CCE SAMPLE QUESTION PAPER

FIRST TERM (SA-I)

MATHEMATICS

(*With Solutions*)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$ is equal to

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) None of these

Solution. Choice (*b*) is correct.

$(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= 1$$

2. The value of tan 1° tan 2° tan 3° .... tan 89° is

(*a*) –1 (*b*) 1

(*c*) 0 (*d*) None of these

Solution. Choice (*b*) is correct.

$\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 44° \tan 45° \tan 46° \ldots\ldots \tan 87° \tan 88° \tan 89°$

$= \tan 1° \tan 2° \tan 3° \ldots\ldots \tan 44° (1) \tan(90°-44°) \ldots\ldots \tan(90°-3°)\tan(90°-2°)\tan(90°-1°)$

$= \tan 1° \tan 2° \tan 3° \ldots\ldots \tan 44° \cot 44° \ldots\ldots \cot 3° \cot 2° \cot 1°$

$= (\tan 1°.\cot 1°)(\tan 2°.\cot 2°)(\tan 3°.\cot 3°) \ldots\ldots (\tan 44° \cot 44°)$

$= (1)(1)(1) \ldots\ldots (1)$

$= \mathbf{1}$

3. If $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles, then the value of θ is

(*a*) 12° (*b*) 15°

(*c*) 18° (*d*) 20°

Solution. Choice (*b*) is correct.

$$\cos 2\theta = \sin 4\theta$$
$$\Rightarrow \sin(90° - 2\theta) = \sin 4\theta$$
$$\Rightarrow 90° - 2\theta = 4\theta$$
$$\Rightarrow 90° = 4\theta + 2\theta$$
$$\Rightarrow 6\theta = 90°$$
$$\Rightarrow \theta = 90° \div 6$$
$$\Rightarrow \boldsymbol{\theta = 15°}$$

4. If $5 \tan \alpha = 4$, then the value of $\dfrac{5 \sin \alpha - 3 \cos \alpha}{\sin \alpha + 2 \cos \alpha}$ is

(*a*) $\frac{5}{14}$ (*b*) $\frac{5}{13}$

(*c*) $\frac{5}{12}$ (*d*) $\frac{5}{11}$

Solution. Choice (*a*) is correct.

$$\frac{5 \sin \alpha - 3 \cos \alpha}{\sin \alpha + 2 \cos \alpha} = \frac{(5 \sin \alpha - 3 \cos \alpha)/\cos \alpha}{(\sin \alpha + 2 \cos \alpha)/\cos \alpha}$$ [Dividing numerator and denominator by $\cos \alpha$]

$$= \frac{\dfrac{5 \sin \alpha}{\cos \alpha} - \dfrac{3 \cos \alpha}{\cos \alpha}}{\dfrac{\sin \alpha}{\cos \alpha} + 2\dfrac{\cos \alpha}{\cos \alpha}}$$

$$= \frac{5 \tan \alpha - 3}{\tan \alpha + 2}$$

$$= \frac{5\left(\dfrac{4}{5}\right) - 3}{\dfrac{4}{5} + 2}$$

$$= \frac{4 - 3}{14/5}$$

$$= \mathbf{\frac{5}{14}}$$

5. If $\Delta ABC \sim \Delta PQR$, area ($\Delta ABC$) = 81 cm^2, area ($\Delta PQR$) = 144 cm^2 and QR = 6 cm, then the length of BC is

(*a*) 4.5 cm (*b*) 5 cm

(*c*) 9 cm (*d*) 10 cm

Solution. Choice (*a*) is correct.

Since $\Delta ABC \sim \Delta PQR$, we have

$$\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{81}{144} = \left(\frac{BC}{QR}\right)^2$$

$$\Rightarrow \quad \left(\frac{9}{12}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

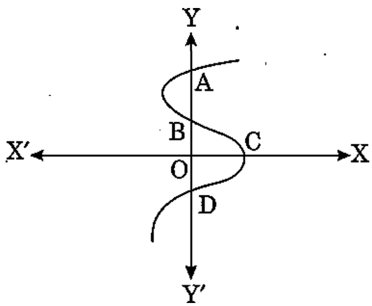
$$\Rightarrow \quad \frac{9}{12} = \frac{BC}{QR}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{BC}{6} \qquad [\because QR = 6 \text{ cm (given)}]$$

$$\Rightarrow \quad BC = \frac{3 \times 6}{4}$$

$$\Rightarrow \quad BC = \frac{18}{4} = \mathbf{4.5\ cm.}$$

6. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Solution. Choice (*a*) is correct.

The number of zeroes is 1 as the graph intersects the x-axis at one point C in figure.

7. The HCF of two numbers is 18 and their product is 12960. Their LCM will be

(*a*) 600 (*b*) 720

(*c*) 420 (*d*) 800

Solution. Choice (*b*) is correct.

LCM × HCF = Product of two positive numbers a and b.

$\Rightarrow \quad \text{LCM} \times 18 = 12960$

$\Rightarrow \quad \text{LCM} = \frac{12960}{18}$

$\Rightarrow \quad \text{LCM} = \mathbf{720.}$

8. The decimal expansion of $\frac{189}{125}$ will terminate after how many places of decimal ?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 1

Solution. Choice (*b*) is correct.

$$\frac{189}{125} = \frac{189}{5^3} = \frac{189 \times 2^3}{2^3 \times 5^3} = \frac{1512}{(2 \times 5)^3} = \frac{1512}{10^3} = 1.512$$

Thus, the decimal expansion of $\frac{189}{125}$ will terminate after **three** places of decimal.

9. The algebraic sum of the deviations of a frequency distribution from its mean is

(*a*) zero (*b*) always positive

(*c*) always negative (*d*) a non-zero number.

Solution. Choice (*a*) is correct.

$$\begin{aligned} \Sigma f_i(x_i - \bar{x}) &= \Sigma f_i x_i - \Sigma f_i \bar{x} \\ &= n\bar{x} - \bar{x}\Sigma f_i \\ &= n\bar{x} - \bar{x}.n \\ &= 0 \end{aligned}$$

10: If the linear pair of equations $10x + 5y - (k - 5) = 0$ and $20x + 10y - k = 0$ has infinitely many solutions, then the value of k is

(*a*) 5 (*b*) 10

(*c*) 12 (*d*) 15

Solution. Choice (*b*) is correct.

For a pair of linear equations to have infinitely many solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{k-5}{k}$$

$$\Rightarrow \quad k = 2k - 10$$

$$\Rightarrow \quad \mathbf{k = 10.}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. What are the quotient and the remainder, when $3x^4 + 5x^3 - 7x^2 + 2x + 2$ is divided by $x^2 + 3x + 1$?

Solution. We have

$$\begin{array}{r|l} & 3x^2 - 4x + 2 \\ x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\ & \underline{\underset{-}{}3x^4 \underset{-}{+} 9x^3 \underset{-}{+} 3x^2} \\ & -4x^3 - 10x^2 + 2x + 2 \\ & \underline{\underset{+}{-}4x^3 \underset{+}{-} 12x^2 \underset{+}{-} 4x} \\ & 2x^2 + 6x + 2 \\ & \underline{\underset{-}{}2x^2 \underset{-}{+} 6x \underset{-}{+} 2} \\ & 0 \end{array}$$

$\left[\text{First term of quotient is } \frac{3x^4}{x^2} = 3x^2\right]$

$\left[\text{Second term of quotient is } \frac{-4x^3}{x^2} = -4x\right]$

$\left[\text{Third term of quotient is } \frac{2x^2}{x^2} = 2\right]$

Clearly, the quotient is $\mathbf{3x^2 - 4x + 2}$ and remainder = **0**.

12. Prove that $\sqrt{3}$ is an irrational number.

Solution. Let us assume, to the contrary, that $\sqrt{3}$ is rational. Then

$$\sqrt{3} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0.$$

Suppose p and q have a common factor other than 1, then we can divide by the common factor, to get

$$\sqrt{3} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are coprime.}$$

So, $\sqrt{3}\, b = a$

Squaring on both sides, and rearranging, we get

$3b^2 = a^2 \Rightarrow a^2$ is divisible by 3 $\Rightarrow a$ is also divisible by 3

[If r (prime) divides a^2, then r divides a]

Let $a = 3m$, where m is an integer.

Substituting $a = 3m$ in $3b^2 = a^2$, we get

$3b^2 = 9m^2 \Rightarrow b^2 = 3m^2$

This means that b^2 is divisible by 3, and so b is also divisible by 3. Therefore, a and b have at least 3 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

So, we conclude that $\sqrt{3}$ is **irrational.**

13. For which value of k will the following system of linear equations have no solution ?

$$3x + y = 1$$
$$(2k - 1)x + (k - 1)y = 2k + 1$$

Solution. We know that the system of linear equations

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

has no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given system of equations will have no solution, if

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1} \text{ and } \frac{1}{k-1} \neq \frac{1}{2k+1}$$

Now, $$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$\Rightarrow \quad 3k-3 = 2k-1$$

$$\Rightarrow \quad k = 2$$

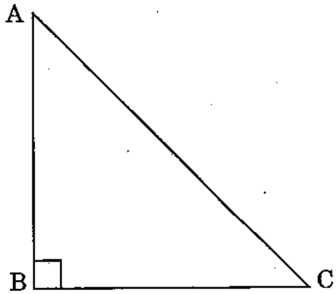
Clearly, for $k = 2$, we have

$$\frac{1}{k-1} \neq \frac{1}{2k+1}$$

Hence, the given system of linear equations will have no solution, if $k = 2$.

14. In a right triangle ABC, right angled at B, if $\tan A = 1$, then verify that $2 \sin A \cos A = 1$.

Solution. In ΔABC,



$$\tan A = \frac{BC}{AB} \text{ (see figure)}$$

$$\Rightarrow \quad 1 = \frac{BC}{AB} \qquad [\because \tan A = 1 \text{ (given)}]$$

$$\Rightarrow \quad AB = BC$$

Let $AB = BC = k$, where k is a positive number.

In ΔABC, $AC^2 = AB^2 + BC^2$

$$\Rightarrow \quad AC^2 = k^2 + k^2 = 2k^2$$

$$\Rightarrow \quad AC = \sqrt{2}\,k$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}} \text{ and } \cos A = \frac{AB}{AC} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}}$$

Now, $2 \sin A \cos A = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{2} = 1$, which is the required value.

Or

Show that :

$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta.$$

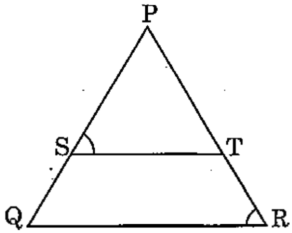
Solution. We have

$$\begin{aligned}
\text{L.H.S.} &= (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 \\
&= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\cdot\operatorname{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\cdot\sec\theta) \\
&= (\sin^2\theta + \cos^2\theta) + \operatorname{cosec}^2\theta + \sec^2\theta + 2\sin\theta\cdot\frac{1}{\sin\theta} + 2\cos\theta\cdot\frac{1}{\cos\theta} \\
&= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2 \qquad \left[\because \begin{array}{l}\sec^2\theta = 1 + \tan^2\theta \\ \operatorname{cosec}^2\theta = 1 + \cot^2\theta\end{array}\right] \\
&= (1 + 1 + 1 + 2 + 2) + \tan^2\theta + \cot^2\theta \\
&= 7 + \tan^2\theta + \cot^2\theta \\
&= \text{R.H.S.}
\end{aligned}$$

15. In figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.

Solution. We have

$$\frac{PS}{SQ} = \frac{PT}{TR}$$



$\Rightarrow \quad ST \parallel QR$ [By using the converse of BPT]

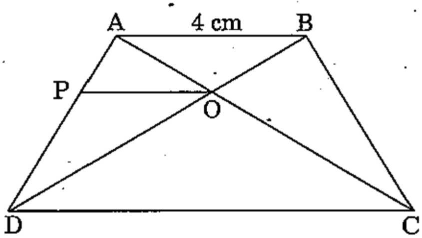
$\Rightarrow \quad \angle PST = \angle PQR$ [Corresponding $\angle s$]

$\Rightarrow \quad \angle PRQ = \angle PQR$ [$\because \angle PST = \angle PRQ$ (given)]

$\Rightarrow \quad PQ = PR$ [$\because$ Sides opposite to equal $\angle$s are equal]

$\Rightarrow \Delta PQR$ is an **isosceles triangle.**

16. In the figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$, and $AB = 4$ cm, find the value of DC.



Solution. In the quadrilateral $ABCD$,

$$\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2} \text{ (given)}$$

$$\Rightarrow \quad \frac{OC}{OA} = \frac{OD}{BO} = \frac{2}{1}$$

$$\Rightarrow \quad 1 + \frac{OC}{OA} = 1 + \frac{OD}{BO} = 1 + \frac{2}{1}$$

$$\Rightarrow \quad \frac{OA + OC}{OA} = \frac{BO + OD}{BO} = \frac{1+2}{1}$$

$$\Rightarrow \quad \frac{AC}{OA} = \frac{BD}{BO} = \frac{3}{1}$$

$$\therefore \quad \frac{AO}{AC} = \frac{BO}{BD} = \frac{1}{3}$$

Through O draw $OP \parallel CD$ and so $\parallel AB$.

Then in ΔACD,

$$\frac{AO}{AC} = \frac{OP}{DC} = \frac{1}{3}$$

$\therefore \quad 3OP = DC$ ...(1)

In ΔABD,

$$\frac{OD}{BD} = \frac{OP}{AB} = \frac{2}{3}$$

$\therefore \quad 3OP = 2AB$ ...(2)

From (1) and (2), we have

$CD = 2AB = 2 \times 4$ cm = **8 cm.** [using $AB = 4$ cm (given)]

17. In a continuous frequency distribution if lower limit of modal class = 125, frequency of modal class = 20, frequency of the class preceding the modal class = 13, frequency of the class succeeding the modal class = 14 and size of each class is 20. *Find the mode of the distribution.*

Solution. Given, $l = 125, f_1 = 20, f_0 = 13, f_2 = 14, h = 20$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 125 + \frac{20 - 13}{2 \times 20 - 13 - 14} \times 20$$

$$= 125 + \frac{7}{40 - 27} \times 20$$

$$= 125 + \frac{140}{13}$$

$$= 125 + 10.76$$

$$= \mathbf{135.76}$$

18. The following cumulative frequency distribution gives daily wages of 60 workers.

Daily wages (in ₹)	*No. of workers*
More than or equal to 300	**0**
More than or equal to 250	**12**
More than or equal to 200	**21**
More than or equal to 150	**44**
More than or equal to 100	**53**
More than or equal to 50	**59**
More than or equal to 0	**60**

Write the above cumulative frequency distribution as frequency distribution.

Solution.

Daily wages (in ₹)	*No. of workers (cf)*	*Daily wages (in ₹)*	*No. of workers (Frequency)*
More than or equal to 300	0	0 – 50	1 (60 – 59)
More than or equal to 250	12	50 – 100	6 (59 – 53)
More than or equal to 200	21	100 – 150	9 (53 – 44)
More than or equal to 150	44	150 – 200	23 (44 – 21)
More than or equal to 100	53	200 – 250	9 (21 – 12)
More than or equal to 50	59	250 – 300	12 (12 – 0)
More than or equal to 0	60		

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Prove that the square of any positive integer is of the form $4q$ or $4q + 1$ for some positive integer.

Solution. Let x be any positive integer then it is of the form $4m$, $4m + 1$, $4m + 2$ or $4m + 3$

When $x = 4m$, then by squaring, we have

$$x^2 = (4m)^2 = 16m^2 = 4(4m)^2 = 4q, \text{ where } q = 4m^2$$

When $x = 4m + 1$, then by squaring, we have

$$x^2 = (4m + 1)^2 = 16m^2 + 8m + 1$$

$\Rightarrow$ $x^2 = 4(4m^2 + 2m) + 1$

$\Rightarrow$ $x^2 = 4q + 1$, where $q = 4m^2 + 2m$

When $x = 4m + 2$, then by squaring, we have

$$x^2 = (4m + 2)^2 = 16m^2 + 16m + 4$$

$\Rightarrow$ $x^2 = 4(4m^2 + 4m + 1)$

$\Rightarrow$ $x^2 = 4q$, where $q = 4m^2 + 4m + 1$

When $x = 4m + 3$, then by squaring, we have

$$x^2 = (4m + 3)^2 = 16m^2 + 24m + 9$$

$\Rightarrow$ $x^2 = 4(4m^2 + 6m + 2) + 1$

$\Rightarrow$ $x^2 = 4q + 1$, where $q = 4m^2 + 6m + 2$

Hence, the square of any positive integer, say x, is of the form $\mathbf{4q}$ or $\mathbf{4q + 1}$ for some positive integer.

Or

Show that $5 - \sqrt{3}$ is irrational.

Solution. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational,

i.e., we can find coprime a and b ($b \neq 0$) such that

$$5 - \sqrt{3} = \frac{a}{b}$$

$\Rightarrow$ $$5 - \frac{a}{b} = \sqrt{3}$$

Since a and b are integers, we get, $5 - \dfrac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is **irrational**.

20. Find the HCF of 65 and 117 and express in the form $65m + 117n$.

Solution. Given integers are 65 and 117.

Apply Euclid's division lemma to 65 and 117, we get

$$117 = 65 \times 1 + 52 \qquad ...(1)$$

$$65 = 52 \times 1 + 13 \qquad ...(2)$$

$$52 = 13 \times 4 + 0 \qquad ...(3)$$

In equation (3), the remainder is zero. So, the last divisor or the non-zero remainder at the earliest stage, *i.e.*, in equation (2) is 13. Therefore, HCF of 65 and 117 is **13.**

From (2), we have

$$\begin{aligned} 13 &= 65 - 52 \times 1 \\ &= 65 - [117 - 65 \times 1] \times 1 \quad \text{[using (1)]} \\ &= 65 - 117 \times 1 + 65 \times 1 \\ &= 65 \times (1 + 1) - 117 \times 1 \\ &= 65 \times 2 - 117 \times 1 \\ &= 65 \times 2 + 117 \times (-1) \end{aligned}$$

Compare $65 \times 2 + 117 \times (-1)$ with $65m + 117n$, we get

$$m = 2 \quad \text{and} \quad n = -1.$$

21. A man travels 400 km partly by train and partly by car. If he covers half the distance by train and the rest by car, it takes him 4 hours 30 minutes. But if he travels 100 km by train and the rest by car, he takes 15 minutes longer. Find the speed of the train and that of the car.

Solution. Let the speed of the train be x km/h and the speed of the car be y km/h.

Total distance covered = 400 km.

When a man travels half the distance by train and the rest by car.

If he covers 200 km by train, then distance covered by car is (400 – 200) km = 200 km.

$\therefore$ Time taken to cover 200 km by train = $\frac{200}{x}$ hours

and time taken to cover 200 km by car = $\frac{200}{y}$ hours

Thus, the total time taken to cover a distance of 400 km = $\left(\frac{200}{x} + \frac{200}{y}\right)$ hours

It is given that the total time of journey is 4 hours and 30 minutes

$$\therefore \quad \frac{200}{x} + \frac{200}{y} = 4\frac{1}{2}$$

$$\Rightarrow \quad \frac{200}{x} + \frac{200}{y} = \frac{9}{2} \quad \ldots(1)$$

When a man travel 100 km by train and the rest by car.

If he covers 100 km by train, then distance covered by car is (400 – 100) km = 300 km

$\therefore$ Time taken to cover 100 km by train = $\frac{100}{x}$ hours

and time taken to cover 300 km by car = $\frac{300}{y}$ hours

Thus, the total time taken to cover a distance of 400 km = $\left(\frac{100}{x} + \frac{300}{y}\right)$ hours

In this case, the total time of the journey is 4 hours and 45 minutes.

$$\therefore \quad \frac{100}{x} + \frac{300}{y} = 4\frac{45}{60}$$

$$\Rightarrow \quad \frac{100}{x} + \frac{300}{y} = 4\frac{3}{4}$$

$$\Rightarrow \quad \frac{100}{x} + \frac{300}{y} = \frac{19}{4} \quad \quad ...(2)$$

From (2), we get

$$\frac{100}{x} = \frac{19}{4} - \frac{300}{y} \quad \quad ...(3)$$

Putting this value of $\frac{100}{x}$ in (1), we obtain

$$2\left(\frac{19}{4} - \frac{300}{y}\right) + \frac{200}{y} = \frac{9}{2}$$

$$\Rightarrow \quad \frac{19}{2} - \frac{600}{y} + \frac{200}{y} = \frac{9}{2}$$

$$\Rightarrow \quad \frac{19}{2} - \frac{9}{2} = \frac{600}{y} - \frac{200}{y}$$

$$\Rightarrow \quad 5 = \frac{400}{y}$$

$$\Rightarrow \quad y = \frac{400}{5} = 80 \text{ km/h}$$

Now, substituting $y = 80$ in (3), we get

$$\frac{100}{x} = \frac{19}{4} - \frac{300}{80}$$

$$\Rightarrow \quad \frac{100}{x} = \frac{380 - 300}{80}$$

$$\Rightarrow \quad \frac{100}{x} = \frac{80}{80}$$

$$\Rightarrow \quad x = 100 \text{ km/h}$$

Hence, speed of the train = **100 km/h** and speed of car = **80 km/h.**

Or

Solve for x and y :

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2$$
$$(a + b)(x + y) = a^2 + b^2$$

Solution. The given linear equations in x and y are

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2 \quad \quad ...(1)$$

$$(a + b)(x + y) = a^2 + b^2 \quad \quad ...(2)$$

Re-writing (2) as

$$(a + b)x + (a + b)y = a^2 + b^2 \quad \quad ...(3)$$

Subtracting (1) from (3), we get

$$[(a + b)x + (a + b)y] - [(a - b)x + (a + b)y] = (a^2 + b^2) - (a^2 - 2ab - b^2)$$

$$\Rightarrow \quad (a + b)x - (a - b)x = a^2 + b^2 - a^2 + 2ab + b^2$$

$$\Rightarrow \quad [(a + b) - (a - b)]x = 2b^2 + 2ab$$

$$\Rightarrow \quad (a + b - a + b)x = 2b(b + a)$$
$$\Rightarrow \quad 2bx = 2b(b + a)$$
$$\Rightarrow \quad x = b + a \qquad \text{[Cancelling } 2b \text{ from both sides]}$$

Substituting $x = a + b$ in (1), we obtain

$$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$$
$$\Rightarrow \quad a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$$
$$\Rightarrow \quad (a + b)y = a^2 - 2ab - b^2 - (a^2 - b^2)$$
$$\Rightarrow \quad (a + b)y = a^2 - 2ab - b^2 - a^2 + b^2$$
$$\Rightarrow \quad (a + b)y = -2ab$$
$$\Rightarrow \quad y = \frac{-2ab}{a + b}$$

Hence, the solution is $\boldsymbol{x = a + b}$ and $\boldsymbol{y = \frac{-2ab}{a + b}}$.

22. Find the zeroes of the quadratic polynomial

$$\boldsymbol{f(x) = abx^2 + (b^2 - ac)x - bc}$$

and verify the relationship between the zeroes and its coefficients.

Solution. We have

$$f(x) = abx^2 + (b^2 - ac)x - bc$$
$$= abx^2 + b^2x - acx - bc$$
$$= bx(ax + b) - c(ax + b)$$
$$\Rightarrow \quad f(x) = (ax + b)(bx - c)$$

The zeroes of $f(x)$ are given by

$$f(x) = 0$$
$$\Rightarrow \quad (ax + b)(bx - c) = 0$$
$$\Rightarrow \quad ax + b = 0 \quad \text{or} \quad bx - c = 0$$
$$\Rightarrow \quad x = \frac{-b}{a} \quad \text{or} \quad x = \frac{c}{b}$$

Thus, the zeroes of $f(x)$ are : $\boldsymbol{\frac{-b}{a}}$ and $\boldsymbol{\frac{c}{b}}$

Now, $\quad$ Sum of the zeroes $= -\frac{b}{a} + \frac{c}{b}$

$$= \frac{-b^2 + ac}{ab}$$
$$= \frac{-(b^2 - ac)}{ab}$$
$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

and $\quad$ product of the zeroes $= \left(\frac{-b}{a}\right) \times \left(\frac{c}{b}\right)$

$$= \frac{-bc}{ab}$$
$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

23. Prove that :

$$\mathbf{2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta}$$

Solution. We have

$$\begin{aligned} \text{L.H.S.} &= 2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta \\ &= (2 \sec^2 \theta - \sec^4 \theta) - (2 \operatorname{cosec}^2 \theta - \operatorname{cosec}^4 \theta) \\ &= \sec^2 \theta(2 - \sec^2 \theta) - \operatorname{cosec}^2 \theta(2 - \operatorname{cosec}^2 \theta) \\ &= \sec^2 \theta[2 - (1 + \tan^2 \theta)] - \operatorname{cosec}^2 \theta[2 - (1 + \cot^2 \theta)] \\ &= \sec^2 \theta[1 - \tan^2 \theta] - \operatorname{cosec}^2 \theta[1 - \cot^2 \theta] \\ &= (1 + \tan^2 \theta)(1 - \tan^2 \theta) - (1 + \cot^2 \theta)(1 - \cot^2 \theta) \\ &= (1 - \tan^4 \theta) - (1 - \cot^4 \theta) \\ &= 1 - \tan^4 \theta - 1 + \cot^4 \theta \\ &= \cot^4 \theta - \tan^4 \theta \\ &= \text{R.H.S.} \end{aligned}$$

24. If $a \cos \theta + b \sin \theta = c$, then prove that $a \sin \theta - b \cos \theta = \pm\sqrt{a^2 + b^2 - c^2}$.

Solution. Given, $a \cos \theta + b \sin \theta = c$

Squaring both sides, we have

$$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = c^2$$

$$\Rightarrow a^2(1 - \sin^2 \theta) + b^2(1 - \cos^2 \theta) + 2ab \sin \theta \cos \theta = c^2$$

$$\Rightarrow a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta = c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta$$

$$\Rightarrow a^2 + b^2 - c^2 = (a \sin \theta)^2 + (b \cos \theta)^2 - 2(a \sin \theta)(b \cos \theta)$$

$$\Rightarrow a^2 + b^2 - c^2 = (a \sin \theta - b \cos \theta)^2$$

$$\Rightarrow \mathbf{a \sin \theta - b \cos \theta = \pm\sqrt{a^2 + b^2 - c^2}}.$$

25. In a trapezium *ABCD*, *O* is the point of intersection of *AC* and *BD*, *AB* || *CD* and *AB* = 2*CD*. If the area of Δ*AOB* = 84 cm², find the area of Δ*COD*.

Solution. *ABCD* is a trapezium in which *O* is the point of intersection of *AC* and *BD* and *AB* || *CD*.

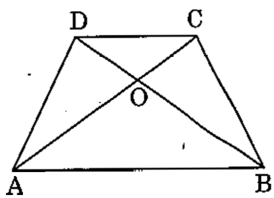
In triangles *AOB* and *COD*, we have

$\angle AOB = \angle COD$ [Vertically opposite ∠s]

and $\angle OAB = \angle OCD$ [Alternate ∠s]

So, by AA-criterion of similarity of triangles, we have

$$\Delta AOB \sim \Delta COD$$



$$\Rightarrow \frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \frac{AB^2}{CD^2}$$

[∵ The ratio of the area of two similar triangles is equal to the ratio of squares of their corresponding sides]

$$\Rightarrow \frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \frac{(2CD)^2}{CD^2}$$ [∵ *AB* = 2*CD* (given)]

$$\Rightarrow \frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \frac{4CD^2}{CD^2}$$

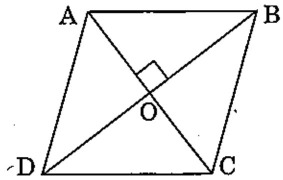
$$\Rightarrow \frac{84}{ar(\Delta COD)} = \frac{4}{1}$$

$$\Rightarrow ar(\Delta COD) = 84 \div 4 = 21 \text{ cm}^2$$

Hence, the area of ΔCOD is equal to **21 cm²**.

26. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Solution. Let $ABCD$ be a rhombus and its diagonals AC and BD intersect at O.



We have to prove that

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Since the diagonals of a rhombus bisect each other at right angles, therefore, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

and $OA = OC$ and $OB = OD$.

In ΔAOB, $\angle AOB = 90°$, then

$$AB^2 = OA^2 + OB^2 \quad \text{[Pythagoras Theorem]}$$

$$\Rightarrow AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 \quad \left[\because OA = OC \Rightarrow OA = \frac{1}{2}AC,\ OB = OD \Rightarrow OB = \frac{1}{2}BD\right]$$

$$\Rightarrow 4AB^2 = AC^2 + BD^2$$

$$\Rightarrow AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

[$ABCD$ is a rhombus in which $AB = BC = CD = DA$]

Hence, $\boldsymbol{AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2}$

27. The mean of the following frequency distribution is 24.4. Find the value of *p*.

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**16**	**24**	**28**	***p***	**18**

Solution. **Calculation of Mean**

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	f_ix_i
0 – 10	16	5	80
10 – 20	24	15	360
20 – 30	28	25	700
30 – 40	p	35	$35p$
40 – 50	18	45	810
Total	$n = \Sigma f_i = 86 + p$		$\Sigma f_ix_i = 1950 + 35p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{(given) } 24.4 = \frac{1950 + 35p}{86 + p}$$

$\Rightarrow \quad 2098.4 + 24.4p = 1950 + 35p$

$\Rightarrow \quad 2098.4 - 1950 = 35p - 24.4p$

$\Rightarrow \quad 148.4 = 10.6p$

$\Rightarrow \quad \mathbf{p = 14}$

Or

Find the mean age in years using step deviation method from the frequency distribution given below :

***Class-interval** (Age in years)*	***Frequency***
25 – 29	**4**
30 – 34	**14**
35 – 39	**22**
40 – 44	**16**
45 – 49	**6**
50 – 54	**5**
55 – 59	**3**
Total	**70**

Solution. **Calculation of Mean**

Class-interval (Age in years)	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 42}{5}$	$f_i u_i$
25 – 29	4	27	– 3	– 12
30 – 34	14	32	– 2	– 28
35 – 39	22	37	– 1	– 22
40 – 44	16	42	0	0
45 – 49	6	47	1	6
50 – 54	5	52	2	10
55 – 59	3	57	3	9
Total	$n = \Sigma f_i = 70$			$\Sigma f_i u_i = -37$

Here the assumed mean be $a = 42$ and $h = 5$.
Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 42 + \frac{(-37)}{70} \times 5$$

$$= 42 - \frac{37}{14}$$

$$= 42 - 2.64$$

$$= \mathbf{39.36.}$$

28. Find the median wage of the workers from the following distribution table :

Wages (in ₹)	*No. of workers*
More than 150	0
More than 140	10
More than 130	29
More than 120	60
More than 110	104
More than 100	134
More than 90	151
More than 80	160

Solution.

Wages (in ₹)	*No. of workers* (f_i)	*Wages (in ₹)*	f	cf
More than 80	160	80 – 90	9 (160 – 151)	9
More than 90	151	90 – 100	17 (151 – 134)	26
More than 100	134	100 – 110	30 (134 – 104)	56 (cf)
More than 110	104	110 – 120	44 (104 – 60) (f)	100
More than 120	60	120 – 130	31 (60 – 29)	131
More than 130	29	130 – 140	19 (29 – 10)	150
More than 140	10	140 – 150	10 (10 – 0)	160
More than 150	0	150 – 160	0	
Total			$n = \Sigma f_i = 160$	

Here, $n = 160 \Rightarrow \frac{n}{2} = 80$

Now; 110 – 120 is the class whose cumulative frequency 100 is greater than $\frac{n}{2} = 80$.

$\therefore$ 110 – 120 is the median class

From the table, $f = 44$, $cf = 56$, $h = 10$

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 110 + \frac{80 - 56}{44} \times 10$$

$$= 110 + \frac{24}{44} \times 10$$

$$= 110 + \frac{60}{11}$$

$$= 110 + 5.45$$

$$= \mathbf{115.45}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial $x^4 - 13x^3 + 52x^2 - 64x + 24$ are $3 \pm \sqrt{5}$, find other zeroes.

Solution. Since two zeroes of a polynomial $x^4 - 13x^2 + 52x^2 - 64x + 24$ are $3 \pm \sqrt{5}$, therefore

$$[x - (3 + \sqrt{5})][x - (3 - \sqrt{5}]$$
$$= [(x-3) - \sqrt{5}][(x-3) + \sqrt{5}]$$
$$= (x-3)^2 - (\sqrt{5})^2$$
$$= (x^2 - 6x + 9) - 5$$
$$= x^2 - 6x + 4$$

is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 6x + 4$

$$\begin{array}{r|l} & x^2 - 7x + 6 \\ \hline x^2 - 6x + 4 & x^4 - 13x^3 + 52x^2 - 64x - 24 \\ & \underline{\overset{-}{}x^4 \mp 6x^3 \pm 4x^2} \\ & -7x^3 + 48x^2 - 64x + 24 \\ & \underline{\mp 7x^3 \pm 42x^2 \mp 28x} \\ & 6x^2 - 36x + 24 \\ & \underline{\overset{-}{}6x^2 \mp 36x \pm 24} \\ & 0 \end{array}$$

$\left[\text{First term of quotient is } \dfrac{x^4}{x^2} = x^2\right]$

$\left[\text{Second term of quotient is } \dfrac{-7x^3}{x^2} = -7x\right]$

$\left[\text{Third term of quotient is } \dfrac{6x^2}{x} = 6\right]$

So, $x^4 - 13x^3 + 52x^2 - 64x + 24 = (x^2 - 6x + 4)(x^2 - 7x + 6)$

Now, by splitting $-7x$ as $-6x - x$, we can write

$$x^2 - 7x + 6 = x^2 - 6x - x + 6$$
$$= (x^2 - 6x) - (x - 6)$$
$$= x(x - 6) - (x - 6)$$
$$= (x - 6)(x - 1)$$

So, the zeroes are given by 6 and 1.

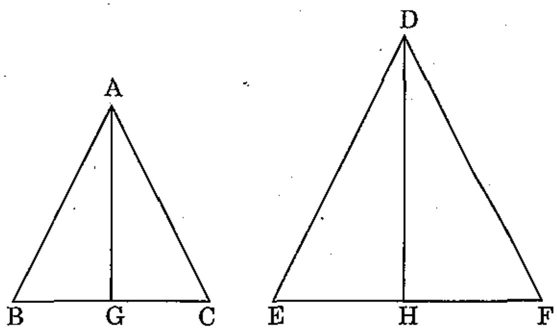
Hence, the other zeroes of the given polynomial are **6** and **1**.

30. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Solution. Given : ΔABC and ΔDEF such that $\Delta ABC \sim \Delta DEF$.

To prove : $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2}$.

Construction : Draw $AG \perp BC$ and $DH \perp EF$.



Proof : $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{\frac{1}{2} BC \times AG}{\frac{1}{2} EF \times DH}$ [$\because$ Area of $\Delta = \frac{1}{2}$(base) × (height)]

$\Rightarrow \quad \dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC}{EF} \times \dfrac{AG}{DH}$ ...(1)

Now, in Δs ABG and DEH, we have

$\angle B = \angle E$ [As $\Delta ABC \sim \Delta DEF$]

$\angle AGB = \angle DHE$ [Each equal to 90°]

3rd $\angle BAG$ = 3rd $\angle EDH$

Thus, ΔABG and ΔDEH are equiangular and hence they are similar.

$\therefore \quad \dfrac{AB}{DE} = \dfrac{AG}{DH}$ [$\because$ If Δ's are similar, the ratio of their corresponding sides is same]

But $\quad \dfrac{AB}{DE} = \dfrac{BC}{EF}$ [As $\Delta ABC \sim \Delta DEF$]

$\Rightarrow \quad \dfrac{AG}{DH} = \dfrac{BC}{EF}$ ...(2)

Now, from (1) and (2), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

Similarly, it can be proved that

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$

Hence, $\boldsymbol{\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2}}$.

Or

Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution. Given : A right triangle ABC, right angled at B.

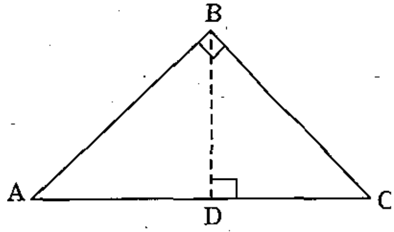
To prove : (Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

i.e., $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]



So, $\quad \dfrac{AD}{AB} = \dfrac{AB}{AC}$ [Sides are proportional]

$\Rightarrow \quad AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\quad \frac{CD}{BC} = \frac{BC}{AC}$ [Sides are proportional]

$\Rightarrow \quad CD.AC = BC^2$ ...(2)

Adding (1) and (2), we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$\Rightarrow \quad AC.AC = AB^2 + BC^2$

Hence, $\quad \mathbf{AC^2 = AB^2 + BC^2}$

31. Without using the trigonometric tables, evaluate the following :

$$\frac{\cos 22^\circ}{\sin 68^\circ} + \frac{\cos \theta}{\sin (90^\circ - \theta)} + \frac{\operatorname{cosec} \theta}{\sec (90^\circ - \theta)} - 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ$$

Solution. We have

$$\frac{\cos 22^\circ}{\sin 68^\circ} + \frac{\cos \theta}{\sin (90^\circ - \theta)} + \frac{\operatorname{cosec} \theta}{\sec (90^\circ - \theta)} - 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 89^\circ$$

$$= \frac{\cos 22^\circ}{\sin (90^\circ - 22^\circ)} + \frac{\cos \theta}{\sin (90^\circ - \theta)} + \frac{\operatorname{cosec} \theta}{\sec (90^\circ - \theta)}$$
$$- 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 87^\circ \tan 88^\circ \tan 89^\circ$$

$$= \frac{\cos 22^\circ}{\cos 22^\circ} + \frac{\cos \theta}{\cos \theta} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta} - 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\ldots \tan 87^\circ \tan 88^\circ \tan 89^\circ$$

[$\because \sin (90^\circ - \theta) = \cos \theta$ and $\sec (90^\circ - \theta) = \operatorname{cosec} \theta$]

$= 1 + 1 + 1 - 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\text{.}\tan (90^\circ - 3^\circ) \tan (90^\circ - 2^\circ) \tan (90^\circ - 1^\circ)$

$= 3 - 2 \tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots\text{.} \tan 45^\circ \ldots\ldots\text{.} \cot 3^\circ \cot 2^\circ \cot 1^\circ$

[$\because \tan (90^\circ - \theta) = \cot \theta$]

$= 3 - 2(\tan 1^\circ \cot 1^\circ)(\tan 2^\circ \cot 2^\circ)(\tan 3^\circ \cot 3^\circ) \ldots\ldots \tan 45^\circ$

$= 3 - 2 [(1)(1)(1) \ldots 1]$ [$\because \tan \theta \cdot \cot \theta = 1$ and $\tan 45^\circ = 1$]

$= 3 - 2$

$= \mathbf{1.}$

Or

Determine the value of x such that :

$$\mathbf{2 \operatorname{cosec}^2 30^\circ + x \sin^2 60^\circ - \frac{3}{4} \tan^2 30^\circ = 10}$$

Solution. We have

$$2 \operatorname{cosec}^2 30^\circ + x \sin^2 60^\circ - \frac{3}{4} \tan^2 30^\circ = 10$$

$$\Rightarrow \quad 2 \times (2)^2 + x \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\frac{1}{\sqrt{3}}\right)^2 = 10 \quad \left[\because \operatorname{cosec} 30^\circ = 2, \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and } \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad 2 \times 4 + x \times \frac{3}{4} - \frac{3}{4} \times \frac{1}{3} = 10$$

$$\Rightarrow \quad 8 + \frac{3x}{4} - \frac{1}{4} = 10$$

$$\Rightarrow \quad \frac{3x}{4} - \frac{1}{4} = 10 - 8$$

$$\Rightarrow \quad \frac{3x - 1}{4} = 2$$

$$\Rightarrow \quad 3x - 1 = 2 \times 4$$

$$\Rightarrow \quad 3x = 8 + 1$$

$$\Rightarrow \quad x = 9 \div 3$$

$$\Rightarrow \quad x = 3$$

32. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

***Length (in* mm)**	***Number of leaves***
118 – 126	**3**
127 – 135	**5**
136 – 144	**9**
145 – 153	**12**
154 – 162	**5**
163 – 171	**4**
172 – 180	**2**

Draw a more than type ogive for the given data. Hence, obtain the median length of the leaves from the graph.

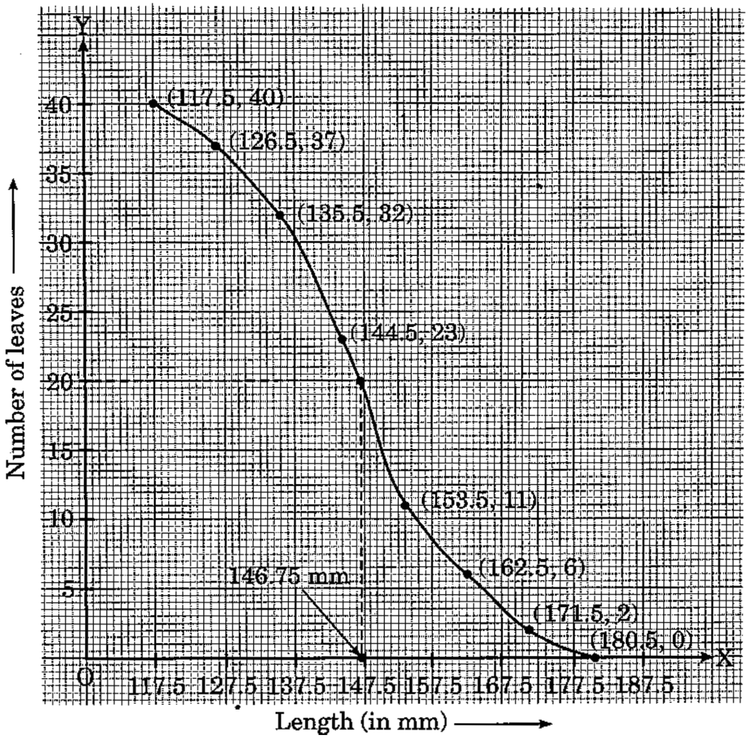
Solution. The given frequency distribution is not continuous. So, we first make it continuous and prepare the cumulative frequency table by more than type given below :

Length (*in* mm)	*Number of leaves* [*Frequency* (*f*)]	*Length more than*	*Cumulative frequency* (*cf*)
117.5 – 126.5	3	117.5	40
126.5 – 135.5	5	126.5	37
135.5 – 144.5	9	135.5	32
144.5 – 153.5	12	144.5	23
153.5 – 162.5	5	153.5	11
162.5 – 171.5	4	162.5	6
171.5 – 180.5	2	171.5	2

Other than the given class-interval, we assume that the class-interval 180.5 – 189.5 with zero frequency. Now, we mark the lower class limits on x-axis and the cumulative frequencies along y-axis on suitable scales to plot the points (117.5, 40), (126.5, 37), (135.5, 32), (144.5, 23), (153.5, 11), (162.5, 6), (171.5, 2). We join these points with a smooth curve to get the **"more than"** ogive as shown in the figure.

Locate $\frac{n}{2} = \frac{40}{2} = 20$ on the y-axis (see figure).

From this point, draw a line parallel to the x-axis cutting the curve at a point. From this point, draw a perpendicular to the x-axis. The point of intersection of this perpendicular with the x-axis determine the median length of the data (see figure.), *i.e.*, median length is **146.75 mm.**



33. If $\text{cosec}\ \theta = x + \frac{1}{4x}$, prove that $\text{cosec}\ \theta + \cot \theta = 2x$ or $\frac{1}{2x}$.

Solution. We know $1 + \cot^2 \theta = \text{cosec}^2 \theta$

$\Rightarrow \quad \cot^2 \theta = \text{cosec}^2 \theta - 1$

$\Rightarrow \quad \cot^2 \theta = \left(x + \frac{1}{4x}\right)^2 - 1 \qquad \left[\because \text{cosec}\ \theta = x + \frac{1}{4x}\right]$

$\Rightarrow \quad \cot^2 \theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$

$\Rightarrow \quad \cot^2 \theta = x^2 \quad \frac{1}{16x^2} \quad \frac{1}{2}$

$\Rightarrow \quad \cot \theta = \pm\sqrt{\left(x - \frac{1}{4x}\right)^2}$

$\Rightarrow \quad \cot \theta = x - \frac{1}{4x} \quad \text{or} \quad -x + \frac{1}{4x}$

$\therefore \quad \text{cosec}\ \theta + \cot \theta = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

Also $\quad \text{cosec}\ \theta + \cot \theta = x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$

Hence $\quad \text{cosec}\ \theta + \cot \theta = \mathbf{2x} \quad \text{or} \quad \mathbf{\frac{1}{2x}}$

34. Draw the graph of the following pair of linear equations

$$x + 3y = 6$$
$$2x - 3y = 12$$

Hence, find the area of the region bounded by

$$x = 0,\ y = 0 \text{ and } 2x - 3y = 12.$$

Solution. Two solutions of each of the equations :

$$x + 3y = 6 \quad \text{...(1)}$$
$$2x - 3y = 12 \quad \text{...(2)}$$

are given in tables below.

$x + 3y = 6$

x	6	0
y	0	2
	A	B

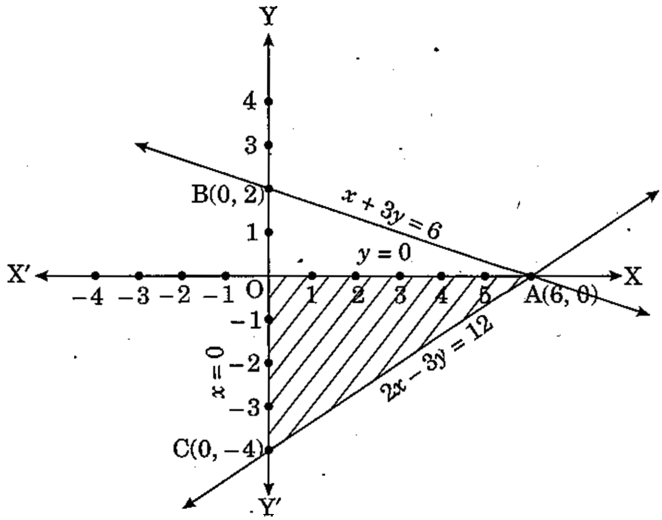
$2x - 3y = 12$

x	6	0
y	0	−4
	A	C

Plot the points $A(6, 0)$, $B(0, 2)$ and $C(0, -4)$ corresponding to the solutions in tables. Now, draw the lines AB and AC representing the equations $x + 3y = 6$ and $2x - 3y = 12$ as shown in figure.

In figure, we observe that the two lines representing the two equations are intersecting at the point $A(6, 0)$.

Hence, $\boldsymbol{x = 6}$ and $\boldsymbol{y = 0}$.



Area of the region bounded by $x = 0$, $y = 0$ and $2x - 3y = 12$ is the triangle AOC.

$$= \frac{1}{2}(\text{Base} \times \text{Height})$$

$$= \frac{1}{2}(OC \times OA)$$

$$= \frac{1}{2}(4 \times 6) \qquad [\because OC = 4 \text{ units}, OA = 6 \text{ units}]$$

$$= \mathbf{12\ sq.\ units}$$