CCE SAMPLE QUESTION PAPER

FIRST TERM (SA-I)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The decimal expansion of π is

(*a*) terminating.

(*b*) non-terminating and non-recurring.

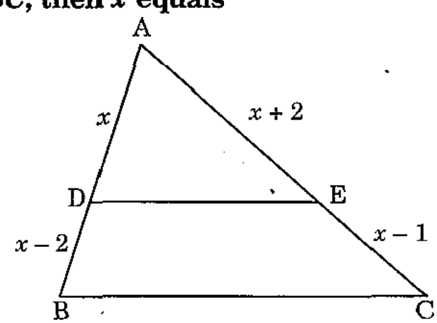
(*c*) non-terminating and recurring.

(*d*) does not exist.

Solution. Choice (*c*) is correct.

$$\pi = \frac{22}{7} = 3.1428571428571 \text{.....} = 3.\overline{142857}$$

2. In figure, if *DE* ∥ *BC*, then *x* equals



(*a*) 4 cm (*b*) 3 cm

(*c*) 5 cm (*d*) 6 cm

Solution. Choice (*a*) is correct.

Since, $DE \parallel BC$, therefore by BPT

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$$

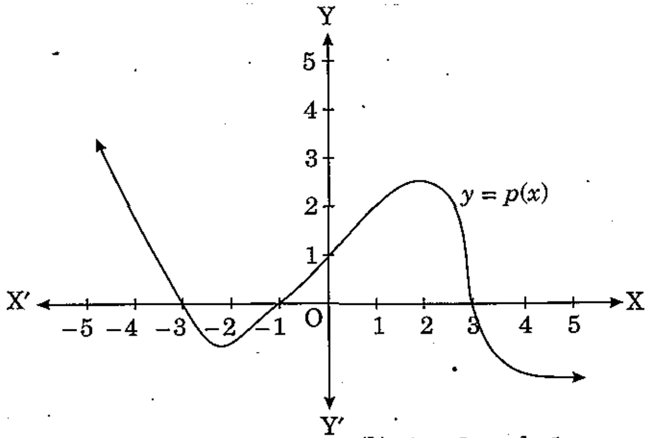
$$\Rightarrow \quad x(x-1) = (x+2)(x-2)$$

$$\Rightarrow \quad x^2 - x = x^2 - 4$$

$$\Rightarrow \quad -x = -4$$

$$\Rightarrow \quad x = 4 \text{ cm.}$$

3. In figure, the graph of some polynomial $p(x)$ is given. The zeroes of the polynomial $p(x)$ are



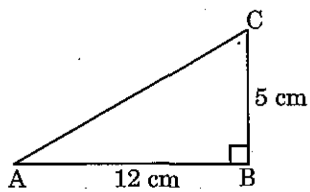
(*a*) 1, 3 and –1 (*b*) 1, – 3 and –1

(*c*) –1, – 3 and 3 (*d*) – 3, 1 and 3

Solution. Choice (*c*) is correct.

From the graph, we can see that – 3, – 1 and 3 are zeroes of the given polynomial as the graph intersects the *x*-axis at three points, *viz.*, at point (– 3, 0), (–1, 0) and (3, 0).

4. In figure, *ABC* is a right angle triangle, right angled at *B*, *AB* = 12 cm and *BC* = 5 cm, then the value of tan *A* + cot *C* is



(*a*) $\frac{6}{5}$ (*b*) $\frac{5}{6}$

(*c*) $\frac{12}{5}$ (*d*) $\frac{13}{5}$

Solution. Choice (*b*) is correct.

$$\tan A + \cot C = \frac{BC}{AB} + \frac{BC}{AB}$$

$$= \frac{5}{12} + \frac{5}{12}$$

$$= \frac{10}{12} = \frac{5}{6}$$

5. If $\sin\theta = \frac{a}{b}$, then in terms of a and b, the value of $\sec\theta + \tan\theta$ is

(a) $\sqrt{\frac{a+b}{a-b}}$ (b) $\frac{a+b}{a-b}$

(c) $\sqrt{\frac{b+a}{b-a}}$ (d) $\frac{b+a}{b-a}$

Solution. Choice (c) is correct.

$$\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1+\frac{a}{b}}{\sqrt{1-\frac{a^2}{b^2}}}$$

$$= \frac{(b+a)/b}{\sqrt{b^2-a^2}/b}$$

$$= \frac{b+a}{\sqrt{b^2-a^2}}$$

$$= \frac{b+a}{\sqrt{(b+a)(b-a)}}$$

$$= \frac{\sqrt{b+a}}{\sqrt{b-a}} = \sqrt{\frac{\boldsymbol{b+a}}{\boldsymbol{b-a}}}$$

6. The value of $\cos 0° \cos 1° \cos 2° \ldots\ldots\ldots\ldots \cos 179° \cos 180°$ is

(a) 1 (b) -1

(c) 0 (d) None of these

Solution. Choice (c) is correct.

$\cos 0° \cos 1° \cos 2° \ldots\ldots\ldots\ldots \cos 89° \cos 90° \cos 91° \ldots\ldots\ldots\ldots \cos 179° \cos 180°$

$= \cos 0° \cos 1° \cos 2° \ldots\ldots\ldots\ldots \cos 89° (0) \cos (91°) \ldots\ldots\ldots\ldots \cos 180°$

$= \mathbf{0}$

7. The prime factorization of a natural number n is $2^3 \times 3^2 \times 5^2 \times 7$, then the number of consecutive zeroes in n are

(a) 2 (b) 3

(c) 4 (d) 5

Solution. Choice (a) is correct.

$$\begin{aligned} n &= 2^3 \times 3^2 \times 5^2 \times 7 \\ &= 2 \times 3^2 \times 7 \times (2^2 \times 5^2) \\ &= 2 \times 9 \times 7 \times (10)^2 \\ &= 12600 \end{aligned}$$

It clearly shows that number of consecutive zeroes in n are **2.**

8. If the lines represented by the pair of linear equations $4x + ky - 8 = 0$ and $3x - 5y + 7 = 0$ are parallel, then the value of k is

(a) $-\frac{3}{20}$ (b) $-\frac{20}{3}$

(c) $-\frac{10}{3}$ (d) $\frac{20}{3}$

Solution. Choice (b) is correct.

Since the lines represented by the given pair of linear equations are parallel, therefore

$$\frac{4}{3} = \frac{k}{-5} \neq \frac{-8}{7}$$

$$\Rightarrow \quad k = -\frac{\mathbf{20}}{\mathbf{3}}$$

9. If $\tan A = \cot B$, then $A + B$ is equal to

(a) 60° (b) 90°

(c) 75° (d) 85°

Solution. Choice (b) is correct.

$$\tan A = \cot B$$

$$\Rightarrow \quad \tan A = \tan (90° - B)$$

$$\Rightarrow \quad A = 90° - B$$

$$\Rightarrow \quad A + B = \mathbf{90°}$$

10. The median and mean of a unimodal statistical data are 42.4 and 40.6 respectively, then its mode is

(a) 45.8 (b) 46

(c) 46.3 (d) 45.3

Solution. Choice (b) is correct.

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$\Rightarrow \quad \text{Mode} = 3 \times 42.4 - 2 \times 40.6$$

$$\Rightarrow \quad \text{Mode} = 127.2 - 81.2$$

$$\Rightarrow \quad \text{Mode} = \mathbf{46}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Explain why $7 \times 11 \times 13 \times 17 + 17$ is a composite number.

Solution. We have

$$7 \times 11 \times 13 \times 17 + 17$$

$$= (7 \times 11 \times 13 + 1) \times 17$$

$= (77 \times 13 + 1) \times 17$
$= (1001 + 1) \times 17$
$= (1002) \times 17$
$= 2 \times 501 \times 17$
$= 2 \times 3 \times 167 \times 17$
$= 2 \times 3 \times 17 \times 167$

$\Rightarrow$ $7 \times 11 \times 13 \times 17 + 17 = 2 \times 3 \times 17 \times 167$ is a **composite number** as powers of prime occur.

12. If 2 is a zero of both the polynomials $3x^2 + ax - 14$ and $2x^3 + bx^2 + x - 2$, find the value of $a - 2b$.

Solution. Let $f(x) = 3x^2 + ax - 14$ and $g(x) = 2x^3 + bx^2 + x - 2$

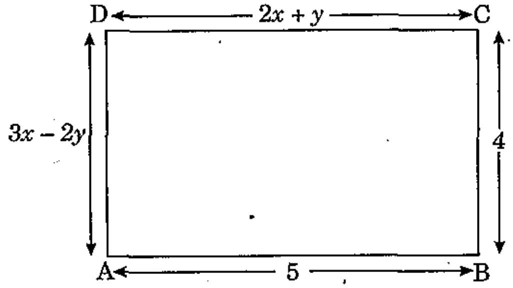
Since 2 is a zero of both polynomials $f(x)$ and $g(x)$, therefore

$f(2) = 0 \Rightarrow 3(2)^2 + a(2) - 14 = 0 \Rightarrow 12 + 2a - 14 = 0 \Rightarrow 2a = 2 \Rightarrow a = 1$

and $g(2) = 0 \Rightarrow 2(2)^3 + b(2)^2 + 2 - 2 = 0 \Rightarrow 16 + 4b = 0 \Rightarrow 4b = -16 \Rightarrow b = -4$

Now, $a - 2b = 1 - 2(-4) = 1 + 8 = $ **9.**

13. In figure, *ABCD* is a rectangle. Find the values of *x* and *y*.



Solution. Since *ABCD* is a rectangle, therefore,

$AB = DC$ and $AD = BC$

$\Rightarrow$ $5 = 2x + y$ ...(1)

and $3x - 2y = 4$ ...(2)

Multiplying (1) by 2 and adding in (2), we get

$2(2x + y) + (3x - 2y) = 2 \times 5 + 4$

$\Rightarrow$ $4x + 2y + 3x - 2y = 10 + 4$

$\Rightarrow$ $7x = 14$

$\Rightarrow$ $x = 2$

Substituting $x = 2$ in (1), we get

$2(2) + y = 5$

$\Rightarrow$ $y = 5 - 4 = 1$

Hence, the values of x and y are **2** and **1**, respectively.

14. If *A* and *B* are acute angles such that $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$ and $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, find $A + B$.

Solution. Given, $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$.

$\therefore$ $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

$$= \frac{\frac{3+2}{6}}{1-\frac{1}{6}}$$

$$= \frac{5/6}{(6-1)/6}$$

$$= \frac{5/6}{5/6}$$

$\Rightarrow$ $\tan(A+B) = 1$

$\Rightarrow$ $\tan(A+B) = \tan 45°$

$\Rightarrow$ $\boldsymbol{A+B = 45°}$

Or

If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$.

Solution. Given, $\cos A + \cos^2 A = 1$

$\Rightarrow$ $\cos A = 1 - \cos^2 A$

$\Rightarrow$ $\cos A = \sin^2 A$ [$\because 1 - \cos^2\theta = \sin^2\theta$]

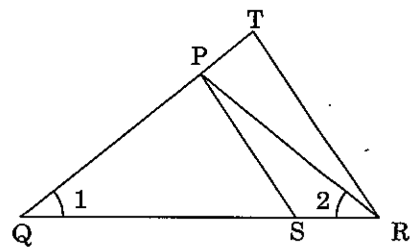
$\Rightarrow$ $\cos^2 A = \sin^4 A$ [Squaring both sides]

$\Rightarrow$ $1 - \sin^2 A = \sin^4 A$

$\Rightarrow$ $\boldsymbol{\sin^2 A + \sin^4 A = 1}$

15. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.

Solution. We have



$$\frac{QR}{QS} = \frac{QT}{PR}$$ [given]

$\Rightarrow$ $$\frac{QT}{QR} = \frac{PR}{QS}$$ ... (1)

We also have

$\angle 1 = \angle 2$ [given]

$\Rightarrow$ $PR = PQ$ ... (2) [Sides opposite to equal angles are equal]

From (1) and (2), we get

$$\frac{QT}{QR} = \frac{PQ}{QS}$$

$\Rightarrow$ $$\frac{PQ}{QT} = \frac{QS}{QR}$$

Thus, in triangles PQS and TQR, we have

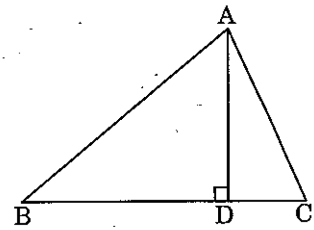
$$\frac{PQ}{QT} = \frac{QS}{QR}$$

and $\angle PQS = \angle TQR = \angle Q$

So, by SAS criterion of similarity of triangles, we have

$$\mathbf{\Delta PQS \sim \Delta TQR}$$

16. The perpendicular from vertex A on the side BC of triangle ABC intersects BC at point D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.



Solution. We have

$DB = 3CD$ [given]

$\Rightarrow CD + DB = CD + 3CD$ [Adding CD to both sides]

$\Rightarrow BC = 4CD$

In right angle ΔABD, right angled at D, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow 2AB^2 = 2AD^2 + 2BD^2$ ...(1)

In right angle ΔACD, right angled at D, we have

$AC^2 = AD^2 + DC^2$

$\Rightarrow 2AC^2 = 2AD^2 + 2DC^2$ ...(2)

Subtracting (2) from (1), we get

$2AB^2 - 2AC^2 = (2AD^2 + 2BD^2) - (2AD^2 + 2DC^2)$

$\Rightarrow 2AB^2 - 2AC^2 = 2(BD^2 - DC^2)$

$\Rightarrow 2AB^2 - 2AC^2 = 2(9DC^2 - DC^2)$ [$\because DB = 3CD \Rightarrow DB^2 = 9DC^2$]

$\Rightarrow 2AB^2 - 2AC^2 = 2(8DC^2)$

$\Rightarrow 2AB^2 - 2AC^2 = 16DC^2$

$\Rightarrow 2AB^2 - 2AC^2 = (4DC)^2$

$\Rightarrow 2AB^2 - 2AC^2 = (BC)^2$ [$\because BC = 4CD$ (proved above)]

$\Rightarrow \mathbf{2AB^2 = 2AC^2 + BC^2}$

17. The following distribution gives the marks of 50 students of a particular school.

Marks	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**
No. of students	**5**	**6**	**8**	**12**	**13**	**6**

Write the above distribution as less than type cumulative frequency distribution.

Solution. We calculate "less than type" cumulative frequency distribution by adding frequencies from top to bottom.

Less Than Type Cumulative Frequency Distribution

Upper Limits	*Frequency*	*Cumulative Frequency*
Less than 10	5	5
Less than 20	6	11 (5 + 6)
Less than 30	8	19 (11 + 8)
Less than 40	12	31 (19 + 12)
Less than 50	13	44 (31 + 13)
Less than 60	6	50 (44 + 6)

18. Find the mode of the following distribution of house-hold expenditure (in ₹) of manual workers in a city :

Expenditure (in ₹)	*Frequency*	*Expenditure (in ₹)*	*Frequency*
1000 – 2000	24	5000 – 6000	30
2000 – 3000	40	6000 – 7000	22
3000 – 4000	33	7000 – 8000	16
4000 – 5000	28	8000 – 9000	7

Solution. Since the maximum frequency is 40, therefore, the modal class is 2000 – 3000. Thus, the lower limit (l) of the modal class = 2000

$\therefore \quad f_1 = 40, f_0 = 24, f_2 = 33, h = 1000$

Using the formula :

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 2000 + \frac{40 - 24}{2 \times 40 - 24 - 33} \times 1000$$

$$= 2000 + \frac{16}{80 - 57} \times 1000$$

$$= 2000 + \frac{16}{23} \times 1000$$

$$= 2000 + 695.65$$

$$= \mathbf{2695.65}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. If d is the HCF of 45 and 27, find x, y satisfying $d = 27x + 45y$.

Solution. Applying Euclid's division lemma to 27 and 45, we get

$45 = 27 \times 1 + 18$ ...(1)

$27 = 18 \times 1 + 9$ ...(2)

$18 = 9 \times 2 + 0$ ...(3)

Since the remainder is zero, therefore, last divisor 9 is the HCF of 27 and 45.

From (2), we get

$9 = 27 - 18 \times 1$

$= 27 - (45 - 27 \times 1) \times 1$ [using (1)]

$= 27 - 45 \times 1 + 27 \times 1 \times 1$

$= 27 - 45 + 27$

$= 54 - 45$

$\Rightarrow \quad 9 = 27 \times 2 - 45 \times 1$ ...(4)

Comparing (4) with $d = 27x + 45y$, we get

$\mathbf{d = 9, x = 2}$ and $\mathbf{y = -1}$

Or

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?

Solution. This can be done by trial and error. But to do it systematically, we find HCF (616, 32). Then this number will give the maximum number of columns in which they, can march and the number of rows will then be the least

Let us use Euclid's division lemma to find their HCF.

We have :

$$616 = 32 \times 19 + 8$$
$$32 = 8 \times 4 + 0$$

So, the HCF of 616 and 32 is 8.

Therefore, the army contingent can march in **8 columns.**

20. Prove that $\frac{1}{2+\sqrt{3}}$ is an irrational number.

Solution. We have

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$$
$$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

Let us assume, to the contrary, that $\frac{1}{2+\sqrt{3}}$ is rational

$\Rightarrow$ $2-\sqrt{3}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that

$$2-\sqrt{3} = \frac{a}{b}$$

$$\Rightarrow \qquad 2-\frac{a}{b} = \sqrt{3}$$

Rearranging the equation, we have

$$\sqrt{3} = 2-\frac{a}{b} = \frac{2b-a}{b}$$

Since a and b are integers, we get $\frac{2b-a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{1}{2+\sqrt{3}}$ is rational.

So, we conclude that $\frac{1}{2+\sqrt{3}}$ is **irrational.**

21. The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.

Solution. Let the unit digit be x and the ten's place digit be y, then

Original number = $10y + x$

New number obtained by reversing the order of the digits = $10x + y$

According to the given information, we have

Original number + New number = 165

$\Rightarrow (10y + x) + (10x + y) = 165$

$\Rightarrow \quad 11x + 11y = 165$

$\Rightarrow \quad x + y = 15$ [Dividing by 11]

and $\quad x - y = \pm 3$ [Digits differ by 3]

Thus, we have the following set of linear equations :

$$x + y = 15 \quad \ldots(1)$$

and $\quad x - y = 3 \quad \ldots(2)$

Adding and subtracting (1) and (2), we get

$$2x = 18 \Rightarrow x = 9$$

and $\quad 2y = 12 \Rightarrow y = 6$

When $x = 9$ and $y = 6$, then

Original number $= 10y + x$

$= 10 \times 6 + 9$

$= 69$

$$x + y = 15 \quad \ldots(1)$$

and $\quad x - y = -3 \quad \ldots(3)$

Adding and subtracting (1) and (3), we get

$$2x = 12 \Rightarrow x = 6$$

and $\quad 2y = 18 \Rightarrow y = 9$

When $x = 6$ and $y = 9$, then

Original number $= 10y + x$

$= 10 \times 9 + 6$

$= 96$

Hence, the number is either **69** or **96**.

Or

Ram scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Ram would have scored 50 marks. How many questions were there in the test ?

Solution. Let x and y denote the number of right (or correct) answer and wrong (or incorrect) answer respectively. Then according to the given condition, we have

$$3x - y = 40 \quad \ldots(1)$$

and $\quad 4x - 2y = 50 \Rightarrow 2x - y = 25 \quad \ldots(2)$

Subtracting (2) from (1), we get

$$(3x - y) - (2x - y) = 40 - 25$$

$\Rightarrow \quad 3x - 2x = 15$

$\Rightarrow \quad x = 15$

Substituting $x = 15$ in (1), we get

$$3 \times 15 - y = 40$$

$\Rightarrow \quad y = 45 - 40$

$\Rightarrow \quad y = 5$

The total number of questions in the test $= x + y = 15 + 5 =$ **20.**

22. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 4x + 3$, find the value of $\alpha^4\beta^2 + \alpha^2\beta^4$.

Solution. Since α and β are the zeroes of the given polynomial $f(x) = x^2 - 4x + 3$, therefore

$$\alpha + \beta = -\frac{(-4)}{1} = 4$$

$$\alpha\beta = \frac{3}{1} = 3$$

Now, $\alpha^4\beta^2 + \alpha^2\beta^4$

$$= \alpha^2\beta^2 (\alpha^2 + \beta^2)$$

$$= \alpha^2\beta^2 [(\alpha^2 + \beta^2 + 2\alpha\beta) - 2\alpha\beta]$$

$$= (\alpha\beta)^2[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$= (3)^2 \times [(4)^2 - 2(3)]$$

$= 9 \times [16 - 6]$
$= 9 \times 10$
$= \mathbf{90}$

23. Prove the following identity :

$$(\mathbf{cosec}\, A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$$

Solution. We have

L.H.S. $= (\text{cosec}\, A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{1}{\sin A \cos A}\right) \qquad \left[\because 1 - \sin^2 A = \cos^2 A, 1 - \cos^2 A = \sin^2 A \text{ and } \sin^2 A + \cos^2 A = 1\right]$$

$$= \frac{\cos^2 A \sin^2 A}{\sin^2 A \cos^2 A}$$

$= 1$

$=$ R.H.S.

24. Prove the identity :

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{2}{1 - 2\cos^2\theta} = \frac{2}{2\sin^2\theta - 1} = \frac{2}{\sin^2\theta - \cos^2\theta}.$$

Solution. We have

$$\text{L.H.S.} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta) + (\sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta)}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{2(\sin^2\theta + \cos^2\theta)}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{2}{\sin^2\theta - \cos^2\theta} = \text{R.H.S.} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2}{(1 - \cos^2\theta) - \cos^2\theta} \qquad [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$= \frac{2}{1 - 2\cos^2\theta} = \text{R.H.S.}$$

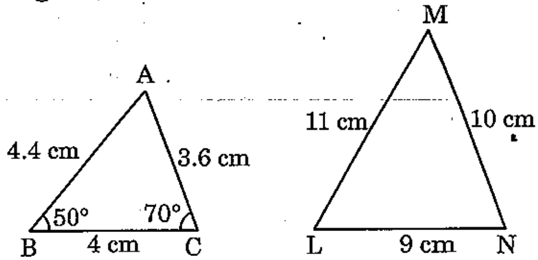
$$= \frac{2}{1-2(1-\sin^2\theta)} \qquad [\because \cos^2\theta = 1 - \sin^2\theta]$$

$$= \frac{2}{1-2+2\sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1}$$

$= \text{R.H.S.}$

25. From the given figure, find $\angle MLN$.



Solution. In triangles ABC and LMN, we have

$$\frac{AB}{LM} = \frac{4.4}{11} = \frac{2}{5}, \frac{BC}{MN} = \frac{4}{10} = \frac{2}{5} \text{ and } \frac{CA}{NL} = \frac{3.6}{9} = \frac{2}{5}$$

Therefore, by SSS-criterion of similarity, we have

$\Delta ABC \sim \Delta LMN$

$\Rightarrow \angle A = \angle L, \angle B = \angle M$ and $\angle C = \angle N$.

Now in ΔABC, we have

$\angle A + \angle B + \angle C = 180°$ [Sum of angles of a $\Delta = 180°$]

$\Rightarrow \angle A + 50° + 70° = 180°$

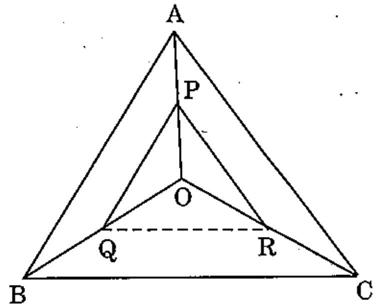
$\Rightarrow \angle A = 180° - 120° = 60°$

$\Rightarrow \angle L = 60°$ $[\because \angle L = \angle A]$

$\Rightarrow$ **$\angle MLN = 60°$** $[\because \angle L = \angle MLN]$

26. In figure, $PQ \parallel AB$ and $PR \parallel AC$. Prove that $QR \parallel BC$.

Solution. In figure, $PQ \parallel AB$ and $PR \parallel AC$. Join QR.



In ΔOAB, we have

$PQ \parallel AB$

$$\Rightarrow \frac{OP}{PA} = \frac{OQ}{QB} \qquad \text{...(1) [By BPT]}$$

In ΔOAC, we have

$PR \parallel AC$

$$\Rightarrow \frac{OP}{PA} = \frac{OR}{RC} \qquad \text{...(2) [By BPT]}$$

From (1) and (2), we have

$$\frac{OQ}{QB} = \frac{OR}{RC}$$

Thus, in ΔOBC, Q and R are points dividing sides OB and OC in the same ratio. Therefore, by converse of BPT, we have

$QR \parallel BC$

27. Find the mean of the following frequency distribution using step-deviation method :

Class Interval	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150	150 – 180
Frequency	8	15	16	20	12	9

Solution. Let the assumed mean $(a) = 105$ and $h = 30$.

Calculation of Mean

Class Interval	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 105}{30}$	$f_i u_i$
0 – 30	8	15	– 3	– 24
30 – 60	15	45	– 2	– 30
60 – 90	16	75	– 1	– 16
90 – 120	20	105	0	0
120 – 150	12	135	1	12
150 – 180	9	165	2	18
Total	$n = \Sigma f_i = 80$			$\Sigma f_i u_i = -40$

By Step-Deviation Method, we have

$$\overline{X} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad \overline{X} = 105 + \frac{(-40)}{80} \times 30$$

$$\Rightarrow \quad \overline{X} = 105 - 15$$

$$\Rightarrow \quad \overline{X} = \mathbf{90}$$

Or

If the mean of the following distribution is 27, find the value of *p*.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	*p*	12	13	10

Solution. **Calculation of Mean**

Class	*Frequency* (f_i)	*Class-mark* (x_i)	$f_i x_i$
0 – 10	8	5	40
10 – 20	p	15	$15p$
20 – 30	12	25	300
30 – 40	13	35	455
40 – 50	10	45	450
Total	$n = \Sigma f_i = 43 + p$		$\Sigma f_i x_i = 1245 + 15p$

Using the formula :

$$\text{Mean } (\overline{X}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 27 = \frac{1245 + 15p}{43 + p}$$

$\Rightarrow \quad 27(43 + p) = 1245 + 15p$
$\Rightarrow \quad 1161 + 27p = 1245 + 15p$
$\Rightarrow \quad 27p - 15p = 1245 - 1161$
$\Rightarrow \quad 12p = 84$
$\Rightarrow \quad \boldsymbol{p = 7}$

28. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

Number of letters	**1 – 4**	**4 – 7**	**7 – 10**	**10 – 13**	**13 – 16**	**16 – 19**
Number of surnames	**6**	**30**	**40**	**16**	**4**	**4**

Determine the median number of letters in the surnames.

Solution. **Cumulative Frequency Distribution**

Number of letters	*Number of surnames* (f)	*Cumulative Frequency* (cf)
1 – 4	6	6
4 – 7	30	36 (cf)
7 – 10	40 (f)	76
10 – 13	16	92
13 – 16	4	96
16 – 19	4	100
Total	$n = \Sigma f_i = 100$	

From the table, $n = \Sigma f_i = 100 \Rightarrow \frac{n}{2} = 50, h = 3.$

Since 7 – 10 is the class whose cumulative frequency 76 is greater than $\frac{n}{2} = 50.$

Therefore, 7 – 10 is the median class. Thus, the lower limit (l) of the median class is 7.
Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 7 + \frac{50 - 36}{40} \times 3$$

$$= 7 + \frac{14}{40} \times 3$$

$$= 7 + \frac{21}{20}$$

$$= 7 + \frac{10.5}{10}$$

$$= 7 + 1.05$$

$$= \mathbf{8.05}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution. Since two zeroes of a polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ therefore,

$$[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})] = [(x - 2) - \sqrt{3}][(x - 2) + \sqrt{3}]$$
$$= (x - 2)^2 - (\sqrt{3})^2$$
$$= x^2 - 4x + 4 - 3$$
$$= x^2 - 4x + 1$$

is a factor of the given polynomial.

Now, we divide the given polynomial by $(x^2 - 4x + 1)$

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & \underline{\overset{-}{}x^4 \overset{+}{-} 4x^3 \overset{-}{+} x^2} \\ & -2x^3 - 27x^2 + 138x - 35 \\ & \underline{\overset{+}{-} 2x^3 \overset{-}{+} 8x^2 \overset{+}{-} 2x} \\ & -35x^2 + 140x - 35 \\ & \underline{\overset{+}{-} 35x^2 \overset{-}{+} 140x \overset{+}{-} 35} \\ & 0 \end{array}$$

$\left[\text{First term of the quotient is } \frac{x^4}{x^2} = x^2\right]$

$\left[\text{Second term of the quotient is } \frac{-2x^3}{x^2} = -2x\right]$

$\left[\text{Third term of the quotient is } \frac{-35x^2}{x^2} = -35\right]$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

Now, by splitting $-2x$, as $-7x + 5x$, we can write

$$x^2 - 2x - 35 = x^2 - 7x + 5x - 35$$
$$= x(x - 7) + 5(x - 7)$$
$$= (x - 7)(x + 5)$$

So, its zeroes are given by $x = 7$ and $x = -5$.

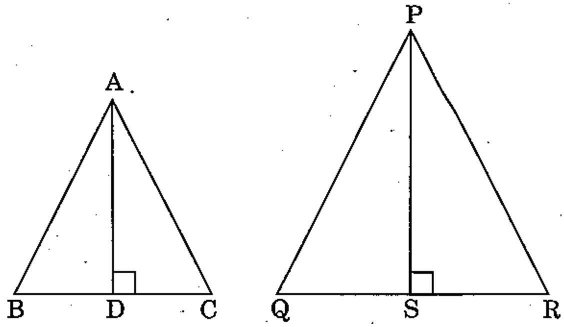
Hence, the other zeroes of the given polynomial are **7** and **– 5.**

30. Prove that the ratio of areas of two similar triangles is equal to the square of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.



Proof : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\frac{AB}{PQ} = \frac{BC}{QR}$ [$\because \Delta ABC \sim \Delta PQR$]

$\Rightarrow \quad \frac{AD}{PS} = \frac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$$

$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$ ...(4)

As $\Delta ABC \sim \Delta PQR$, therefore

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(5)

Hence, $\frac{\textbf{ar}(\Delta ABC)}{\textbf{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$ [From (4) and (5)]

Or

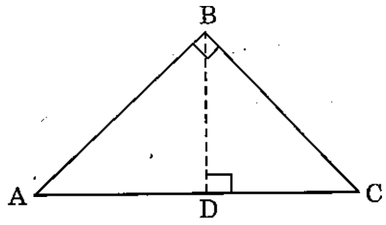
Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Solution. Given : A right triangle ABC, right angled at B.

To prove : (Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

i.e., $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$.



Proof : $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]

So, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ [Sides are proportional]

$\Rightarrow \quad AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$ [Sides are proportional]

$\Rightarrow \quad CD.AC = BC^2$ ...(2)

Adding (1) and (2) , we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$AC.AC = AB^2 + BC^2$

Hence, $\boldsymbol{AC^2 = AB^2 + BC^2}$

31. If $\sec\theta = x + \dfrac{1}{4x}$, prove that

$$\sec\theta + \tan\theta = 2x \text{ or } \frac{1}{2x}.$$

Solution. We have

$$\sec\theta = x + \frac{1}{4x}$$

$$\Rightarrow \quad \sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad 1 + \tan^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \tan^2\theta = \left(x + \frac{1}{4x}\right)^2 - 1$$

$$\Rightarrow \quad \tan^2\theta = \left(x^2 + \frac{1}{16x^2} + 2\cdot x\cdot\frac{1}{4x}\right) - 1$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - 2(x)\cdot\left(\frac{1}{4x}\right)$$

$$\Rightarrow \qquad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

$$\Rightarrow \qquad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

When $\tan\theta = x - \frac{1}{4x}$, then

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right)$$

$$\Rightarrow \quad \sec\theta + \tan\theta = 2x$$

When $\tan\theta = -\left(x - \frac{1}{4x}\right)$, then

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) - \left(x - \frac{1}{4x}\right)$$

$$\Rightarrow \quad \sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x}$$

$$\Rightarrow \quad \sec\theta + \tan\theta = \frac{2}{4x}$$

$$\Rightarrow \quad \sec\theta + \tan\theta = \frac{1}{2x}$$

Hence, $\sec\theta + \tan\theta = \mathbf{2x}$ or $\mathbf{\frac{1}{2x}}$.

32. Prove that

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A.$$

Solution. We have

$$\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\operatorname{cosec}^2 A} \qquad [\because \sec^2 A = 1 + \tan^2 A \text{ and } \operatorname{cosec}^2 A = 1 + \cot^2 A]$$

$$= \frac{1}{\cos^2 A} \cdot \frac{\sin^2 A}{1}$$

$$= \left(\frac{\sin A}{\cos A}\right)^2$$

$$= \tan^2 A \qquad \text{...(1)}$$

$$\text{Again, } \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1 - \frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}} \right)^2$$

$$= \left[-\left(\frac{\sin A - \cos A}{\cos A} \right) \cdot \left(\frac{\sin A}{\sin A - \cos A} \right) \right]^2$$

$$= \left(-\frac{\sin A}{\cos A} \right)^2$$

[Cancelling $\sin A - \cos A$ from numerator and denominator]

$$= (-\tan A)^2$$

$$= \tan^2 A \qquad \text{...(2)}$$

From (1) and (2), we conclude that

$$\left(\frac{\mathbf{1 + \tan^2 A}}{\mathbf{1 + \cot^2 A}} \right) = \left(\frac{\mathbf{1 - \tan A}}{\mathbf{1 - \cot A}} \right)^2 = \mathbf{\tan^2 A.}$$

Or

Without using trigonometric tables evaluate :

$$\frac{\sec^2 36° - \cot^2 54°}{\text{cosec}^2 33° - \tan^2 57°} + \cot\theta\,(\tan 90° - \theta) - \sec\,(90° - \theta)\,\text{cosec}\,\theta.$$

Solution. We have

$$\frac{\sec^2 36° - \cot^2 54°}{\text{cosec}^2 33° - \tan^2 57°} + \cot\theta\tan(90° - \theta) - \sec(90° - \theta)\,\text{cosec}\,\theta$$

$$= \frac{\sec^2 36° - \cot^2(90° - 36°)}{\text{cosec}^2 33° - \tan^2(90° - 33°)} + \cot\theta\tan(90° - \theta) - \sec(90° - \theta)\,\text{cosec}\,\theta$$

$$= \frac{\sec^2 36° - \tan^2 36°}{\text{cosec}^2 33° - \cot^2 33°} + \cot\theta\cot\theta - \text{cosec}\,\theta\,\text{cosec}\,\theta$$

[$\because$ $\cot(90° - A) = \tan A$, $\tan(90° - A) = \cot A$, $\sec(90° - A) = \text{cosec}\,A$]

$$= \frac{(1 + \tan^2 36°) - \tan^2 36°}{(1 + \cot^2 33°) - \cot^2 33°} + \cot^2\theta - \text{cosec}^2\theta$$

[$\because \sec^2 A = 1 + \tan^2 A$; $\text{cosec}^2 A = 1 + \cot^2 A$]

$$= \frac{1}{1} + \cot^2\theta - (1 + \cot^2\theta)$$

$$= 1 - 1$$

$$= 0.$$

33. Form a pair of linear equations in two variables using the following information and solve it graphically :

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also three years from now, I shall be three times as old as you will be".

Find their present ages. What was the age of Aftab when his daughter was born ?

Solution. Let Aftab's present age = x years

Daughter's present age = y years

Seven years ago, Aftab's age was = $(x - 7)$ years

and his daughter's age was = $(y - 7)$ years.

After three years, Aftab's age will be $(x + 3)$ years and daughter's age will be $(y + 3)$ years.

According to the given condition :

(*i*) Seven years ago, Aftab's age = 7 times the age of the daughter

$\Rightarrow \quad x - 7 = 7(y - 7)$

$\Rightarrow \quad x - 7 = 7y - 49$

$\Rightarrow \quad x - 7y + 42 = 0 \qquad ...(1)$

(*ii*) Three years from now, Aftab's age = 3 times the age of the daughter

$\Rightarrow \quad x + 3 = 3(y + 3)$

$\Rightarrow \quad x + 3 = 3y + 9$

$\Rightarrow \quad x - 3y - 6 = 0 \qquad ...(2)$

Thus, the algebraic representation is given by the equations (1) and (2), where x represents the present age of Aftab and y represents the present age of his daughter.

To obtain the equivalent graphical representation, we find two points on the line representing each equation, *i.e.*, we find two solutions of each equation.

These solutions are given below in Tables.

$x - 7y + 42 = 0$

x	-42	0
y	0	6
	A	B

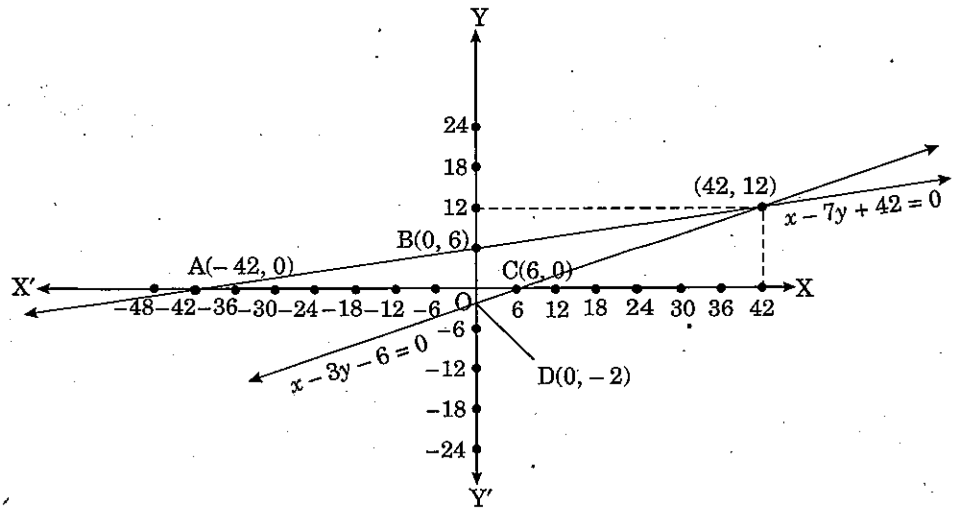
$x - 3y - 6 = 0$

x	6	0
y	0	-2
	C	D

We plot these points $A(-42, 0)$ and $B(0, 6)$ to get the line AB and the points $C(6, 0)$ and $D(0, -2)$ to get the line CD.

In figure, we see that the two lines representing the two equations intersecting at the point (42, 12).

Hence, $x = 42$ and $y = 12$. Therefore, Aftab's present age is **42 years** and his daughter's present age is **12 years**. Also, Aftab was $(42 - 12)$ = **30 years** old when his daughter was born.



34. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	*Frequency*
0 – 100	2
100 – 200	5
200 – 300	x
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	y
700 – 800	9
800 – 900	7
900 – 1000	4

Solution. Here, the missing frequencies are x and y.

Class Interval	*Frequency*	*Cumulative Frequency*
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$

It is given that, $n = 100$ = Total frequency.

$\Rightarrow \quad 76 + x + y = 100$

$\Rightarrow \quad x + y = 100 - 76$

$\Rightarrow \quad x + y = 24$ ...(1)

The median is 525 (given), which lies in the class 500 – 600.

So, l = lower limit of median class = 500

f = frequency of median class = 20

cf = cumulative frequency of class preceding the median class = $36 + x$

h = class size = 100

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 525 = 500 + \left[\frac{50 - (36 + x)}{20}\right] \times 100$$

$$\Rightarrow \quad 525 - 500 = \left[\frac{50 - 36 - x}{20}\right] \times 100$$

$$\Rightarrow \quad 25 = (14 - x) \times 5$$

$$\Rightarrow \quad 5 = 14 - x$$

$$\Rightarrow \quad x = 14 - 5$$

$$\Rightarrow \quad x = 9$$

Now, from (1), we get, $9 + y = 24 \Rightarrow y = 24 - 9 = 15$

Hence, $\boldsymbol{x = 9}$ and $\boldsymbol{y = 15}$.