle and $AD \perp BC$.



In ΔADB and ΔADC, we have

$AB = AC$ [given]

$\angle B = \angle C$ [Each = 60°]

and $\angle ADB = \angle ADC$ [Each = 90°]

$\therefore$ $\Delta ADB \cong \Delta ADC$

$\Rightarrow$ $BD = DC$ ...(1)

$\therefore$ $BC = BD + DC = DC + DC = 2DC$ ...(2) [using (1)]

In right angled ΔADC, we have

$$AC^2 = AD^2 + DC^2$$

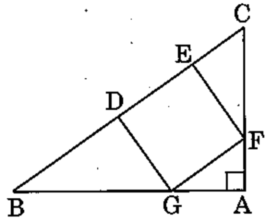
$\Rightarrow$ $BC^2 = AD^2 + DC^2$ [$\because$ $AC = BC$, sides of an equilateral Δ]

$\Rightarrow$ $(2DC)^2 = AD^2 + DC^2$ [using (2)]

$\Rightarrow$ $AD^2 = 4DC^2 - DC^2$

$\Rightarrow$ $\boldsymbol{AD^2 = 3DC^2}$

26. In figure, *DEFG* is a square and $\angle BAC = 90°$. Show that $DE^2 = BD \times EC$.



Solution. In ΔAGF and ΔDBG, we have

$\angle GAF = \angle BDG$ [Each = 90°]

$\angle AGF = \angle DBG$ [Corresponding $\angle s$]

So, by AA-criterion of similarity of triangles, we have

$\Delta AGF \sim \Delta DBG$ ...(1)

In ΔAGF and ΔEFC, we have

$\angle FAG = \angle CEF$ [Each = 90°]

$\angle AFG = \angle ECF$ [Corresponding $\angle s$]

So, by AA-criterion of similarity of triangles, we have

$\Delta AGF \sim \Delta EFC$ ...(2)

From (1) and (2), we obtain

$\Delta DBG \sim \Delta EFC$

$\Rightarrow$ $\dfrac{BD}{FE} = \dfrac{DG}{EC}$

$\Rightarrow$ $\dfrac{BD}{DE} = \dfrac{DE}{EC}$ [$\because$ *DEFG* is a square. $\therefore EF = DE, DG = DE$]

$\Rightarrow$ $\boldsymbol{DE^2 = BD \times EC}$.

27. Find the mean of the following distribution, using step-deviation method :

Classes	*Number of students*
4 – 8	2
8 – 12	12
12 – 16	15
16 – 20	25
20 – 24	18
24 – 28	12
28 – 32	13
32 – 36	3

Solution. Let the assumed mean be $a = 18$ and $h = 4$.

Calculation of Mean

Classes	*Number of students* (f)	*Class-mark* x_i	$d_i = x_i - 18$	$u_i = \frac{x_i - 18}{4}$	$f_i u_i$
4 – 8	2	6	– 12	– 3	– 6
8 – 12	12	10	– 8	– 2	– 24
12 – 16	15	14	– 4	– 1	– 15
16 – 20	25	18	0	0	0
20 – 24	18	22	4	1	18
24 – 28	12	26	8	2	24
28 – 32	13	30	12	3	39
32 – 36	3	34	16	4	12
Total	$n = \Sigma f_i = 100$				$\Sigma f_i u_i = 48$

From the table, $n = \Sigma f_i = 100$ and $\Sigma f_i u_i = 48$

Using the formula :

$$\text{Mean } (\overline{X}) = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad \overline{X} = 18 + \frac{48}{100} \times 4$$

$$\Rightarrow \quad \overline{X} = 18 + \frac{192}{100}$$

$$\Rightarrow \quad \overline{X} = 18 + 1.92$$

$$\Rightarrow \quad \overline{X} = 19.92$$

Hence, mean is **19.92.**

Or

The mean of the following distribution is 196.8. Find the value of *p*.

Classes	**0 – 80**	**80 – 160**	**160 – 240**	**240 – 320**	**320 – 400**
Frequency	**22**	**35**	**44**	***p***	**24**

Solution. **Calculation of Mean**

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	$f_i x_i$
0 – 80	22	40	880
80 – 160	35	120	4200
160 – 240	44	200	8800
240 – 320	p	280	$280p$
320 – 400	24	360	8640
Total	$n = \Sigma f_i = 125 + p$		$\Sigma f_i x_i = 22520 + 280p$

From the table, $n = \Sigma f_i = 125 + p$, $\Sigma f_i x_i = 22520 + 280p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 196.8 \text{ (given)} = \frac{22520 + 280p}{125 + p}$$

$$\Rightarrow \quad 196.8(125 + p) = 22520 + 280p$$

$$\Rightarrow \quad 24600 + 196.8p = 22520 + 280p$$

$$\Rightarrow \quad 280p - 196.8p = 24600 - 22520$$

$$\Rightarrow \quad 83.2p = 2080$$

$$\Rightarrow \quad p = \frac{2080}{83.2}$$

$$\Rightarrow \quad p = 25$$

Hence, the value of p is **25**.

28. Find the median of the following distribution :

Marks	***Frequency***
0 – 100	**2**
100 – 200	**5**
200 – 300	**9**
300 – 400	**12**
400 – 500	**17**
500 – 600	**20**
600 – 700	**15**
700 – 800	**9**
800 – 900	**7**
900 – 1000	**4**

Solution. **Calculation of Median**

Marks	*Frequency* (f)	*Cumulative frequency* (cf)
0 – 100	2	2
100 – 200	5	7
200 – 300	9	16
300 – 400	12	28
400 – 500	17	45 (cf)
500 – 600	20 (f)	65
600 – 700	15	80
700 – 800	9	89
800 – 900	7	96
900 – 1000	4	100
Total	$n = \Sigma f_i = 100$	

Here, $n = 100$ and $\frac{n}{2} = \frac{100}{2} = 50$.

Now, 500 – 600 is the class whose cumulative frequency is 65 is greater than $\frac{n}{2} = 50$.

From the table, $f = 20$, $cf = 45$, $h = 100$

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 500 + \frac{50 - 45}{20} \times 100$$

$$= 500 + \frac{500}{20}$$

$$= 500 + 25 = \mathbf{525}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

Solution. Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$, therefore $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 2$.

$$\begin{array}{r|l} & x^2 + 3x - 18 \\ x^2 - 2 & x^4 + 3x^3 - 20x^2 - 6x + 36 \\ & \underline{\overset{-}{x^4} \qquad \overset{+}{-} 2x^2} \\ & 3x^3 - 18x^2 - 6x + 36 \\ & \underline{\overset{-}{3x^3} \qquad \quad 6x} \\ & -18x^2 + 36 \\ & \underline{-18x^2 + 36} \\ & 0 \end{array}$$

[First term of quotient is $\frac{x^4}{x^2} = x^2$]

[Second term of quotient is $\frac{3x^3}{x^2} = 3x$]

[Third term of quotient is $\frac{-18x^2}{x^2} = -18$]

So, $x^4 + 3x^3 - 20x^2 - 6x + 36 = (x^2 - 2)(x^2 + 3x - 18)$

$$= (x - \sqrt{2})(x + \sqrt{2})[x^2 + 6x - 3x - 18]$$

$$= (x - \sqrt{2})(x + \sqrt{2})[x(x + 6) - 3(x + 6)]$$

$$= (x - \sqrt{2})(x + \sqrt{2})(x + 6)(x - 3)$$

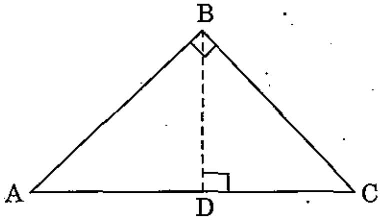
So, the zeroes of $x^2 + 3x - 18 = (x + 6)(x - 3)$ are given by $x = -6$ and $x = 3$.

Hence, the other zeroes of the given polynomial are **– 6** and **3**.

30. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution. Given : A right triangle ABC, right angled at B.

To prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$



i.e., $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]

So, $\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

$\Rightarrow AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\frac{CD}{BC} = \frac{BC}{AC}$ [Sides are proportional]

$\Rightarrow CD.AC = BC^2$ ...(2)

Adding (1) and (2), we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$\Rightarrow AC.AC = AB^2 + BC^2$

Hence, $\mathbf{AC^2 = AB^2 + BC^2}$

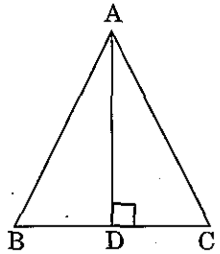
Or

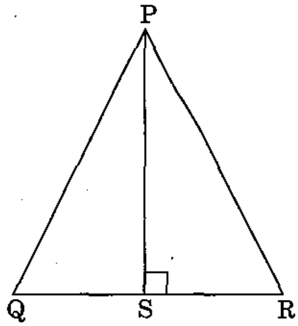
Prove that the ratio of areas of two similar triangles is equal to the square of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.





Proof : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\frac{AB}{PQ} = \frac{BC}{QR}$ [$\because \Delta ABC \sim \Delta PQR$]

$\Rightarrow \frac{AD}{PS} = \frac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2} \quad ...(4)$$

As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad ...(5)$$

Hence, $\boldsymbol{\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}}$. [From (4) and (5)]

31. If $x = \tan A + \sin A$ and $y = \tan A - \sin A$, show that

$$x^2 - y^2 = 4\sqrt{xy}$$

Solution. Given, $\tan A + \sin A = x$ ...(1)

and $\tan A - \sin A = y$ ...(2)

Adding (1) and (2), we get

$$2 \tan A = x + y \quad ...(3)$$

Subtracting (2) from (1), we get

$$2 \sin A = x - y \quad ...(4)$$

Multiplying (3) and (4), we get

$$(x + y)(x - y) = (2 \tan A)(2 \sin A)$$

$$\Rightarrow x^2 - y^2 = 4 \tan A \sin A \quad ...(5)$$

Now, $4\sqrt{xy} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$ [using (1) and (2)]

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\sin^2 A\left(\frac{1}{\cos^2 A} - 1\right)}$$

$$= 4\sqrt{\sin^2 A\left(\frac{1-\cos^2 A}{\cos^2 A}\right)}$$

$$= 4\sqrt{\frac{\sin^2 A.\sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A\cdot\sin A}{\cos A}$$

$$= 4\left(\frac{\sin A}{\cos A}\right).\sin A$$

$$= 4\tan A\sin A \qquad \ldots(6)$$

From (5) and (6), we obtain

$$x^2 - y^2 = 4\tan A\sin A = 4\sqrt{xy}$$

$$\Rightarrow \qquad \boldsymbol{x^2 - y^2 = 4\sqrt{xy}}$$

Or

Evaluate :

$$\frac{2}{3}\textbf{cosec}^2\,\textbf{58°} - \frac{2}{3}\textbf{cot 58° tan 32°} - \frac{5}{3}\textbf{tan 13° tan 37° tan 45° tan 53° tan 77°}$$

Solution. We have

$$\frac{2}{3}\text{cosec}^2\,58° - \frac{2}{3}\cot 58°\tan 32° - \frac{5}{3}\tan 13°\tan 37°\tan 45°\tan 53°\tan 77°$$

$$= \frac{2}{3}\text{cosec}^2\,58° - \frac{2}{3}\cot 58°\tan(90° - 58°)$$

$$- \frac{5}{3}\tan 13°\tan 37°\tan 45°\tan(90° - 37°)\tan(90° - 13°)$$

$$= \frac{2}{3}\text{cosec}^2\,58° - \frac{2}{3}\cot 58°\cot 58° - \frac{5}{3}\tan 13°\tan 37°\tan 45°\cot 37°\cot 13°$$

$[\because \tan(90° - \theta) = \cot\theta]$

$$= \frac{2}{3}\text{cosec}^2\,58° - \frac{2}{3}\cot^2 58° - \frac{5}{3}(\tan 13°\cot 13°)\tan 45°(\tan 37°\cot 37°)$$

$$= \frac{2}{3}(1 + \cot^2 58°) - \frac{2}{3}\cot^2 58° - \frac{5}{3}(1)\tan 45°(1) \qquad [\because \text{cosec}^2\theta = 1 + \cot^2\theta \text{ and } \tan\theta\cot\theta = 1]$$

$$= \frac{2}{3} + \frac{2}{3}\cot^2 58° - \frac{2}{3}\cot^2 58° - \frac{5}{3}(1) \qquad [\because \tan 45° = 1]$$

$$= \frac{2}{3} - \frac{5}{3}$$

$$= \frac{-3}{3}$$

$$= -\mathbf{1}.$$

32. Prove that :

$$\sin\theta\,(1+\tan\theta) + \cos\theta\,(1+\cot\theta) = \sec\theta + \operatorname{cosec}\theta.$$

Solution. We have

$$\begin{aligned}
\text{L.H.S.} &= \sin\theta\,(1+\tan\theta) + \cos\theta\,(1+\cot\theta) \\
&= (\sin\theta + \sin\theta\tan\theta) + (\cos\theta + \cos\theta\cot\theta) \\
&= \left(\sin\theta + \sin\theta\frac{\sin\theta}{\cos\theta}\right) + \left(\cos\theta + \cos\theta\frac{\cos\theta}{\sin\theta}\right) \\
&= (\sin\theta + \cos\theta) + \left(\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}\right) \\
&= (\sin\theta + \cos\theta) + \left(\frac{\sin^3\theta + \cos^3\theta}{\sin\theta\cos\theta}\right) \\
&= (\sin\theta + \cos\theta) + \frac{(\sin\theta+\cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{\sin\theta\cos\theta}
\end{aligned}$$

$[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$$\begin{aligned}
&= (\sin\theta + \cos\theta)\left[1 + \frac{\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta}\right] \\
&= (\sin\theta + \cos\theta)\left[1 + \frac{1 - \sin\theta\cos\theta}{\sin\theta\cos\theta}\right] \\
&= (\sin\theta + \cos\theta)\left[\frac{\sin\theta\cos\theta + 1 - \sin\theta\cos\theta}{\sin\theta\cos\theta}\right] \\
&= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\,(1) \\
&= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta} \\
&= \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \\
&= \sec\theta + \operatorname{cosec}\theta \\
&= \text{R.H.S.}
\end{aligned}$$

33. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, he buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.

Solution. Let the price of a bat be ₹ x and that of a ball be ₹ y.

It is given that the coach buys 3 bats and 6 balls for ₹ 3900.

$\therefore \quad 3x + 6y = 3900$

Also, it is given that the coach buys another bat and 3 more balls for ₹ 1300.

$\therefore \quad x + 3y = 1300$

The **algebraic representation** of the given conditions is

$$3x + 6y = 3900 \quad ...(1)$$

$$x + 3y = 1300 \quad ...(2)$$

Geometrical representation : Let us draw the graphs of the equations (1) and (2). For this, we find two solutions of each of the equations which are given in tables.

$3x + 6y = 3900$

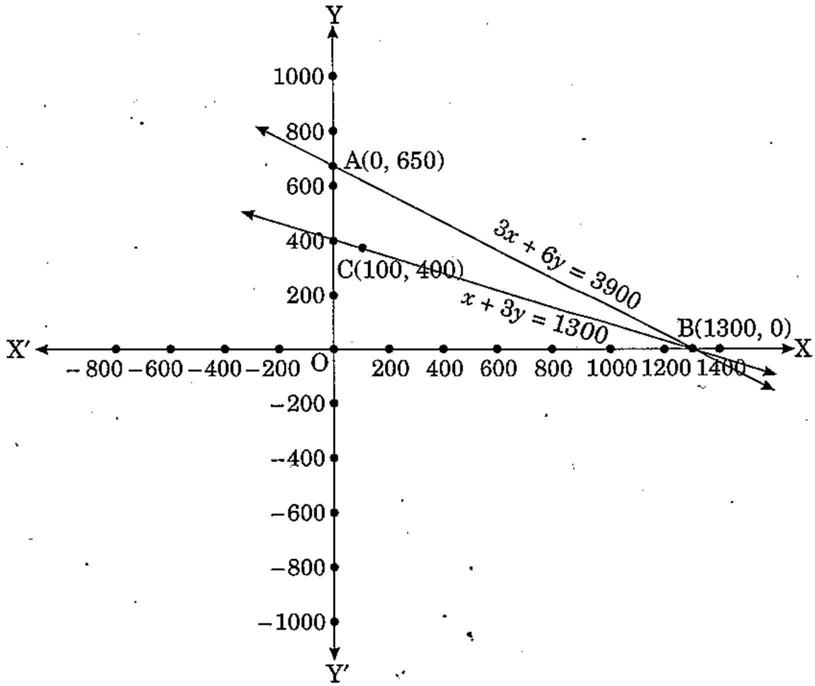
x	0	1300
y	650	0

$x + 3y = 1300$

x	100	1300
y	400	0

Plot the points $A(0, 650)$, $B(1300, 0)$, $C(100, 400)$ and $B(1300, 0)$ on graph paper and join the points to form the lines AB and BC as shown in figure.

The two lines (1) and (2) intersect at the point $B(1300, 0)$. So, $\boldsymbol{x = 1300, y = 0}$ is the required solution of the pair of linear equations.



34. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length **(*in* mm)**	***Number of leaves***
118 – 126	**3**
127 – 135	**5**
136 – 144	**9**
145 – 153	**12**
154 – 162	**5**
163 – 171	**4**
172 – 180	**2**

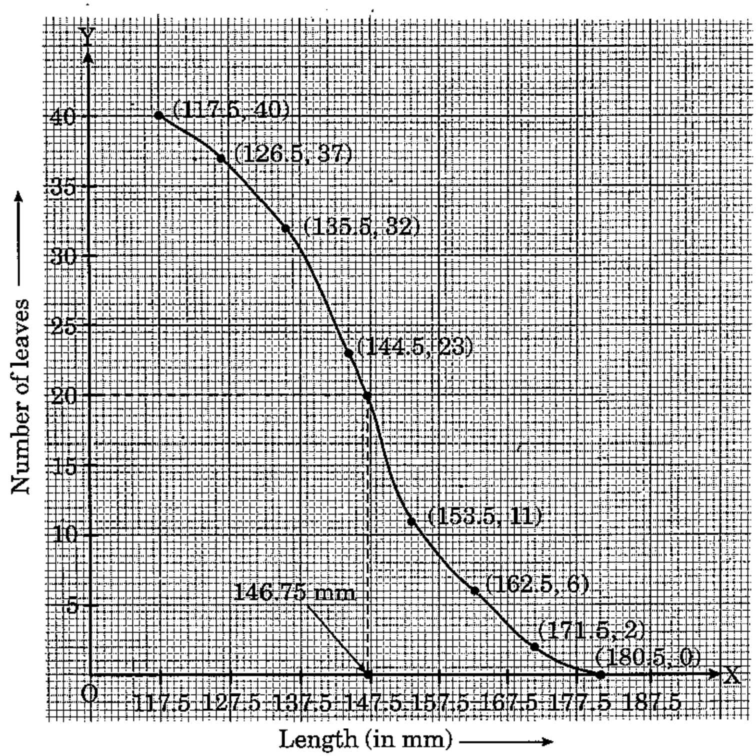
Draw a more than type ogive for the given data. Hence, obtain the median length of the leaves from the graph and verify the result by using the formula.

Solution. The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency table by more than type given below :

Length (*in* mm)	*Number of leaves* [*Frequency* (*f*)]	*Length more than*	*Cumulative frequency* (*cf*)
117.5 – 126.5	3	117.5	40
126.5 – 135.5	5	126.5	37
135.5 – 144.5	9	135.5	32
144.5 – 153.5	12	144.5	23
153.5 – 162.5	5	153.5	11
162.5 – 171.5	4	162.5	6
171.5 – 180.5	2	171.5	2

Other than the given class-interval, we assume that the class-interval 180.5 – 189.5 with zero frequency. Now, we mark the lower class limits on x-axis and the cumulative frequencies along y-axis on suitable scales to plot the points (117.5, 40), (126.5, 37), (135.5, 32), (144.5, 23), (153.5, 11), (162.5, 6), (171.5, 2). We join these points with a smooth curve to get the **"more than"** ogive as shown in the figure.

Locate $\frac{n}{2} = \frac{40}{2} = 20$ on the y-axis (see figure).



From this point, draw a line parallel to the x-axis cutting the curve at a point. From this point, draw a perpendicular to the x-axis. The point of intersection of this perpendicular with the x-axis determine the median length of the data (see figure), *i.e.*, median length is **146.75 mm.**

To calculate the median length, we need the frequency distribution table with the given cumulative frequency.

Table

Class interval	*Frequency (f)*	*Cumulative frequency (cf)*
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40

Now, 144.5 – 153.5 is the class whose cumulative frequency is 29 is greater than $\frac{n}{2} = \frac{40}{2} = 20$

$\therefore$ 144.5 – 153.5 is the median class.

From the table : $f = 12$, $cf = 17$, $h = 9$

Using the formula :

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 144.5 + \frac{(20 - 17)}{12} \times 9$$

$$= 144.5 + \frac{27}{12}$$

$$= 144.50 + 2.25$$

$$= 146.75 \text{ mm}$$

So, about half the leaves have length less than **146.75 mm** and the other half have length more than **146.75 mm.**