

# MODEL TEST PAPER

## FIRST TERM (SA-I)

### MATHEMATICS

(With Answers)

### CLASS X

Time Allowed : 3 to 3½ Hours

Maximum Marks : 80

#### General Instructions :

- All questions are compulsory.
- The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.
- Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.
- There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

#### Section 'A'

Question numbers 1 to 10 are of one mark each.

- The HCF of two numbers is 16 and their product is 3072. Their LCM is  
(a) 192 (b) 186  
(c) 188 (d) 194
- If the sum of the zeroes of the polynomial  $p(x) = 2x^3 - 3kx^2 + 4x - 5$  is 6, then the value of  $k$  is  
(a) -2 (b) 2  
(c) 4 (d) -4
- Two poles of heights 6 m and 11 m stand vertically upward on a plane ground. If the distance between their feet is 12 m, the distance between their tops is  
(a) 11 m (b) 12 m  
(c) 13 m (d) 14 m
- If  $\sin 2\theta = \cos 4\theta$ , where  $2\theta$  and  $4\theta$  are acute angles, then the value of  $\theta$  is  
(a)  $20^\circ$  (b)  $18^\circ$   
(c)  $15^\circ$  (d)  $12^\circ$
- If  $\operatorname{cosec} \theta - \cot \theta = k$  then, the value of  $\operatorname{cosec} \theta + \cot \theta$  is  
(a)  $1 - \frac{1}{k}$  (b)  $1 - k$   
(c)  $1 + k$  (d)  $\frac{1}{k}$
- If  $\operatorname{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) = k$ , then the value of  $k$  is  
(a) 0 (b) 1  
(c) 2 (d) 3

7. If  $\cot \theta = \frac{1}{\sqrt{3}}$ , then the value of  $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$  is

(a)  $\frac{1}{5}$

(b)  $\frac{2}{5}$

(c)  $\frac{3}{5}$

(d)  $\frac{4}{5}$

8. The decimal expansion of  $\frac{189}{125}$  will terminate after how many places of decimal ?

(a) 1

(b) 2

(c) 3

(d) 4

9. If  $am \neq bl$ , then the system of equations

$$ax + by = c$$

$$lx + my = n$$

(a) has a unique solution

(b) has no solution

(c) has infinitely many solutions

(d) may or may not have a solution

10. The mean of  $n$  observations is  $\bar{x}$ , if the first term is decreased by 1, second is decreased by 2 and so on, then the new mean is

(a)  $\bar{x} - n$

(b)  $\bar{x} - \frac{n}{2}$

(c)  $\bar{x} + \frac{n+1}{2}$

(d)  $\bar{x} - \frac{n+1}{2}$

### Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Prove that area of an equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

12.  $P$  and  $Q$  respectively the points on the sides  $AB$  and  $AC$  of a  $\triangle ABC$ . If  $AP = 2$  cm,  $PB = 6$  cm,  $AQ = 3$  cm and  $QC = 9$  cm, prove that  $BC = 4PQ$ .

13. Express 3825 as the product of prime factors.

14. Show that :

$$\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} + \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{2 \sec^2 \theta}{\tan^2 \theta - 1}$$

Or

Simplify :  $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} + \sin \theta \cos \theta$ .

15. Find the value of  $k$  for which the pair of linear equations

$$x + (k + 1)y = 4$$

$$(k + 1)x + 9y = 5k + 2$$

have infinitely many solutions.

16. Find the zeroes of the quadratic polynomial  $8x^2 - 21 - 22x$  and verify the relationship between the zeroes and the coefficients of the polynomial.

17. The mode of the following distribution is 55. Find the value of  $x$ .

<i>Class Interval</i>	0 – 15	15 – 30	30 – 45	45 – 60	60 – 75	75 – 90
<i>Frequency</i>	6	7	$x$	15	10	8

18. The following distribution gives the daily income of 100 workers of a factory.

<i>Income (in ₹)</i>	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600
<i>Number of workers</i>	7	15	35	28	10	5

Write the above distribution as more than type cumulative frequency distribution.

**Section 'C'**

*Question numbers 19 to 28 carry 3 marks each.*

19. Show that any even integer is of the form  $6q$  or  $6q + 2$  or  $6q + 4$ , where  $q$  is a positive integer.

20. Prove that  $\sqrt{2} + \sqrt{5}$  is irrational.

*Or*

Prove that  $\frac{3\sqrt{3}}{5}$  is irrational.

21. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

*Or*

Yash scored 40 marks in a test getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

22. If  $\alpha$  and  $\beta$  are the zeroes of the quadratic polynomial :  $p(x) = 3x^2 - 4x + 1$ , find a quadratic polynomial whose zeroes are  $\frac{\alpha^2}{\beta}$  and  $\frac{\beta^2}{\alpha}$ .

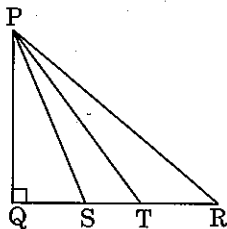
23. If  $\sin \theta + \cos \theta = a$  and  $\sec \theta + \operatorname{cosec} \theta = b$ , show that  $b(a^2 - 1) = 2a$ .

24. Prove that :

$$2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$$

25. In figure,  $S$  and  $T$  trisect the side  $QR$  of a right triangle  $PQR$ . Prove that :

$$8PT^2 = 3PR^2 + 5PS^2$$



26. In  $\Delta ABC$ ,  $XY$  is parallel to  $BC$  and it divides  $\Delta ABC$  into two parts of equal area. Prove that

$$\frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

27. Find the mean of the following frequency distribution using step-deviation method :

<i>Class Interval</i>	0 - 30	30 - 60	60 - 90	90 - 120	120 - 150	150 - 180
<i>Frequency</i>	8	15	16	20	12	9

**Or**

The following table gives the output (in units) of workers in a certain factory. The frequency of the class-interval 490 - 520 is missing.

<i>Output (in units)</i>	400 - 430	430 - 460	460 - 490	490 - 520	520 - 550
<i>No. of workers</i>	31	58	60	—	27

It is known that the mean of the above frequency distribution is 472. Find the missing frequency.

28. The following distribution shows the number of runs scored by some top batsman of the world in one day cricket matches :

<i>Runs Scored</i>	<i>Number of batsman</i>
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

Find the mode.

### Section D

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial  $x^4 + 3x^3 - 20x^2 - 6x + 36$  are  $\sqrt{2}$  and  $-\sqrt{2}$ , find the other zeroes of the polynomial.

30. Form a pair of linear equations in two variables using the following information and solve it graphically.

Five years ago, Sagar was twice as old Tiru. Ten years later Sagar's age will be ten years more than Tiru's age. Find their present ages. What was the age of Sagar when Tiru was born?

31. Prove the following identity :

$$\left( \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} \right)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

**Or**

Without using trigonometric tables, evaluate the following :

$$\frac{\sec^2(90^\circ - \theta) - \cot^2 \theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} + \frac{2 \cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)} + \frac{1}{\sqrt{3}} (\tan 10^\circ \tan 30^\circ \tan 45^\circ \tan 80^\circ)$$

32. If  $\tan A = n \tan B$  and  $\sin A = m \sin B$ , prove that

$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

33. During the medical check-up of 35 students of a class, their weights were recorded as follows :

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

34. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

**Or**

Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

### ANSWERS

#### Section 'A'

- |         |        |        |
|---------|--------|--------|
| 1. (a)  | 2. (c) | 3. (c) |
| 4. (c)  | 5. (d) | 6. (b) |
| 7. (c)  | 8. (c) | 9. (a) |
| 10. (d) |        |        |

#### Section 'B'

- |   |          |             |
|---|----------|-------------|
| 13. $3825 = 3^2 \times 5^2 \times 17$ .           | 14. Or 1 | 15. $k = 2$ |
| 16. Zeroes are $\frac{7}{2}$ and $\frac{-3}{4}$ . |          |             |
| 17. $x = 5$                                       |          |             |

18.

<i>Income (in ₹)</i>	<i>No. of workers (f)</i>	<i>Income</i>	<i>Cumulative frequency</i>
0 – 100	7	More than 0	100 (93 + 7)
100 – 200	15	More than 100	93 (78 + 15)
200 – 300	35	More than 200	78 (43 + 35)
300 – 400	28	More than 300	43 (28 + 15)
400 – 500	10	More than 400	15 (10 + 5)
500 – 600	5	More than 500	5
<i>Total</i>	100		

**Section 'C'**

21. 36 students Or 20    22.  $f(x) = k\left(x^2 - \frac{28}{9}x + \frac{1}{3}\right)$  where  $k$  is non-zero real number

27. Mean = 90 Or 44

28. Mode = 4608.70

**Section 'D'**

29. Other zeroes are - 6 and 3

30. Sagar's present age is 25 years and Tiru's present age is 15 years, Sagar was ten years old when Tiru was born.

31. Or 1

33. Median = 46.5