CCE SAMPLE QUESTION PAPER

FIRST TERM (SA–I)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. Which of the following numbers has non-terminating repeating decimal expansion ?

(*a*) $\frac{7}{80}$ (*b*) $\frac{17}{320}$

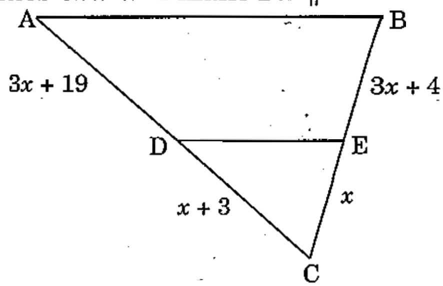
(*c*) $\frac{84}{400}$ (*d*) $\frac{93}{420}$

Solution. Choice (*d*) is correct.

$$\frac{93}{420} = \frac{31}{140} = \frac{31}{2^2 \times 5^1 \times 7^1}$$

$\because$ The denominator has a factor other than 2 or 5.

2. In figure, what values of *x* will make *DE* || *AB* ?



(*a*) 3 (*b*) 2
(*c*) 5 (*d*) 4

Solution. Choice (*b*) is correct.

In triangle *CAB*, if *DE* divides *CA* and *CB* in the same ratio, then $DE \parallel AB$.

$$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB}$$

$$\Rightarrow \quad \frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$$\Rightarrow \quad (x+3)(3x+4) = x(3x+19)$$

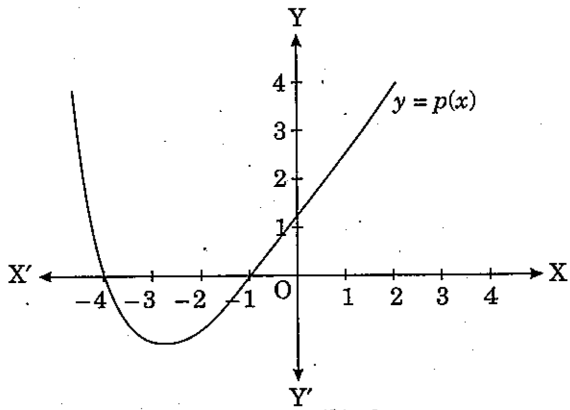
$$\Rightarrow \quad 3x^2 + 4x + 9x + 12 = 3x^2 + 19x$$

$$\Rightarrow \quad 19x - 4x - 9x = 12$$

$$\Rightarrow \quad 6x = 12$$

$$\Rightarrow \quad x = 2$$

3. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 1

Solution. Choice (*a*) is correct.

The number of zeroes of $p(x)$ is **2** as the graph intersects the *x*-axis at two points *viz.*, (– 4, 0) and (– 1, 0) in figure.

4. If sin 5θ = cos 4θ, where 5θ and 4θ are acute angles, then the value of θ is

(*a*) 15° (*b*) 8°
(*c*) 10° (*d*) 12°

Solution. Choice (*c*) is correct.

We have

$$\sin 5\theta = \cos 4\theta$$

$$\Rightarrow \quad \cos(90^\circ - 5\theta) = \cos 4\theta$$

$$\Rightarrow \quad 90^\circ - 5\theta = 4\theta$$

$$\Rightarrow \quad 4\theta + 5\theta = 90^\circ$$

$$\Rightarrow \quad 9\theta = 90^\circ$$

$$\Rightarrow \quad \boldsymbol{\theta = 10^\circ}$$

5. If $\tan\theta = \dfrac{12}{13}$, then the value of $\dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ is

(a) $\dfrac{307}{25}$ (b) $\dfrac{312}{25}$

(c) $\dfrac{309}{25}$ (d) $\dfrac{316}{25}$

Solution. Choice (b) is correct.

We have

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\sin\theta\cos\theta/\cos^2\theta}{(\cos^2\theta - \sin^2\theta)/\cos^2\theta}$$ [Dividing numerator and denominator by $\cos^2\theta$]

$$= \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{2 \times \frac{12}{13}}{1 - \left(\frac{12}{13}\right)^2}$$

$$= \frac{24/13}{1 - \frac{144}{169}}$$

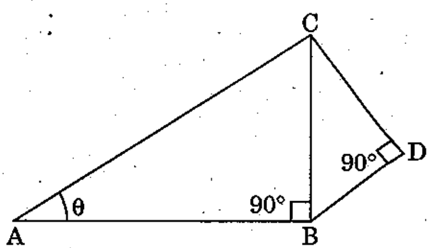
$$= \frac{24}{13} \times \frac{169}{169 - 144}$$

$$= \frac{24}{13} \times \frac{169}{25}$$

$$= \frac{24 \times 13}{25}$$

$$= \mathbf{\frac{312}{25}}$$

6. In figure, $AB = 5\sqrt{3}$ cm, $DC = 4$ cm, $BD = 3$ cm, then $\tan\theta$ is



(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{4}{\sqrt{3}}$

(d) $\frac{5}{\sqrt{3}}$

Solution. Choice (*a*) is correct.

In ΔCBD, we have

$$BC^2 = BD^2 + DC^2$$

$$\Rightarrow \quad BC^2 = (3)^2 + (4)^2 = 25 = (5)^2$$

$$\Rightarrow \quad BC = 5$$

In ΔABC, $\tan\theta = \frac{BC}{AB} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$.

7. If HCF (96, 404) = 4, then LCM (96, 404) is

(a) 9626

(b) 9696

(c) 9656

(d) 9676

Solution. Choice (*b*) is correct.

We know that ;

HCF × LCM = Product of two positive numbers

$$\Rightarrow \quad 4 \times \text{LCM} = 96 \times 404$$

$$\Rightarrow \quad \text{LCM} = \frac{96 \times 404}{4}$$

$$\Rightarrow \quad \text{LCM} = 96 \times 101$$

$$\Rightarrow \quad \textbf{LCM} = \mathbf{9696}$$

8. If the pair of linear equations $10x + 5y - (k - 5) = 0$ and $20x + 10y - k = 0$ have infinitely many solutions, then the value of k is

(a) 2

(b) 5

(c) 10

(d) 8

Solution. Choice (*c*) is correct.

For a pair of linear equations to have infinitely many solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} = \frac{k-5}{k}$$

$$\Rightarrow \quad k = 2k - 10$$

$$\Rightarrow \quad \mathbf{k = 10}$$

9. If $\tan\theta = \frac{3}{2}$, then the value of $\frac{(2 + 2\sec\theta)(1 - \sec\theta)}{(2 + 2\operatorname{cosec}\theta)(1 - \operatorname{cosec}\theta)}$ is

(a) $\frac{81}{16}$

(b) $\frac{75}{16}$

(c) $\frac{83}{16}$

(d) $\frac{77}{16}$

Solution. Choice (a) is correct.

$$\frac{(2+2\sec\theta)(1-\sec\theta)}{(2+2\,\text{cosec}\,\theta)(1-\text{cosec}\,\theta)}$$

$$= \frac{2(1+\sec\theta)(1-\sec\theta)}{2(1+\text{cosec}\,\theta)(1-\text{cosec}\,\theta)}$$

$$= \frac{2(1-\sec^2\theta)}{2(1-\text{cosec}^2\theta)}$$

$$= \frac{1-\sec^2\theta}{1-\text{cosec}^2\theta}$$

$$= \frac{1-(1+\tan^2\theta)}{1-(1+\cot^2\theta)}$$

$$= \frac{-\tan^2\theta}{-\cot^2\theta}$$

$$= \tan^2\theta \times \tan^2\theta$$

$$= \tan^4\theta$$

$$= \left(\frac{3}{2}\right)^4$$

$$= \mathbf{\frac{81}{16}}$$

10. The mean of first 20 natural numbers is

(a) 7.5 (b) 8.5

(c) 9.5 (d) 10.5

Solution. Choice (d) is correct.

Mean of first 20 natural numbers

$$= \frac{\text{Sum of observations from 1 to 20}}{\text{Number of observations}}$$

$$= \frac{1+2+\ldots\ldots+20}{20}$$

$$= \frac{\frac{20(20+1)}{2}}{20} \qquad \left[\because \text{Sum of first `}n\text{' natural numbers} = \frac{n(n+1)}{2}\right]$$

$$= \frac{21}{2}$$

$$= \mathbf{10.5}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Check whether 6^n can end with the digit 0 for any natural number n.

Solution. We know that any positive integer ending with the digit 0 is divisible by 5 and so its prime factorisation must contain the prime 5.

We have

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

$\Rightarrow$ There are two prime in the factorisation of $6^n = 2^n \times 3^n$

$\Rightarrow$ 5 does not occur in the prime factorisation of 6^n for any n.

[By uniqueness of the Fundamental Theorem of Arithmetic]

Hence, 6^n can never end with the digit **0** for any natural number.

12. Find the zeroes of the quadratic polynomial $8x^2 - 21 - 22x$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution. We have

$$\begin{aligned} 8x^2 - 21 - 22x &= 8x^2 - 22x - 21 \\ &= 8x^2 - 28x + 6x - 21 \qquad [8 \times (-21) = 6 \times (-28) \text{ and } -28 + 6 = -22] \\ &= 4x(2x - 7) + 3(2x - 7) \\ &= (2x - 7)(4x + 3) \end{aligned}$$

So, the value of $8x^2 - 22x - 21$ is zero, when $2x - 7 = 0$ or $4x + 3 = 0$ *i.e.*, when $x = \frac{7}{2}$ or $x = -\frac{3}{4}$.

Therefore, the zeroes of $8x^2 - 22x - 21$ are $\frac{7}{2}$ and $-\frac{3}{4}$.

$$\begin{aligned} \text{Now, sum of zeroes} &= \frac{7}{2} + \left(-\frac{3}{4}\right) \\ &= \frac{14 - 3}{4} \\ &= \frac{11}{4} \\ &= \frac{22}{8} \\ &= \frac{-(-22)}{8} \\ &= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \end{aligned}$$

$$\begin{aligned} \text{Product of zeroes} &= \frac{7}{2} \times \left(-\frac{3}{4}\right) \\ &= \frac{-21}{8} \\ &= \frac{(-21)}{8} \\ &= \frac{\text{Constant term}}{\text{Coefficient of } x^2}. \end{aligned}$$

13. *A* and *B* each have certain number of oranges. *A* says to *B*, "If you give me 10 of your oranges, I will have twice the number of oranges left with you". *B* replies, "If

you give me 10 of your oranges, I will have the same number of oranges as left with you". Find the number of oranges with A and B separately.

Solution. Let A has x number of oranges and B has y number of oranges.

Then, according to the given condition, we have.

$$x + 10 = 2(y - 10)$$

$$\Rightarrow \quad x + 10 = 2y - 20$$

$$\Rightarrow \quad x = 2y - 30 \qquad ...(1)$$

and $\quad y + 10 = x - 10$

$$\Rightarrow \quad x = y + 20 \qquad ...(2)$$

From (1) and (2), we have

$$2y - 30 = y + 20$$

$$\Rightarrow \quad 2y - y = 30 + 20$$

$$\Rightarrow \quad y = 50$$

Substituting $y = 50$ in (2), we get

$$x = 50 + 20$$

$$\Rightarrow \quad x = 70$$

Hence, A has **70 oranges** and B has **50 oranges.**

14. Without using trigonometric tables, find the value of

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 57^\circ \operatorname{cosec} 33^\circ - 2\cos 60^\circ$$

Solution. We have

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 57^\circ \operatorname{cosec} 33^\circ - 2\cos 60^\circ$$

$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \cos(90^\circ - 33^\circ).\operatorname{cosec} 33^\circ - 2\cos 60^\circ$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \sin 33^\circ.\operatorname{cosec} 33^\circ - 2\cos 60^\circ \qquad [\because \cos(90^\circ - \theta) = \sin\theta]$$

$$= 1 + 1 - 2 \times \frac{1}{2} \qquad \left[\because \sin\theta \cdot \operatorname{cosec}\theta = 1, \cos 60^\circ = \frac{1}{2}\right]$$

$$= 1 + 1 - 1$$

$$= \mathbf{1.}$$

Or

If A, B, C are interior angles of ΔABC, then show that

$$\cos\left(\frac{B + C}{2}\right) = \sin\frac{A}{2}$$

Solution. If A, B, C are interior angles of ΔABC, then

$$A + B + C = 180^\circ$$

$$\Rightarrow \quad B + C = 180^\circ - A$$

$$\Rightarrow \quad \frac{B + C}{2} = \frac{180^\circ - A}{2}$$

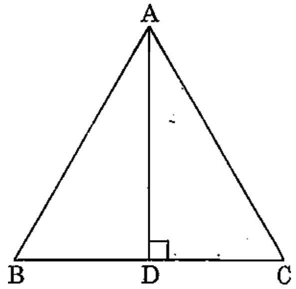
$$\Rightarrow \quad \frac{B + C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \quad \cos\left(\frac{B + C}{2}\right) = \cos\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \quad \cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}.$$

15. If ABC is an equilateral triangle with $AD \perp BC$, then prove $AD^2 = 3DC^2$.

Solution. Let ABC be an equilateral triangle and $AD \perp BC$.



In ΔADB and ΔADC, we have

$AB = AC$ [given]

$\angle B = \angle C$ [Each = 60°]

and $\angle ADB = \angle ADC$ [Each 90°]

$\therefore \quad \Delta ADB \cong \Delta ADC$

$\Rightarrow \quad BD = DC$ ...(1)

$\therefore \quad BC = BD + DC = DC + DC = 2DC$ ...(2) [using (1)]

In right angled ΔADC, we have

$AC^2 = AD^2 + DC^2$

$\Rightarrow \quad BC^2 = AD^2 + DC^2$ [$\because AC = BC$, sides of an equilateral Δ]

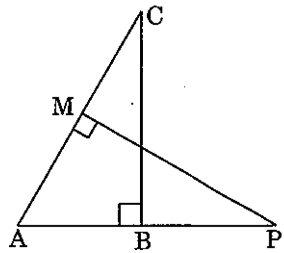
$\Rightarrow \quad (2DC)^2 = AD^2 + DC^2$ [using (2)]

$\Rightarrow \quad AD^2 = 4DC^2 - DC^2$

$\Rightarrow \quad \boldsymbol{AD^2 = 3DC^2}$

16. If in figure, ΔABC and ΔAMP are right angled at B and M respectively, prove that

$$CA \times MP = PA \times BC$$



Solution. In ΔABC and ΔAMP. we have

$\angle ABC = \angle AMP = 90°$

and $\angle BAC = \angle MAP$ [Each equal to $\angle A$]

Therefore, by AA-criterion of similarity, we have

$\Delta ABC \sim \Delta AMP$

$$\Rightarrow \quad \frac{CA}{BC} = \frac{PA}{MP}$$

$\Rightarrow \quad \boldsymbol{CA \times MP = PA \times BC}$

17. Given below is the distribution of marks obtained by 229 students :

Marks	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**	***Total***
No. of students	**12**	**30**	**34**	**65**	**45**	**25**	**18**	**229**

Write the above distribution as more than type cumulative frequency distribution.

Solution. Cumulative frequency table as more than type is given below :

Marks	*No. of students [Frequency (f)]*	*Marks more than*	*Cumulative frequency (cf)*
10 – 20	12	10	229 (217 + 12)
20 – 30	30	20	217 (187 + 30)
30 – 40	34	30	187 (153 + 34)
40 – 50	65	40	153 (65 + 88)
50 – 60	45	50	88 (45 + 43)
60 – 70	25	60	43 (25 + 18)
70 – 80	18	70	18

18. The mode of the following distribution is 55. Find the value of x.

Class-interval	**0 – 15**	**15 – 30**	**30 – 45**	**45 – 60**	**60 – 75**	**75 – 90**
Frequency	**6**	**7**	$\boldsymbol{x}$	**15**	**10**	**8**

Solution. Since mode = 55 (given), therefore, the modal class is 45 – 60. The lower limit (l) of the modal class is 45.

$$f_1 = 15, f_0 = x, f_2 = 10, h = 15$$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow \quad 55 = 45 + \frac{15 - x}{2 \times 15 - x - 10} \times 15$$

$$\Rightarrow \quad 55 - 45 = \frac{15 - x}{30 - x - 10} \times 15$$

$$\Rightarrow \quad 10 = \frac{15 - x}{20 - x} \times 15$$

$\Rightarrow \quad 200 - 10x = 225 - 15x$

$\Rightarrow \quad 15x - 10x = 225 - 200$

$\Rightarrow \quad 5x = 25$

$\Rightarrow \quad x = 5$

Hence, the value of x is **5**.

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $n^2 - n$ is divisible by 2 for any positive integer n.

Solution. We know that any positive integer is of the form $2m$ or $2m + 1$ for some positive integer m.

When $n = 2m$, then

$$\begin{aligned} n^2 - n &= (2m)^2 - 2m \\ &= 4m^2 - 2m \\ &= 2m(2m - 1) \\ &= 2p, \text{ where } p = m(2m - 1) \end{aligned}$$

$\Rightarrow \quad n^2 - n$ is divisible by 2

When $n = 2m + 1$, then

$$\begin{aligned} n^2 - n &= (2m + 1)^2 - (2m + 1) \\ &= (4m^2 + 4m + 1) - 2m - 1 \\ &= 4m^2 + 2m \\ &= 2m(2m + 1) \\ &= 2q, \text{ where } q = m(2m + 1) \end{aligned}$$

$\Rightarrow \quad n^2 - n$ is divisible by 2.

Hence, **$n^2 - n$ is divisible by 2 for any positive integer n.**

20. Prove that $\frac{7}{3}\sqrt{5}$ is irrational number.

Solution. Let us assume to the contrary that $\frac{7}{3}\sqrt{5}$ is rational.

Therefore, there exist co-prime positive integers p and q such that

$$\frac{7}{3}\sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \qquad \sqrt{5} = \frac{3p}{7q}$$

Since p and q are integers, we get $\frac{3p}{7q}$ is rational, and so $\frac{7}{3}\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{7}{3}\sqrt{5}$ is rational.

So, we conclude that $\frac{7}{3}\sqrt{5}$ is **irrational.**

Or

Show that $5 - 2\sqrt{3}$ is an irrational number.

Solution. Let us assume, to contrary, that $5 - 2\sqrt{3}$ is rational.
That is, we can find coprime a and b ($b \neq 0$) such that

$$5 - 2\sqrt{3} = \frac{a}{b}$$

Therefore, $2\sqrt{3} = 5 - \frac{a}{b}$

$$\Rightarrow \qquad 2\sqrt{3} = \frac{5b - a}{b}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{5b - a}{2b}$$

Since a and b are integers, we get $\frac{5b - a}{2b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - 2\sqrt{3}$ is rational.

So, we conclude that $5 - 2\sqrt{3}$ is **irrational.**

21. A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the numbers.

Solution. Let the unit's place digit be x and the ten's place digit be y.
Then, number = $10y + x$
According to the given condition, we have

$$10y + x = 8(x + y) + 1$$

$\Rightarrow \quad 8x - x = 10y - 8y - 1$

$\Rightarrow \quad 7x = 2y - 1$ ...(1)

and $\quad 10y + x = 13(y - x) + 2$

$\Rightarrow \quad x + 13x = 13y - 10y + 2$

$\Rightarrow \quad 14x = 3y + 2$ ...(2)

From (1) and (2), we get

$2(7x) = 3y + 2$

$\Rightarrow \quad 2(2y - 1) = 3y + 2$

$\Rightarrow \quad 4y - 2 = 3y + 2$

$\Rightarrow \quad 4y - 3y = 2 + 2$

$\Rightarrow \quad y = 4$

Substituting $y = 4$ in (1), we get

$7x = 2(4) - 1$

$\Rightarrow \quad 7x = 7$

$\Rightarrow \quad x = 1$

Hence, the number $= 10y + x$

$= 10(4) + 1$

$= \mathbf{41}$

Or

The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹ 200 and for journey of 15 km the charge paid is ₹ 275. What will a person have to pay for travelling a distance of 25 km ?

Solution. Let the fixed charges of taxi be ₹ x and the running charges of taxi be ₹ y per km.

Then, according to the given condition, we have

$x + 10y = 200$ ...(1)

and $\quad x + 15y = 275$ ...(2)

Subtracting (1) from (2), we get

$(x + 15y) - (x + 10y) = 275 - 200$

$\Rightarrow \quad 15y - 10y = 75$

$\Rightarrow \quad 5y = 75$

$\Rightarrow \quad y = 15$

Substituting $y = 15$ in (1), we get

$x + 10(15) = 200$

$\Rightarrow \quad x = 200 - 150$

$\Rightarrow \quad x = 50$

$\therefore$ Total charges for travelling a distance of 25 km

$= x + 25y$

$=$ ₹ $(50 + 25 \times 15)$

$=$ ₹ $(50 + 375)$

$=$ ₹ **425**

22. If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate $\dfrac{\alpha^2}{\beta^2} + \dfrac{\beta^2}{\alpha^2}$.

Solution. Since α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$.

$$\therefore \qquad \alpha + \beta = -\frac{b}{a} \qquad ...(1)$$

$$\text{and} \qquad \alpha\beta = \frac{c}{a} \qquad ...(2)$$

Now, $$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$= \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha^2 + \beta^2)^2 - 2\left(\frac{c}{a}\right)^2}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\frac{c^2}{a^2}}{\frac{c^2}{a^2}}$$

$$= \frac{\left[\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right]^2 - \frac{2c^2}{a^2}}{\frac{c^2}{a^2}}$$

$$= \frac{\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)^2 - \frac{2c^2}{a^2}}{\frac{c^2}{a^2}}$$

$$= \frac{\left(\frac{b^4}{a^4} + \frac{4c^2}{a^2} - \frac{4b^2c}{a^3}\right) - \frac{2c^2}{a^2}}{\frac{c^2}{a^2}}$$

$$= \frac{\frac{b^4}{a^4} + \frac{2c^2}{a^2} - \frac{4b^2c}{a^3}}{\frac{c^2}{a^2}}$$

$$= \frac{b^4 + 2c^2a^2 - 4b^2ca}{a^4} \times \frac{a^2}{c^2}$$

$$= \frac{b^4 + 2c^2a^2 - 4acb^2}{a^2c^2}$$

23. If cosec θ + cot θ = p, prove that $\cos\theta = \dfrac{p^2-1}{p^2+1}$.

Solution. We have

$$\operatorname{cosec}\theta + \cot\theta = p$$

$$\Rightarrow \quad \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = p$$

$$\Rightarrow \quad \frac{1+\cos\theta}{\sin\theta} = p$$

Squaring both sides, we have

$$\frac{(1+\cos\theta)^2}{\sin^2\theta} = p^2 \qquad \text{...(1)}$$

Now,

$$\frac{p^2-1}{p^2+1} = \frac{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2 - 1}{\left(\frac{1+\cos\theta}{\sin\theta}\right)^2 + 1}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{[(1+\cos\theta)^2 - \sin^2\theta]/\sin^2\theta}{[(1+\cos\theta)^2 + \sin^2\theta]/\sin^2\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{(1+\cos\theta)^2 - \sin^2\theta}{(1+\cos\theta)^2 + \sin^2\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{1+\cos^2\theta + 2\cos\theta - \sin^2\theta}{1+\cos^2\theta + 2\cos\theta + \sin^2\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{\cos^2\theta + 2\cos\theta + (1-\sin^2\theta)}{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{\cos^2\theta + 2\cos\theta + \cos^2\theta}{1+1+2\cos\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{2\cos^2\theta + 2\cos\theta}{2+2\cos\theta}$$

$$\Rightarrow \quad \frac{p^2-1}{p^2+1} = \frac{2\cos\theta(\cos\theta+1)}{2(1+\cos\theta)}$$

$$\Rightarrow \quad \mathbf{\frac{p^2-1}{p^2+1} = \cos\theta}$$

24. Show that

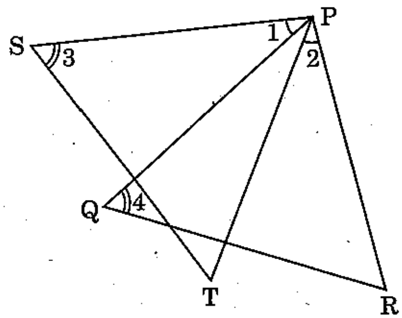
$$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$$

Solution.

$$\begin{aligned}
\text{L.H.S.} &= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 \\
&= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1 \\
&= 2[(\sin^2\theta + \cos^2\theta)\{(\sin^2\theta)^2 - (\sin^2\theta)(\cos^2\theta) + (\cos^2\theta)^2\}] - 3(\sin^4\theta + \cos^4\theta) + 1 \\
&\qquad [\text{using } a^3 + b^3 = (a+b)(a^2 - ab + b^2)] \\
&= 2[1(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)] - 3(\sin^4\theta + \cos^4\theta) + 1 \quad [\because \sin^2\theta + \cos^2\theta = 1] \\
&= 2\sin^4\theta - 2\sin^2\theta\cos^2\theta + 2\cos^4\theta - 3\sin^4\theta - 3\cos^4\theta + 1 \\
&= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1 \\
&= -[\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta] + 1 \\
&= -[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2(\sin^2\theta)(\cos^2\theta)] + 1 \\
&= -(\sin^2\theta + \cos^2\theta)^2 + 1 \qquad [\text{using } a^2 + b^2 + 2ab = (a+b)^2] \\
&= -1 + 1 \\
&= 0 \\
&= \text{R.H.S.}
\end{aligned}$$

25. In the figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$. Show that

$PT.QR = PR.ST$



Solution. In ΔPST and ΔPQR, we have

$\angle 1 = \angle 2$ [given]

$\Rightarrow \angle 1 + \angle QPT = \angle 2 + \angle QPT$ [Adding $\angle QPT$ on both sides]

$\Rightarrow \angle TPS = \angle RPQ$

and $\angle 3 = \angle 4$ [given]

$\Rightarrow \angle PST = \angle PQR$

$\therefore$ 3rd $\angle PTS$ = 3rd $\angle PRQ$ [$\because$ Sum of three angles of a triangle is 180°]

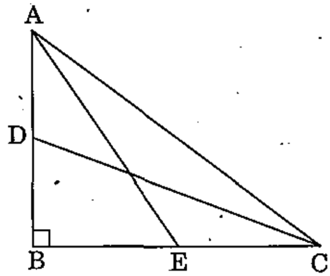
Thus, ΔPST and ΔPQR are equiangular, hence similar

$\therefore \Delta PST \sim \Delta PQR$

$\Rightarrow \dfrac{PT}{PR} = \dfrac{ST}{QR}$ [$\because$ Corresponding sides of similar Δs are proportional]

$\Rightarrow$ $\boldsymbol{PT.QR = PR.ST}$

26. In the figure, ABC is a triangle with $\angle B = 90°$. Medians AE and CD of respective lengths $\sqrt{40}$ cm and 5 cm are drawn. Find the length of the hypotenuse AC.



Solution. In right-angled ΔABE, we have

$$AE^2 = AB^2 + BE^2 \Rightarrow 40 = AB^2 + BE^2 \quad [\because AE = \sqrt{40}]$$

$$\Rightarrow \quad AB^2 = 40 - BE^2 = 40 - \left(\frac{BC}{2}\right)^2 \quad \left[\because BE = \frac{1}{2}BC\right]$$

$$\Rightarrow \quad AB^2 = 40 - \frac{BC^2}{4} \quad ...(1)$$

Also in right-angled ΔCBD, we have

$$CD^2 = BC^2 + BD^2 \Rightarrow 25 = BC^2 + BD^2 \quad [\because CD = 5]$$

$$\Rightarrow \quad BC^2 = 25 - BD^2 = 25 - \left(\frac{AB}{2}\right)^2 \quad \left[\because BD = \frac{1}{2}AB\right]$$

$$\Rightarrow \quad BC^2 = 25 - \frac{AB^2}{4} \quad ...(2)$$

In right-angled ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad AC^2 = 40 - \frac{BC^2}{4} + 25 - \frac{AB^2}{4} \quad \text{[using (1) and (2)]}$$

$$\Rightarrow \quad AC^2 = 65 - \frac{1}{4}(BC^2 + AB^2) = 65 - \frac{1}{4} \times AC^2$$

$$\Rightarrow \quad 4AC^2 = 260 - AC^2$$

$$\Rightarrow \quad 5AC^2 = 260$$

$$\Rightarrow \quad AC^2 = 260 \div 5 = 52$$

Hence, $AC = \sqrt{52} = \mathbf{2\sqrt{13}}$ **cm.**

27. Find mean of the following frequency distribution using step-deviation method :

Class-Interval	0 – 60	60 – 120	120 – 180	180 – 240	240 – 300
Frequency	22	35	44	25	24

Solution. Let the assumed mean be $a = 150$ and $h = 60$.

Class-Interval	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 150}{60}$	f_iu_i
0 – 60	22	30	– 2	– 44
60 – 120	35	90	– 1	– 35
120 – 180	44	$150 = a$	0	0
180 – 240	25	210	1	25
240 – 300	24	270	2	48
Total	$n = \Sigma f_i = 150$			$\Sigma f_iu_i = -6$

By step-deviation method,

$$\text{Mean} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$= 150 + \frac{(-6)}{150} \times 60$$

$$= 150 - \frac{12}{5}$$

$$= 150 - 2.4$$

$$= 147.6$$

Hence, the mean is **147.6.**

Or

The mean of the following distribution is 52.5. Find the value of *p*.

Classes	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**	**80 – 100**
Frequency	**15**	**22**	**37**	***p***	**21**

Solution.

Calculation of Mean

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	f_ix_i
0 – 20	15	10	150
20 – 40	22	30	660
40 – 60	37	50	1850
60 – 80	p	70	$70p$
80 – 100	21	90	1890
Total	$n = \Sigma f_i = 95 + p$		$\Sigma f_ix_i = 4550 + 70p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 52.5 = \frac{4550 + 70p}{95 + p}$$

$$\Rightarrow \quad 4987.5 + 52.5p = 4550 + 70p$$

$$\Rightarrow \quad 70p - 52.5p = 4987.5 - 4550$$

$\Rightarrow \quad 17.5p = 437.5$

$\Rightarrow \quad p = 437.5 \div 17.5$

$\Rightarrow \quad \mathbf{p = 25}$

28. A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained :

Height (in cm)	***Number of girls***
Less than 140	**4**
Less than 145	**11**
Less than 150	**29**
Less than 155	**40**
Less than 160	**46**
Less than 165	**51**

Find the median height.

Solution. To calculate the median height, we need to find the class-interval and their corresponding frequencies.

Height (in cm)	*No. of girls (f)*	*Cumulative frequency (cf)*
135 – 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

Here $\frac{n}{2} = \frac{51}{2} = 25.5$. Now 145 – 150 is the class whose cumulative frequency 29 is greater than $\frac{n}{2} = 25.5$.

$\therefore$ 145 – 150 is the median class.

From the table, $f = 18$, $cf = 11$, $h = 5$

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 145 + \frac{25.5 - 11}{18} \times 5$$

$$= 145 + \frac{14.5}{18} \times 5$$

$$= 145 + \frac{72.5}{18}$$

$$= 145 + 4.03$$

$$= 149.03$$

Hence, the median height is **149.03 cm.**

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If the median of the distribution given below is 28.5, find the values of x and y, if the total frequency is 60.

Class interval	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	***Total***
Frequency	**5**	x	**20**	**15**	y	**5**	**60**

Solution. Here the missing frequencies are x and y :

Class interval	*Frequency*	*Cumulative frequency*
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$
Total	60	

It is given that, $n = 60$ = Total frequency

$\Rightarrow \quad 45 + x + y = 60$

$\Rightarrow \quad x + y = 60 - 45$

$\Rightarrow \quad x + y = 15$ ...(1)

The median is 28.5 (given), which lies in the class 20 – 30.

So, l = lower limit of median class = 20

f = frequency of median class = 20

cf = cumulative frequency of class preceeding the median class = $5 + x$

h = class size = 10

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 28.5 = 20 + \left[\frac{\frac{60}{2} - (5 + x)}{20}\right] \times 10$$

$$\Rightarrow \quad 28.5 - 20 = \frac{30 - 5 - x}{2}$$

$\Rightarrow \quad 8.5 \times 2 = 25 - x$

$\Rightarrow \quad 17 = 25 - x$

$\Rightarrow \quad x = 25 - 17$

$\Rightarrow \quad x = 8$

Now, from (1), we get $8 + y = 15 \Rightarrow y = 15 - 8 = 7$

Hence, $\boldsymbol{x = 8}$ and $\boldsymbol{y = 7}$.

30. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = \dfrac{m^2 - 1}{n^2 - 1}$.

Solution. We have to find $\cos^2 A$ in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now,

$$\tan A = n \tan B \Rightarrow \tan B = \frac{1}{n} \tan A \Rightarrow \cot B = \frac{n}{\tan A}$$

and,
$$\sin A = m \sin B \Rightarrow \sin B = \frac{1}{m} \sin A \Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$$

Substituting the values of $\cot B$ and $\operatorname{cosec} B$ in $\operatorname{cosec}^2 B - \cot^2 B = 1$, we get

$$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \quad \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \quad m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow \quad m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow \quad m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow \quad m^2 - 1 = (n^2 - 1) \cos^2 A$$

$$\Rightarrow \quad \boldsymbol{\frac{m^2 - 1}{n^2 - 1} = \cos^2 A}$$

Or

Prove the identity :

$$\sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = 2 \sec\theta$$

Solution. L.H.S.

$$= \sqrt{\frac{(1 + \sin\theta)}{(1 - \sin\theta)} \times \frac{(1 + \sin\theta)}{(1 + \sin\theta)}} + \sqrt{\frac{(1 - \sin\theta)}{(1 + \sin\theta)} \times \frac{(1 - \sin\theta)}{(1 - \sin\theta)}}$$

$$= \frac{1 + \sin\theta}{\sqrt{1 - \sin^2\theta}} + \frac{1 - \sin\theta}{\sqrt{1 - \sin^2\theta}}$$

$$= \frac{1 + \sin\theta}{\sqrt{\cos^2\theta}} + \frac{1 - \sin\theta}{\sqrt{\cos^2\theta}}$$

$$= \frac{1 + \sin\theta}{\cos\theta} + \frac{1 - \sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta + \sec\theta - \tan\theta$
$= 2\sec\theta$
$=$ R.H.S.

31. Form the pair of linear equations in the following problem, and find their solutions graphically.

10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Solution. Let x and y be the number of girls and number of boys respectively who took part in a Mathematics quiz, then according to the given information, we have the required pair of linear equations as

$$x - y = 4 \quad \ldots(1)$$
$$x + y = 10 \quad \ldots(2)$$

Let us draw the graphs of the equations (1) and (2). For this, we find two solutions of each of the equations which are given in tables.

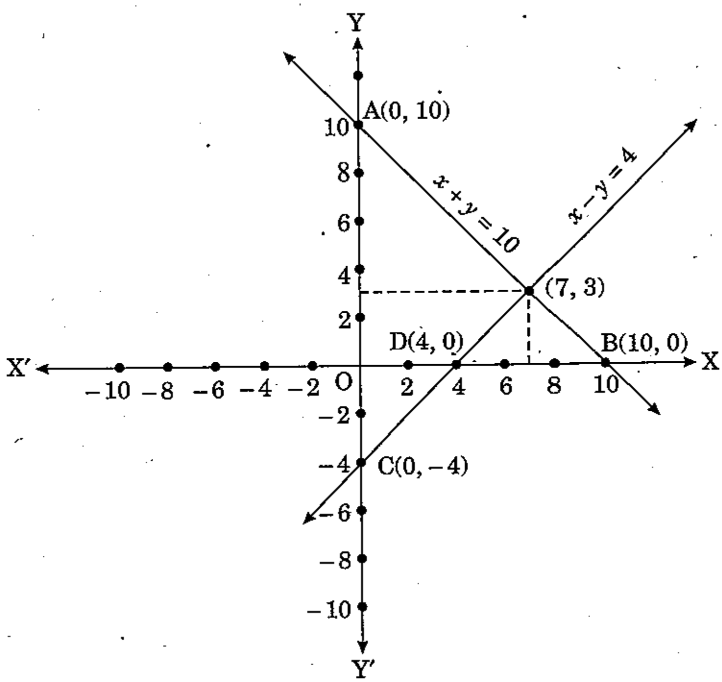
$x + y = 10$

x	0	10
$y = 10 - x$	10	0
	A	B

$x - y = 4$

x	0	4
$y = x - 4$	-4	0
	C	D

Plot the points $A(0, 10)$, $B(10, 0)$, $C(0, -4)$ and $D(4, 0)$ on graph paper, and join the points to form the lines AB and CD as shown in the figure.



The two lines (1) and (2) intersect at the point (7, 3). So, $x = 7, y = 3$ is the required solution of the pair of linear equations, *i.e.*, the number of girls and boys who took part in the quiz are **7** and **3**, respectively.

32. Prove that :

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\cos\theta+\sin\theta.$$

Solution. We have

$$\text{L.H.S.} = \frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}$$

$$= \frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\left(\dfrac{\cos\theta-\sin\theta}{\cos\theta}\right)}+\frac{\sin\theta}{\left(\dfrac{\sin\theta-\cos\theta}{\sin\theta}\right)}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta}+\frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{1}{\cos\theta-\sin\theta}[\cos^2\theta-\sin^2\theta]$$

$$= \frac{1}{\cos\theta-\sin\theta}[(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)]$$

$$= \cos\theta+\sin\theta$$

$$= \text{R.H.S.}$$

33. Find all zeroes of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if its two zeroes are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Solution. Since zeroes of a polynomial $f(x)$ are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$, therefore

$$\left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)=x^2-\frac{3}{2}$$

$$\Rightarrow \quad \left(x+\sqrt{\frac{3}{2}}\right)\left(x-\sqrt{\frac{3}{2}}\right)=\frac{1}{2}(2x^2-3)$$

is a factor of the given polynomial.

Now, we divide the given polynomial by $2x^2 - 3$.

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\ & \underline{\underset{-}{2x^4} \qquad \underset{+}{\mp} 3x^2} \\ & -2x^3 - 4x^2 + 3x + 6 \\ & \underline{\underset{+}{\mp} 2x^3 \qquad \underset{-}{\pm} 3x} \\ & -4x^2 \qquad + 6 \\ & \underline{\underset{+}{-} 4x^2 \qquad \underset{-}{+} 6} \\ & 0 \end{array}$$

$\left[\text{First term of the quotient is } \dfrac{2x^4}{2x^2} = x^2\right]$

$\left[\text{Second term of the quotient is } \dfrac{-2x^3}{2x^2} = -x\right]$

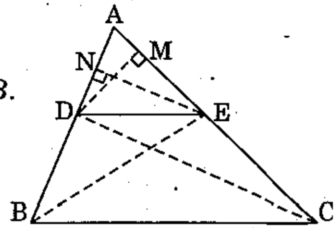
$\left[\text{Third term of the quotient is } \dfrac{-4x^2}{2x^2} = -2\right]$

So,
$$\begin{aligned} 2x^4 - 2x^3 - 7x^2 + 3x + 6 &= (2x^2 - 3)(x^2 - x - 2) \\ &= (2x^2 - 3)[x^2 - 2x + x - 2] \\ &= (2x^2 - 3)[x(x - 2) + (x - 2)] \\ &= 2\left(x^2 - \frac{3}{2}\right)(x + 1)(x - 2) \\ &= 2\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)(x + 1)(x - 2) \end{aligned}$$

Hence, all the zeroes of the given polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$ are $\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$, **$-1$ and 2.**

34. Prove that in a triangle, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution. **Given :** A triangle ABC in which a line parallel to BC intersects other two sides AB and AC at D and E respectively.



To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Construction : Join BE, CD and draw $DM \perp AC$ and $EN \perp AB$.

Proof : Since EN is perpendicular to AB, therefore, EN is the height of triangles ADE and BDE.

$$\therefore \quad ar(\Delta ADE) = \frac{1}{2}(\text{base} \times \text{height})$$

$$= \frac{1}{2}(AD \times EN) \qquad \ldots(1)$$

and
$$ar(\Delta BDE) = \frac{1}{2}(\text{base} \times \text{height})$$

$$= \frac{1}{2}(DB \times EN) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}(AD \times EN)}{\frac{1}{2}(DB \times EN)} \quad \text{[using (1) and (2)]}$$

$$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad ...(3)$$

$$\text{Similarly,} \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2}(AE \times DM)}{\frac{1}{2}(EC \times DM)} = \frac{AE}{EC} \quad ...(4)$$

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

$\therefore \quad ar(\Delta BDE) = ar(\Delta DEC)$ ...(5)

From (4) and (5), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

Again from (3) and (6), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, $\boldsymbol{\frac{AD}{DB} = \frac{AE}{EC}}$.

Or

Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution. Given : A right triangle ABC, right angled at B.

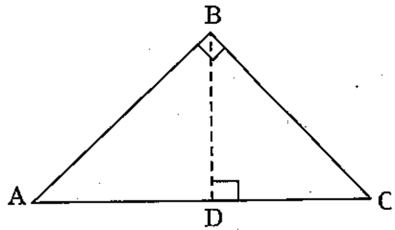
To prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]



So, $\frac{AD}{AB} = \frac{AB}{AC}$ [Sides are proportional]

$\Rightarrow AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\frac{CD}{BC} = \frac{BC}{AC}$ [Sides are proportional]

$\Rightarrow CD.AC = BC^2$ ...(2)

Adding (1) and (2) , we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$\Rightarrow AC.AC = AB^2 + BC^2$

Hence, $\boldsymbol{AC^2 = AB^2 + BC^2}$