MODEL TEST PAPER
FIRST TERM (SA-I)
MATHEMATICS
(With Answers)
CLASS X

Time Allowed : 3 to 3½ Hours]   [*Maximum Marks* : 80

General Instructions :
   (*i*) *All questions are compulsory.*
   (*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*
   (*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*
   (*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*
   (*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If θ and $3\theta - 30°$ are acute angles such that $\sin \theta = \cos (3\theta - 30°)$, then $\tan \theta$ is equal to
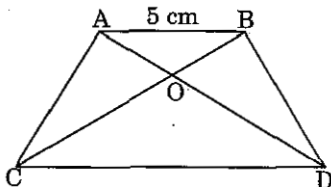(*a*) 1   (*b*) -1

(*c*) $\sqrt{3}$ (*d*) $\dfrac{1}{\sqrt{3}}$

2. If $x \tan 45° \cos 60° = \sin 60° \cot 60°$, then x is equal to

(*a*) 1   (*b*) $\sqrt{3}$

(*c*) $\dfrac{1}{2}$ (*d*) $\dfrac{1}{\sqrt{2}}$

3. In figure, $\dfrac{AO}{OC} = \dfrac{BO}{OD} = \dfrac{1}{2}$ and $AB = 5$ cm, then the value of DC is

(*a*) 2.5 cm   (*b*) 10 cm
(*c*) 7.5 cm   (*d*) 12.5 cm
4. The value of $3 \tan^2 32° - 3 \csc^2 58°$ is
(*a*) 0   (*b*) 3
(*c*) -3 (*d*) -4

5. Given that $\tan \theta = \dfrac{1}{\sqrt{3}}$, the value of $\dfrac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta}$ is

(a) -1 (b) 1

(c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{2}$

6. The decimal expansion of $\dfrac{147}{120}$ will terminate after how many places of decimal ?

(a) 2 (b) 1

(c) 3 (d) 4

7. If a, b are both positive rational numbers, then $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ is
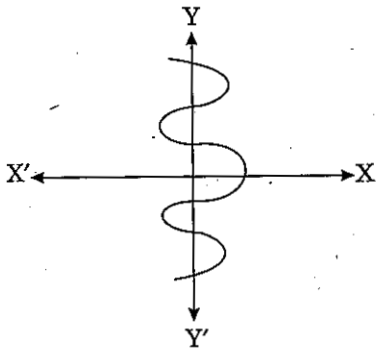
(a) a rational number   (b) an irrational number

(c) neither rational nor irrational number   (d) both rational as well as irrational number

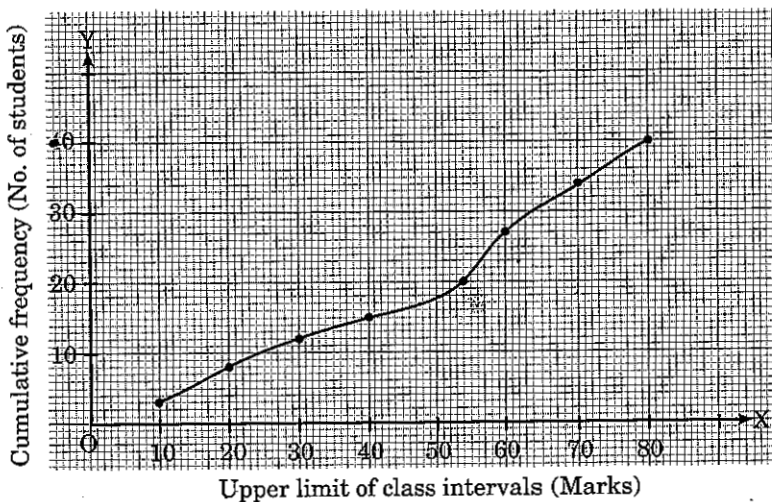**8.** In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is

(a) 5 (b) 1

(c) 7 (d) 3

9. A student draws a cumulative frequency curve for the marks obtained by 40 students of a class as shown below. Then the median marks obtained by the student of the class is

Cumulative frequency (No. of students)

Upper limit of class intervals (Marks)

(a) 51   (b) 52

(c) 53   (d) 54

10. The pair of linear equations

$$5x + 7y = 29 \text{ and } 35x + 33y = 171 \text{ have}$$

(a) one solution   (b) two solutions

(c) many solutions   (d) no solution

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. If the product of zeroes of the polynomial $kx^2 + 9x + 20$ is 6, find the value of k.

12. Prove that $\sqrt{5}$ is an irrational.

13. If $7 \cos^2 \theta + 3 \sin^2 \theta = 6$, show that $\tan \theta = \dfrac{1}{\sqrt{3}}$.

Or

Prove that : $(\text{cosec } \theta - \cot \theta)^2 = \dfrac{1 - \cos \theta}{1 + \cos \theta}$.

14. For what value of k will the following system of equations has infinitely many solutions :

$$3x + 2y = 1$$
$$(2k + 1)x + (k + 2)y = k - 1$$

15. ABC is a right triangle right angled at C. If p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that (i) $pc = ab$ (ii) $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

16. BL and CM are medians of a triangle ABC right-angled at A. Prove that

$$4(BL^2 + CM^2) = 5BC^2$$
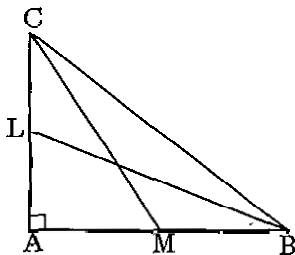
17. The following cumulative frequency distribution gives daily wages of 111 workers.

Daily wages (in ₹)	Number of workers
Less than 55	6
Less than 60	16
Less than 65	38
Less than 70	68
Less than 75	84
Less than 80	96
Less than 100	111

Write the above cumulative frequency distribution as frequency distribution.

20. Prove that $\dfrac{5}{3}\sqrt{7}$ is an irrational number.

Or

Prove that $\sqrt{5} + \sqrt{3}$ is an irrational number.

21. 37 pens and 53 pencils together cost ₹ 320, while 53 pens and 37 pencils together cost ₹ 400. Find the cost of a pen and that of a pencil.

Or

The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and the breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area is increased by 67 square units. Find the length and breadth of the rectangle.

22. If α and β are the zeroes of the quadratic polynomial :

$p(x) = ax^2 + bx + c$, then evaluate : $\alpha^4 + \beta^4$

23. Prove that :

$$\frac{\sin \theta}{1 - \cos \theta} + \frac{\tan \theta}{1 + \cos \theta} = \sec \theta \ \csc \theta + \cot \theta$$
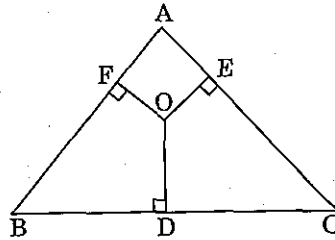
24. Prove that :

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A.$$

25. ABC is a triangle in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. Prove that $BD = BC$.

26. From a point O in the interior of a $\triangle ABC$, perpendicular OD, OE and OF are drawn on the sides BC, CA and AB respectively. Prove that

   (i) $AF^2 + BD^2 + CE^2 = OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

   (ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

27. Find mean of the following frequency distribution using step-deviation method :

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	7	12	13	10	8

Or

The mean of the following distribution is 52.5. Find the value of x.

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	15	22	27	x	21

The mean of the following distribution is 54. Find the value of p.

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	8	7	p	13	10

28. 100 surnames were randomly picked up from local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

Number of letters	1 – 4	4 – 7	7 – 10	10 – 13	13 – 16	16 – 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames.

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

30. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Or

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove this theorem.

31. Prove that :

$$\frac{\sec\theta + \tan\theta - 1}{\sec\theta - \tan\theta + 1} = \frac{\cos\theta}{1 - \sin\theta}$$

Or

Evaluate : $\dfrac{\csc\theta \cdot \sec(90° - \theta) - \cot\theta \cdot \tan(90° - \theta) + \sin^2 32° + \sin^2 58°}{\tan 3° \tan 22° \tan 45° \tan 68° \tan 87° - \tan^2 32° + \csc^2 58°}$

32. The median of the following data is 52.5. Find the values of x and y if the total frequency is 100.

Class Interval	Frequency
0 – 10	3
10 – 20	5
20 – 30	x
30 – 40	12
40 – 50	17
50 – 60	20
60 – 70	y
70 – 80	9
80 – 90	7
90 – 100	4

33. Prove that :

$$\frac{1}{\csc x - \cot x} - \frac{1}{\sin x} = \frac{1}{\sin x} - \frac{1}{\csc x + \cot x}.$$

34. Draw the graph of the following equations on the same graph paper :

$$2x - y = 2$$
$$4x - y = 8$$

Also, find the coordinates of the points where the lines meet the axis of x.

ANSWERS

Section 'A'

1.	(d)	**2.**	(a)	**3.**	(b)
4.	(c)	**5.**	(c)	**6.**	(c)
7.	(a)	**8.**	(b)	**9.**	(d)
10.	(a)				

Section 'B'

11. $k = \dfrac{10}{3}$ **14.** $k = 4$

17.

Daily wages (in ₹)	No. of students frequency
50 – 55	6
55 – 60	10
60 – 65	22
65 – 70	30
70 – 75	16
75 – 80	12
80 – 100	15

18. Mode = 33

Section 'C'

21. Cost of one pen = ₹ 6.50, Cost of one pencil = ₹ 1.50
.Or   Length = 17 units, Breadth = 9 units

22. $\alpha^4 + \beta^4 = \dfrac{(b^2 - 2ac)^2 - 2a^2c^2}{a^4}$ **27.** Mean = 25   Or $p = 12$

28. Median = 8.05

Section 'D'

29. Other zeroes are 7 and – 5.   **31.** Or 1
32. $x = 9$ and $y = 15$
34. $x = 3, y = 4$; (1, 0) and (2, 0)