CCE SAMPLE QUESTION PAPER

FIRST TERM (SA–I)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The solution of the pair of linear equations $x - y - 5 = 0$ and $4x - 3y - 17 = 0$ is

(*a*) $x = -3, y = 2$ (*b*) $x = 2, y = -3$

(*c*) $x = -2, y = 3$ (*d*) $x = 3, y = -2$

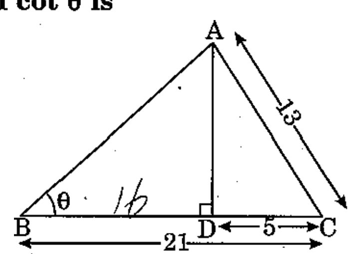
Solution. Choice (*b*) is correct.

Clearly, $x = 2$ and $y = -3$ satisfies the given pair of linear equations, *i.e.*,

$$x - y - 5 = 0 \Rightarrow 2 + 3 - 5 = 0 \Rightarrow 0 = 0$$

and $$4x - 3y - 17 = 0 \Rightarrow 8 + 9 - 17 = 0 \Rightarrow 0 = 0$$

2. In figure, the value of cot θ is



(*a*) $\frac{4}{3}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{13}{21}$ (*d*) $\frac{5}{21}$

Solution. Choice (*a*) is correct.

In right ΔADC, $AD^2 = AC^2 - DC^2$

$\Rightarrow \quad AD^2 = (13)^2 - (5)^2$

$\Rightarrow \quad AD^2 = 169 - 25 = 144 = (12)^2$

$\Rightarrow \quad AD = 12$

$BD = BC - DC = 21 - 5 = 16$

Now in ΔADB, $\cot\theta = \dfrac{BD}{AD} = \dfrac{16}{12} = \dfrac{4}{3}$.

3. If the class-marks of a continuous frequency distribution are 22, 30, 38, 46, 54, 62, then the class corresponding to the class-mark 46 is

(*a*) 41 – 49 (*b*) 41 – 50

(*c*) 41.5 – 49.5 (*d*) 42 – 50

Solution. Choice (*d*) is correct.

Here, the class-marks are uniformly spaced.

∴ Width or size of each class-interval

= The difference between the class-mark of two adjacent classes

= 30 – 22

= 8

For the class-mark 46

Lower class boundary = Class-mark – $\frac{1}{2}$(size of class-interval)

$= 46 - \frac{1}{2}(8)$

$= 46 - 4 = 42$

Upper class boundary = Class-mark + $\frac{1}{2}$(size of class-interval)

$= 46 + \frac{1}{2}(8)$

$= 46 + 4 = 50$

The class-interval corresponding to the class-mark 46 is **42 – 50.**

4. The decimal expansion of the rational number $\frac{47}{2^3 5^4}$, will terminate after how many places of decimal ?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 8

Solution. Choice (*c*) is correct.

$$\frac{47}{2^3 5^4} = \frac{47}{(2^3 \cdot 5^3) \cdot 5} = \frac{47 \div 5}{(10)^3}$$

$$= \frac{9.4}{1000} = 0.0094$$

Thus, decimal expansion of the rational number $\frac{47}{2^3 5^4}$, will terminate after **four** places of decimal.

5. The value $\dfrac{\sin 60°}{\cos^2 45°} - \cot 30° + 15 \cos 90°$ is

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) 2

Solution. Choice (*a*) is correct.

$$\frac{\sin 60°}{\cos^2 45°} - \cot 30° + 15 \cos 90°$$

$$= \frac{(\sqrt{3}/2)}{(1/\sqrt{2})^2} - \sqrt{3} + 15 \times 0$$

$$= \frac{\sqrt{3}}{2} \times 2 - \sqrt{3} + 0$$

$$= \sqrt{3} - \sqrt{3} = \mathbf{0}$$

6. If $\tan 2\theta = \cot (\theta + 6°)$, where (2θ) and $(\theta + 6°)$ are acute angles, the value of θ is

(*a*) 24° (*b*) 25°

(*c*) 28° (*d*) 27°

Solution. Choice (*c*) is correct.

$$\tan 2\theta = \cot (\theta + 6°)$$

$$\Rightarrow \cot (90° - 2\theta) = \cot (\theta + 6°) \quad [\because \cot (90° - A) = \tan A]$$

$$\Rightarrow 90° - 2\theta = \theta + 6°$$

$$\Rightarrow 90° - 6° = \theta + 2\theta$$

$$\Rightarrow 84° = 3\theta$$

$$\Rightarrow \theta = \mathbf{28°}$$

7. If θ is an acute angle such that $\sec^2 \theta = 3$, then the value of $\dfrac{\tan^2 \theta - \text{cosec}^2 \theta}{\tan^2 \theta + \text{cosec}^2 \theta}$ is

(*a*) $\dfrac{4}{7}$ (*b*) $\dfrac{3}{7}$

(*c*) $\dfrac{2}{7}$ (*d*) $\dfrac{1}{7}$

Solution. Choice (*d*) is correct.

Given, $\sec^2 \theta = 3 \Rightarrow 1 + \tan^2 \theta = 3$

$\Rightarrow \tan^2 \theta = 2$

Now, $\text{cosec}^2 \theta = 1 + \cot^2 \theta = 1 + \dfrac{1}{\tan^2 \theta}$

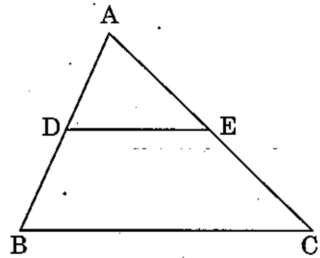
$$= 1 + \frac{1}{2} = \frac{3}{2}$$

$$\therefore \frac{\tan^2 \theta - \text{cosec}^2 \theta}{\tan^2 \theta + \text{cosec}^2 \theta} = \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}}$$

$$= \frac{(4-3)/2}{(4+3)/2}$$

$$= \frac{1}{7}$$

8. In a given ΔABC, $DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$, if $AC = 5.6$ cm, then AE is equal to



(*a*) 2 cm (*b*) 2.1 cm

(*c*) 2.2 cm (*d*) 2.3 cm

Solution. Choice (*b*) is correct.

In ΔABC, $DE \parallel BC$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \text{(given) } \frac{3}{5} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{3}{5} = \frac{AE}{AC - AE}$$

$$\Rightarrow \quad \frac{3}{5} = \frac{AE}{5.6 - AE}$$

$$\Rightarrow \quad 3 \times (5.6 - AE) = 5AE$$

$$\Rightarrow \quad 16.8 - 3AE = 5AE$$

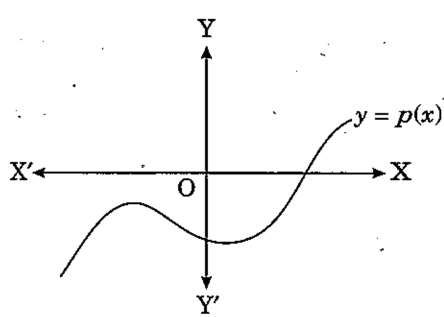
$$\Rightarrow \quad 3AE + 5AE = 16.8$$

$$\Rightarrow \quad 8AE = 16.8$$

$$\Rightarrow \quad AE = 16.8 \div 8$$

$$\Rightarrow \quad AE = \mathbf{2.1 \text{ cm}}$$

9. In figure, the graph of some polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



(*a*) 1 (*b*) 2
(*c*) 4 (*d*) 3

Solution. Choice (*a*) is correct.

The number of zeroes is 1 as the graph intersects the x-axis in one point only.

10. The prime factorisation of 468 is

(*a*) $2^2 \times 7 \times 13$ (*b*) $3^2 \times 2 \times 13$
(*c*) $2^2 \times 3^2 \times 13$ (*d*) $3^3 \times 2^2 \times 13$

Solution. Choice (*c*) is correct.

$$\begin{aligned} 468 &= 2 \times 234 \\ &= 2 \times 2 \times 117 \\ &= 2^2 \times 3 \times 39 \\ &= 2^2 \times 3 \times 3 \times 13 \\ &= \mathbf{2^2 \times 3^2 \times 13} \end{aligned}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Explain why $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number.

Solution. We have

$$\begin{aligned} 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 &= (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5 \\ &= (42 \times 24 + 1) \times 5 \\ &= (1008 + 1) \times 5 \\ &= 1009 \times 5 \end{aligned}$$

$\Rightarrow$ $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times 1009$ is a **composite number** as product of prime occur.

12. What must be subtracted from $14x^3 - 5x^2 + 9x - 1$ so that the resulting polynomial is exactly divisible by $2x - 1$?

Solution. Dividing $14x^3 - 5x^2 + 9x - 1$, by $2x - 1$, we get

$$\begin{array}{r|l} & 7x^2 + x + 5 \\ \hline 2x - 1 & 14x^3 - 5x^2 + 9x - 1 \\ & \underline{14x^3 - 7x^2} \\ & \quad\ \ 2x^2 + 9x - 1 \\ & \quad\ \ \underline{2x^2 - x} \\ & \qquad\quad 10x - 1 \\ & \qquad\quad \underline{10x - 5} \\ & \qquad\qquad\ \ 4 \end{array}$$

$\therefore$ Quotient $= 7x^2 + x + 5$ and Remainder $= 4$

Thus, if we subtract the remainder 4 from $14x^3 - 5x^2 + 9x - 1$, it will be divisible by $2x - 1$.

13. Solve the following system of equations :

$$\mathbf{x + 2y = -1}$$
$$\mathbf{2x - 3y = 12}$$

Solution. The given system of equations is

$$x + 2y = -1 \qquad \ldots(1)$$
$$2x - 3y = 12 \qquad \ldots(2)$$

From equation (1), we get

$$x = -1 - 2y \quad ...(3)$$

Substituting $x = -1 - 2y$ in equation (2), we get

$2(-1 - 2y) - 3y = 12$

$\Rightarrow \quad -2 - 4y - 3y = 12$

$\Rightarrow \quad -7y = 12 + 2$

$\Rightarrow \quad -7y = 14$

$\Rightarrow \quad y = -2$

Substituting $y = -2$ in (3), we get

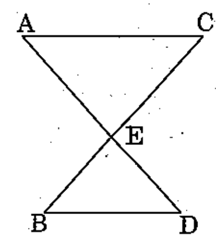
$x = -1 - 2(-2)$

$\Rightarrow \quad x = -1 + 4$

$\Rightarrow \quad x = 3$

Hence, the solution of the given system of equations is $\boldsymbol{x = 3, y = -2}$.

14. In the figure given below, $AC \parallel BD$, is $\dfrac{AE}{CE} = \dfrac{DE}{BE}$? Justify your answer.



Solution. In the figure, $AC \parallel BD$

In ΔACE and ΔBDE

$\angle CAE = \angle BDE$ [Alternate angles, $\because AC \parallel BD$]

$\angle ACE = \angle DBE$ [Alternate angles, $\because AC \parallel BD$]

$\angle AEC = \angle BED$ [Vertically opposite angles]

$\Rightarrow \quad \Delta ACE \sim \Delta BDE$ [AA-similarity]

Now, $\quad \dfrac{AC}{BD} = \dfrac{CE}{BE} = \dfrac{AE}{DE}$

Considering $\quad \dfrac{CE}{BE} = \dfrac{AE}{DE}$

$\Rightarrow \quad \dfrac{AE}{CE} = \dfrac{DE}{BE}$

15. Prove that : $(\sec\theta - \tan\theta)^2(1 + \sin\theta) = 1 - \sin\theta$.

Solution. L.H.S. $= (\sec\theta - \tan\theta)^2(1 + \sin\theta)$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 (1 + \sin\theta)$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2 (1 + \sin\theta)$$

$$= \frac{(1 - \sin\theta)[(1 - \sin\theta)(1 + \sin\theta)]}{\cos^2\theta}$$

$$= \frac{(1 - \sin\theta)(1 - \sin^2\theta)}{\cos^2\theta}$$

$$= \frac{(1 - \sin\theta)\cos^2\theta}{\cos^2\theta} \quad [\because 1 - \sin^2\theta = \cos^2\theta]$$

$$= 1 - \sin\theta$$
$$= \text{R.H.S.}$$

Or

If A, B, C are interior angles of ΔABC, show that

$$\mathbf{cosec^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1}$$

Solution. If A, B, C are interior angles of ΔABC, then

$$A + B + C = 180°$$

$$\Rightarrow \quad B + C = 180° - A$$

$$\Rightarrow \quad \frac{B+C}{2} = \frac{180° - A}{2}$$

$$\Rightarrow \quad \frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\Rightarrow \quad \text{cosec}^2\left(\frac{B+C}{2}\right) = \text{cosec}^2\left(90° - \frac{A}{2}\right)$$

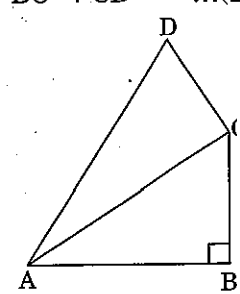
$$\Rightarrow \quad \text{cosec}^2\left(\frac{B+C}{2}\right) = \sec^2\frac{A}{2} \quad [\because \text{cosec}(90° - \theta) = \sec\theta]$$

$$\Rightarrow \quad \text{cosec}^2\left(\frac{B+C}{2}\right) = 1 + \tan^2\frac{A}{2} \quad [\because \sec^2\theta = 1 + \tan^2\theta]$$

$$\Rightarrow \quad \mathbf{cosec^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1}$$

16. In a quadrilateral $ABCD$, $\angle B = 90°$. If $AD^2 = AB^2 + BC^2 + CD^2$, prove that $\angle ACD = 90°$.

Solution. A quadrilateral $ABCD$ in which $\angle B = 90°$ and $AD^2 = AB^2 + BC^2 + CD^2$ ...(1)



Join AC.

In right triangle ABC, we have

$AB^2 + BC^2 = AC^2$ ...(2) [By Pythagoras Theorem]

From (1), we have

$AD^2 = (AB^2 + BC^2) + CD^2$

$\Rightarrow \quad AD^2 = AC^2 + CD^2$ [using (2)]

Thus, in ΔACD, we have

$AD^2 = AC^2 + CD^2$

Hence ΔACD is a right triangle right angled at C, *i.e.*, $\angle C = 90°$.

17. The following cumulative frequency distribution is given :

Less than 20	Less than 30	Less than 40	Less than 50	Less than 60	Less than 70	Less than 80	Less than 90	Less than 100
0	4	16	30	46	66	82	92	100

Write the ordinary frequency distribution.

Solution. **Ordinary Frequency Distribution**

Classes	*Frequency*	*Cumulative frequency (cf less than)*
20 – 30	4	4
30 – 40	12 = (16 – 4)	16
40 – 50	14 = (30 – 16)	30
50 – 60	16 = (46 – 30)	46
60 – 70	20 = (66 – 46)	66
70 – 80	16 = (82 – 66)	82
80 – 90	10 = (92 – 82)	92
90 – 100	8 = (100 – 92)	100

18. Find the mode of the following distribution of the ages of the patients admitted in a hospital during a year :

Ages (in years)	**5 – 15**	**15 – 25**	**25 – 35**	**35 – 45**	**45 – 55**	**55 – 65**
No. of patients	**6**	**11**	**21**	**23**	**14**	**5**

Solution. Since the class 35 – 45 has the maximum frequency 23, therefore 35 – 45 is the modal class.

$\therefore \quad l = 35, h = 10, f_1 = 23, f_0 = 21, f_2 = 14$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 35 + \frac{23 - 21}{2 \times 23 - 21 - 14} \times 10$$

$$= 35 + \frac{2}{46 - 35} \times 10$$

$$= 35 + \frac{20}{11}$$

$$= 35 + 1.81$$

$$= \mathbf{36.81}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Use Euclid's division lemma to show that the square of any positive integer is of the form $4q$ or $4q + 1$ for some positive integer q.

Solution. Let x be any positive integer than it is of the form. $4m$, $4m + 1$, $4m + 2$ or $4m + 3$.

When $x = 4m$, then by squaring, we have

$x^2 = (4m)^2 = 16m^2 = 4(4m^2) = 4q$, where $q = 4m^2$

When $x = 4m + 1$, then by squaring, we have

$x^2 = (4m + 1)^2 = 16m^2 + 8m + 1 = 8m(2m + 1) + 1 = 4q + 1$, where $q = 2m(2m + 1)$

When $x = 4m + 2$, then by squaring, we have

$x^2 = (4m + 2)^2 = 16m^2 + 16m + 4 = 4(4m^2 + 4m + 1) = 4q$, where $q = 4m^2 + 4m + 1$

When $x = 4m + 3$, then by squaring, we have

$$x^2 = (4m+3)^2 = 16m^2 + 24m + 9 = 4(4m^2 + 6m + 2) + 1 = 4q + 1, \text{ where } q = 4m^2 + 6m + 2$$

Hence, the square of any positive integer, say x, is either of the form $\mathbf{4q}$ or $\mathbf{4q + 1}$ for some positive integer q.

20. Prove that $\sqrt{6}$ is an irrational number.

Solution. Let us assume, to the contrary, that $\sqrt{6}$ is rational number. Then

$$\sqrt{6} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are coprime and } q \neq 0.$$

Squaring on both sides, and rearranging, we get

$$p^2 = 6q^2 \qquad \ldots(1)$$

$\Rightarrow$ 3 divides $6q^2$

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p

Let $p = 3m$, where m is a positive integer.

Substituting $p = 3m$ in (1), we get

$$9m^2 = 6q^2$$

$\Rightarrow$ $3m^2 = 2q^2$

As 3 divides $3m^2 \Rightarrow$ 3 divides $2q^2$ but 3 does not divide 2

$\Rightarrow$ 3 divides q^2

$\Rightarrow$ 3 divides q

Therefore, p and q have at least 3 as a common factor.

But this contradicts the fact that p and q are co-prime.

This contradiction arises because of our incorrect assumption that $\sqrt{6}$ is rational.

So, we conclude that $\sqrt{6}$ is an **irrational.**

Or

Show that $5 - 2\sqrt{3}$ is an irrational number.

Solution. Let us assume, to contrary, that $5 - 2\sqrt{3}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that

$$5 - 2\sqrt{3} = \frac{a}{b}$$

Therefore, $2\sqrt{3} = 5 - \dfrac{a}{b}$

$$\Rightarrow \quad 2\sqrt{3} = \frac{5b - a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5b - a}{2b}$$

Since a and b are integers, we get $\dfrac{5b - a}{2b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - 2\sqrt{3}$ is rational.

So, we conclude that $5 - 2\sqrt{3}$ is **irrational.**

21. 20 students of class X took part in mathematics quiz. If the number of girls exceed the number of boys by 8, find the number of boys and girls who took part in quiz.

Solution. Let x and y denote the number of girls and boys respectively.

Then, according to given condition, we have

$$x + y = 20 \quad \ldots(1)$$

and

$$x - y = 8 \quad \ldots(2)$$

Adding (1) and (2), we get

$$(x + y) + (x - y) = 20 + 8$$

$\Rightarrow$ $2x = 28$

$\Rightarrow$ $x = 14$

Substituting $x = 14$ in (1), we get

$$14 + y = 20$$

$\Rightarrow$ $y = 20 - 14$

$\Rightarrow$ $y = 6$

Hence, the number of girls are **14** and number of boys are **6.**

Or

In a competitive examination 2 marks is awarded for each correct answer while 1 mark is deducted for each wrong answer. Ram answered 150 questions and got 240 marks. How many questions did he answer correctly ?

Solution. Let x and $150 - x$ denote the number of correct and wrong answers respectively.

Then, according to the given condition, we have

$$2x - (150 - x) = 240$$

$\Rightarrow$ $2x - 150 + x = 240$

$\Rightarrow$ $3x = 240 + 150$

$\Rightarrow$ $3x = 390$

$\Rightarrow$ $x = 130$

Hence, the number of correctly answered questions = **130.**

22. If α and β are zeroes of the polynomial $x^2 - 2x + 3$, then form a quadratic polynomial whose zeroes are α + 2 and β + 2.

Solution. Since α and β are zeroes of the polynomial $x^2 - 2x + 3$, then

$$\alpha + \beta = -\left(\frac{-2}{1}\right) = 2 \quad \ldots(1)$$

$$\therefore \quad \alpha\beta = \frac{3}{1} = 3 \quad \ldots(2)$$

Let S and P denote respectively the sum and product of zeroes of the required polynomial, then

$$S = (\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 2 + 4 = 6 \quad \text{[using (1)]}$$

$$P = (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4 \quad \text{[using (1) and (2)]}$$

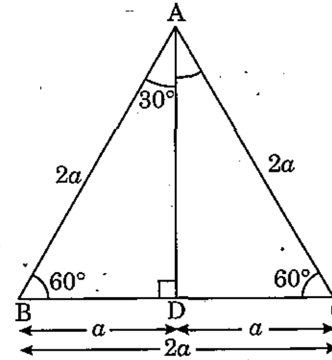
$\Rightarrow$ $P = 3 + 2(2) + 4 = 3 + 4 + 4 = 11$

Hence, the required polynomial is

$$p(x) = k(x^2 - Sx + P)$$

or $p(x) = \boldsymbol{k(x^2 - 6x + 11)}$, where k is any non-zero constant.

23. Find the value of sin 30° geometrically.



Solution. Consider an equilateral triangle ABC with each side of length $2a$. Since each angle of an equilateral triangle is 60°, therefore, each angle of ΔABC is of 60°. Let AD be the perpendicular from A on BC. Since the triangle is an equilateral, therefore, AD is the bisector of $\angle A$ and D is the mid-point of side $BC = 2a$.

$\therefore \quad BD = DC = a$ and $\angle BAD = 30°$

Thus, in ΔABD, $\angle D$ is a right angle, hypotenuse $AB = 2a$ and $BD = a$.

In right ΔADB, we have

$$\sin 30° = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}.$$

24. Prove that :

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$$

Solution. We have

$$\text{L.H.S.} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{[(\sin A + \cos A) - 1][(\sin A + \cos A) + 1]}{\sin A \cos A}$$

$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A} \qquad [\because (x - y)(x + y) = x^2 - y^2]$$

$$= \frac{(\sin^2 A + \cos^2 A + 2\sin A \cos A) - 1}{\sin A \cos A}$$

$$= \frac{(1 + 2\sin A \cos A) - 1}{\sin A \cos A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

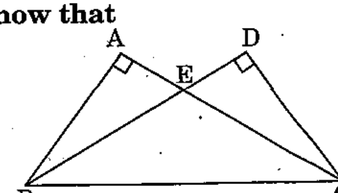
$$= \frac{2\sin A \cos A}{\sin A \cos A}$$

$= 2$

$=$ R.H.S.

25. Two Δ's ABC and DBC are on the same base BC and on the same side of BC in which $\angle A = \angle D = 90°$. If CA and BD meet each other at E, show that

$AE.EC = BE.ED$.



Solution. In ΔAEB and ΔDEC

$\angle A = \angle D = 90°$ [given]

and $\angle AEB = \angle DEC$ [Vertically opposite $\angle s$]

Therefore, by AA-criterion of similarity, we have

$\Delta AEB \sim \Delta DEC$

$\Rightarrow \quad \frac{AE}{DE} = \frac{BE}{CE}$

$\Rightarrow \quad \mathbf{AE.EC = BE.ED.}$ [$\because DE = ED$ and $CE = EC$]

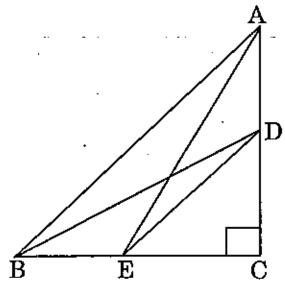
26. *D* and *E* are points on the sides *CA* and *CB* respectively of ΔABC right-angled at *C*. Prove that

$$AE^2 + BD^2 = AB^2 + DE^2.$$

Solution. Given : A triangle *ABC*, right-angled at *C*. *D* and *E* are points on the sides *CA* and *CB* respectively.

To prove : $AE^2 + BD^2 = AB^2 + DE^2$.

Proof : In right triangle *ACB*, we have



$AC^2 + BC^2 = AB^2$ ...(1) [using Pythagoras Theorem]

In right triangle *ACE*, we have

$AC^2 + CE^2 = AE^2$ ...(2) [using Pythagoras Theorem]

In right triangle *BCD*, we have

$BC^2 + CD^2 = BD^2$ ...(3) [using Pythagoras Theorem]

In right triangle *ECD*, we have

$CE^2 + CD^2 = DE^2$ ...(4) [using Pythagoras Theorem]

Adding (2) and (3), we have

$$AE^2 + BD^2 = (AC^2 + CE^2) + (BC^2 + CD^2)$$
$$= (AC^2 + BC^2) + (CE^2 + CD^2)$$
$$= AB^2 + DE^2 \quad \text{[using (1) and (4)]}$$

Hence, $\mathbf{AE^2 + BD^2 = AB^2 + DE^2}$.

27. Find the mean of the following frequency distribution using step-deviation method.

Class-interval	**0 – 50**	**50 – 100**	**100 – 150**	**150 – 200**	**200 – 250**	**250 – 300**	***Total***
Frequency	**10**	**15**	**30**	**35**	**25**	**15**	**130**

Solution. Let the assumed mean $a = 175$ and $h = 50$.

Calculation of Mean

Class-interval	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 175}{50}$	$f_i u_i$
0 – 50	10	25	– 3	– 30
50 – 100	15	75	– 2	– 30
100 – 150	30	125	– 1	– 30
150 – 200	35	175	0	0
200 – 250	25	225	1	25
250 – 300	15	275	2	30
Total	$n = \Sigma f_i = 130$			$\Sigma f_i u_i = -35$

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 175 + \frac{-35}{130} \times 50$$

$$= 175 - \frac{175}{13}$$

$$= 175 - 13.46$$

$$= \mathbf{161.54}$$

Or

The following table gives the output (in units) of workers in a certain factory. The frequency of the class-interval 490 – 520 is missing.

Output (in units)	400 – 430	430 – 460	460 – 490	490 – 520	520 – 550
No. of workers	31	58	60	—	27

It is known that the mean of the above frequency distribution is 472. Find the missing frequency.

Solution. Let the frequency of the class-interval 490 – 520 be x.

Output (in units)	*No. of workers [Frequency (f_i)]*	*Class-mark (x_i)*	$f_i x_i$
400 – 430	31	415	12865
430 – 460	58	445	25810
460 – 490	60	475	28500
490 – 520	x	505	$505x$
520 – 550	27	535	14445
Total	$n = \Sigma f_i = 176 + x$		$\Sigma f_i x_i = 81620 + 505x$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 472 = \frac{81620 + 505x}{176 + x}$$

$$\Rightarrow \quad 472 \times 176 + 472x = 81620 + 505x$$

$$\Rightarrow \quad 83072 + 472x = 81620 + 505x$$

$$\Rightarrow \quad 505x - 472x = 83072 - 81620$$

$$\Rightarrow \quad 33x = 1452$$

$$\Rightarrow \quad x = 1452 \div 33$$

$$\Rightarrow \quad x = 44$$

28. Find the median of the following data :

Classes	***Frequency***
0 – 10	**3**
10 – 20	**8**
20 – 30	**10**
30 – 40	**15**
40 – 50	**7**
50 – 60	**4**
60 – 70	**3**

Solution. The cumulative frequency distribution table with the given frequency becomes :

Classes	*Frequency* (f_i)	*Cumulative Frequency* (*cf*)
0 – 10	3	3
10 – 20	8	11
20 – 30	10	21 (*cf*)
30 – 40	15 (*f*)	36
40 – 50	7	43
50 – 60	4	47
60 – 70	3	50
Total	$n = \Sigma f_i = 50$	

From the table, $n = \Sigma f_i = 50 \Rightarrow \frac{n}{2} = 25, h = 10$

Now, 30 – 40 is the class whose cumulative frequency 36 is greater than $\frac{n}{2} = 25$.

Therefore, 30 – 40 is the median class. Thus, the lower limit (l) of the median class is 30.

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 30 + \frac{25 - 21}{15} \times 10$$

$$= 30 + \frac{4}{15} \times 10$$

$$= 30 + \frac{8}{3}$$

$$= 30 + 2.67$$

$$= \mathbf{32.67}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Find the zeroes of the polynomial $p(x) = x^3 - 5x^2 - 2x + 24$, if it is given that the product of its two zeroes is 12.

Solution. Let α, β, γ be the zeroes of the polynomial $p(x)$ such that

$$\alpha\beta = 12 \text{ (suppose)} \qquad ...(1)$$

We have :

$$\alpha + \beta + \gamma = -\left(\frac{-5}{1}\right) = 5 \qquad ...(2)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-2}{1} = -2 \qquad ...(3)$$

$$\alpha\beta\gamma = -\left(\frac{24}{1}\right) = -24 \qquad ...(4)$$

From (1) and (4), we get

$$12\gamma = -24 \Rightarrow \gamma = -2 \quad ...(5)$$

From (2) and (5), we get

$$\alpha + \beta + (-2) = 5$$

$$\Rightarrow \quad \alpha + \beta = 7 \quad ...(6)$$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow \quad (\alpha - \beta)^2 = (7)^2 - 4(12)$ [using (6) and (1)]

$\Rightarrow \quad (\alpha - \beta)^2 = 49 - 48 = 1$

$\Rightarrow \quad \alpha - \beta = \pm 1$

When $\alpha - \beta = 1$, and $\alpha + \beta = 7$

Solving these, we get

$$\alpha = 4 \text{ and } \beta = 3$$

When $\alpha - \beta = -1$, and $\alpha + \beta = 7$

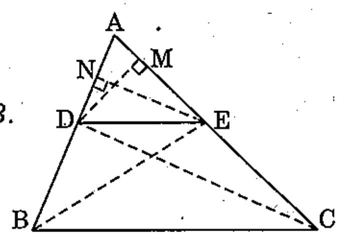
Solving these, we get

$$\alpha = 3 \text{ and } \beta = 4$$

Hence, the zeroes of the given polynomial $p(x)$ are **3, 4** and **– 2.**

30. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove this theorem.

Solution. Given : A triangle ABC in which a line parallel to BC intersects other two sides AB and AC at D and E respectively.



To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$.

Construction : Join BE, CD and draw $DM \perp AC$ and $EN \perp AB$.

Proof : Since EN is perpendicular to AB, therefore, EN is the height of triangles ADE and BDE.

$$\therefore \quad \text{ar}(\Delta ADE) = \frac{1}{2}(\text{base} \times \text{height})$$

$$= \frac{1}{2}(AD \times EN) \quad ...(1)$$

and

$$\text{ar}(\Delta BDE) = \frac{1}{2}(\text{base} \times \text{height})$$

$$= \frac{1}{2}(DB \times EN) \quad ...(2)$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{\frac{1}{2}(AD \times EN)}{\frac{1}{2}(DB \times EN)} \quad \text{[using (1) and (2)]}$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta BDE)} = \frac{AD}{DB} \quad ...(3)$$

$$\text{Similarly,} \quad \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta DEC)} = \frac{\frac{1}{2}(AE \times DM)}{\frac{1}{2}(EC \times DM)} = \frac{AE}{EC} \quad ...(4)$$

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

$\therefore \quad ar(\Delta BDE) = ar(\Delta DEC)$ ...(5)

From (4) and (5), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

Again from (3) and (6), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, $\boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$

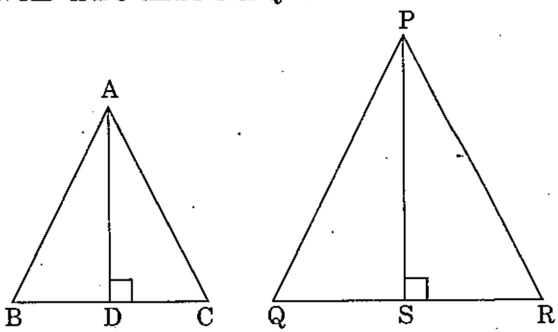
Or

Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.



Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently, $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\quad \frac{AB}{PQ} = \frac{BC}{QR}$

$$\Rightarrow \quad \frac{AD}{PS} = \frac{BC}{QR} \qquad \text{...(3) [using (2)]}$$

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} \qquad \text{[using (3)]}$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2} \qquad \text{...(4)}$$

As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \qquad \text{...(5)}$$

Hence, $\dfrac{\textbf{ar}(\Delta \boldsymbol{ABC})}{\textbf{ar}(\Delta \boldsymbol{PQR})} = \dfrac{\boldsymbol{AB}^2}{\boldsymbol{PQ}^2} = \dfrac{\boldsymbol{BC}^2}{\boldsymbol{QR}^2} = \dfrac{\boldsymbol{CA}^2}{\boldsymbol{RP}^2}$ [From (4) and (5)]

31. Prove that :

$$\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1 + \cos\theta}{\sin\theta}$$

Solution. We have

$$\text{L.H.S.} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - (\operatorname{cosec}^2\theta - \cot^2\theta)}{(\cot\theta - \operatorname{cosec}\theta) + 1} \qquad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)[1 - (\operatorname{cosec}\theta - \cot\theta)]}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)(\cot\theta - \operatorname{cosec}\theta + 1)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

$= \text{R.H.S.}$

Or

Evaluate : $\dfrac{\operatorname{cosec}\theta \sec(90^\circ - \theta) - \cot\theta \tan(90^\circ - \theta) + \sin^2 37^\circ + \sin^2 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

Solution. We have

$$\frac{\operatorname{cosec}\theta \sec(90° - \theta) - \cot\theta \tan(90° - \theta) + \sin^2 37° + \sin^2 53°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}$$

$$= \frac{\operatorname{cosec}\theta \cdot \operatorname{cosec}\theta - \cot\theta \cdot \cot\theta + \sin^2 37° + \sin^2(90° - 37°)}{\tan 5° \tan 25° (1) \tan(90° - 25°) \tan(90° - 5°)}$$

[$\because \sec(90° - \theta) = \operatorname{cosec}\theta, \tan(90° - \theta) = \cot\theta$]

$$= \frac{\operatorname{cosec}^2\theta - \cot^2\theta + \sin^2 37° + \cos^2 37°}{\tan 5° \tan 25° \cot 25° \cot 5°}$$

$$= \frac{(1 + \cot^2\theta) - \cot^2\theta + (\sin^2 37° + \cos^2 37°)}{(\tan 5° \cot 5°)(\tan 25° \cot 25°)}$$

$$= \frac{1+1}{(1)(1)}$$ [$\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta, \sin^2\theta + \cos^2\theta = 1, \tan\theta \cdot \cot\theta = 1$]

$= \mathbf{2}$

32. Prove that :

$$\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$$

Solution. We have

$$\text{L.H.S.} = \frac{1}{\sec x - \tan x} - \frac{1}{\cos x}$$

$$= \frac{\sec x + \tan x}{(\sec x - \tan x)(\sec x + \tan x)} - \frac{1}{\cos x}$$

[Multiplying numerator and denominator by $\sec x + \tan x$]

$$= \frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} - \sec x$$

$$= \frac{\sec x + \tan x}{(1 + \tan^2 x) - \tan^2 x} - \sec x$$

$$= \frac{\sec x + \tan x}{1} - \sec x$$

$= (\sec x + \tan x) - \sec x$

$= \sec x + (\tan x - \sec x)$

$$= \frac{1}{\cos x} - (\sec x - \tan x)$$

$$= \frac{1}{\cos x} - \frac{(\sec x - \tan x)}{\sec^2 x - \tan^2 x}$$ [$\because \sec^2 x = 1 + \tan^2 x \Rightarrow \sec^2 x - \tan^2 x = 1$]

$$= \frac{1}{\cos x} - \frac{(\sec x - \tan x)}{(\sec x - \tan x)(\sec x + \tan x)}$$

$$= \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$$

= R.H.S.

33. Draw the graph of the following pair of linear equations :

$$x - y + 1 = 0$$
$$3x + 2y - 12 = 0$$

Hence, find the area of the region bounded by $x = 0$, $y = 0$ and $3x + 2y - 12 = 0$.

Solution. Two solutions of each of the equations :

$$x - y + 1 = 0 \quad ...(1)$$
$$3x + 2y - 12 = 0 \quad ...(2)$$

are given in Tables below :

$x - y + 1 = 0$

x	-1	0
y	0	1
	A	B

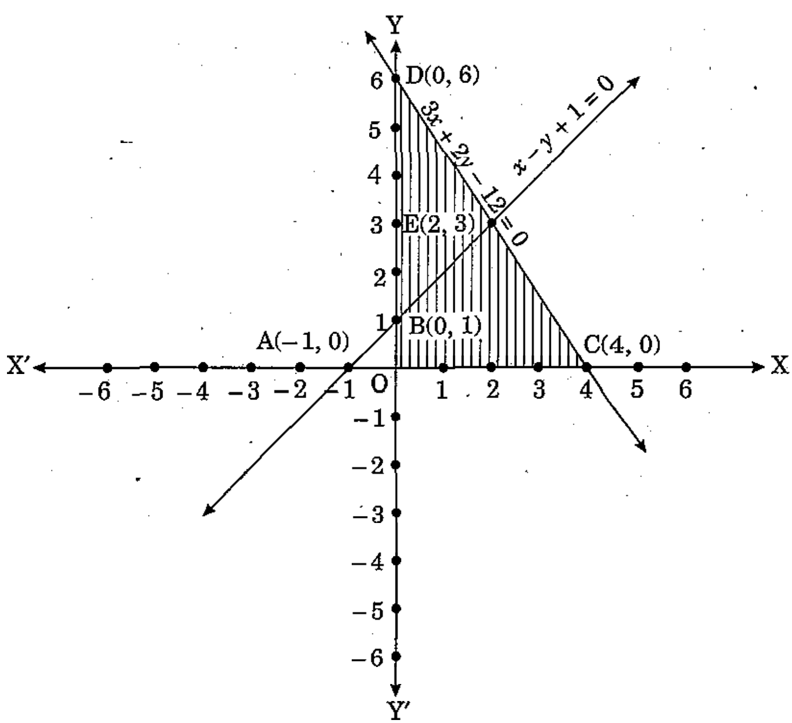
$3x + 2y - 12 = 0$

x	4	0
y	0	6
	C	D

Plot the points $A(-1, 0)$, $B(0, 1)$ and $C(4, 0)$, $D(0, 6)$ corresponding to the solutions in Tables. Now draw the lines AB and CD, representing the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$, as shown in figure.

In figure, we observe that the two lines representing the two equations are intersecting at the point (2, 3).

Hence, $x = 2$ and $y = 3$.



The shaded region of ΔOCD is the area of the region bounded by $x = 0, y = 0$ and $3x + 2y - 12 = 0$.

$\therefore$ Area of the shaded region $= \frac{1}{2}(\text{Base} \times \text{Height})$

$= \frac{1}{2}(OC \times OD)$

$= \frac{1}{2} \times 4 \times 6$ [$\because$ $OC = 4$ units, $OD = 6$ units]

$=$ **12 sq. units.**

34. The mean of the following data is 46.2. Find the missing frequencies f_1 and f_2:

Classes	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	***Total***
Frequencies	**6**	f_1	**16**	**13**	f_2	**2**	**50**

Solution. Here, the missing frequencies are f_1 and f_2

Calculation of Mean

Classes	*Mid-point* (x_i)	*Frequencies* (f_i)	($f_i \times x_i$)
20 – 30	25	6	150
30 – 40	35	f_1	$35f_1$
40 – 50	45	16	720
50 – 60	55	13	715
60 – 70	65	f_2	$65f_2$
70 – 80	75	2	150
Total		$n = 37 + f_1 + f_2$	$\sum_{i=1}^{6} f_i x_i = 1735 + 35f_1 + 65f_2$

We have, $n = 50 =$ Total frequency $= \sum_{i=1}^{6} f_i$

$\Rightarrow \quad 37 + f_1 + f_2 = 50$

$\Rightarrow \quad f_1 + f_2 = 50 - 37$

$\Rightarrow \quad f_1 + f_2 = 13$

$\Rightarrow \quad f_2 = 13 - f_1$ ...(1)

Also, Mean = 46.2 (given)

$\therefore \quad 46.2 = \sum_{i=1}^{6} \frac{f_i x_i}{n}$

$\Rightarrow \quad 46.2 = \frac{1735 + 35f_1 + 65f_2}{50}$ [$\because$ $n = 50$ (given)]

$\Rightarrow \quad 46.2 \times 50 = 1735 + 35f_1 + 65f_2$

$\Rightarrow \quad 2310 - 1735 = 35f_1 + 65(13 - f_1)$ [using (1)]

$\Rightarrow \quad 575 = 35f_1 + 845 - 65f_1$

$\Rightarrow \quad 575 - 845 = 35 f_1 - 65 f_1$

$\Rightarrow \quad -270 = -30 f_1$

$\Rightarrow \quad f_1 = \frac{270}{30} = 9$

Substituting $f_1 = 9$ in (1), we get

$$f_2 = 13 - 9 = 4$$

Hence, the missing frequencies f_1 and f_2 are **9** and **4** respectively.