## Exercise 11.1

**Question 1:**

Find the equation of the circle with centre (0, 2) and radius 2

Answer

The equation of a circle with centre ($h$, $k$) and radius $r$ is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre ($h$, $k$) = (0, 2) and radius ($r$) = 2.

Therefore, the equation of the circle is

$(x - 0)^2 + (y - 2)^2 = 2^2$

$x^2 + y^2 + 4 - 4y = 4$

$x^2 + y^2 - 4y = 0$

**Question 2:**

Find the equation of the circle with centre (–2, 3) and radius 4

Answer

The equation of a circle with centre ($h$, $k$) and radius $r$ is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre ($h$, $k$) = (–2, 3) and radius ($r$) = 4.

Therefore, the equation of the circle is

$(x + 2)^2 + (y - 3)^2 = (4)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$

$x^2 + y^2 + 4x - 6y - 3 = 0$

**Question 3:**

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Answer

The equation of a circle with centre ($h$, $k$) and radius $r$ is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre ($h$, $k$) = $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius ($r$) = $\frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}=\frac{1}{144}$$
$$x^2-x+\frac{1}{4}+y^2-\frac{y}{2}+\frac{1}{16}-\frac{1}{144}=0$$
$$144x^2-144x+36+144y^2-72y+9-1=0$$
$$144x^2-144x+144y^2-72y+44=0$$
$$36x^2-36x+36y^2-18y+11=0$$
$$36x^2+36y^2-36x-18y+11=0$$

**Question 4:**

Find the equation of the circle with centre (1, 1) and radius $\sqrt{2}$

Answer

The equation of a circle with centre (*h*, *k*) and radius *r* is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (*h*, *k*) = (1, 1) and radius (*r*) = $\sqrt{2}$ .

Therefore, the equation of the circle is

$$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$$
$$x^2-2x+1+y^2-2y+1=2$$
$$x^2+y^2-2x-2y=0$$

**Question 5:**

Find the equation of the circle with centre (–*a*, –*b*) and radius $\sqrt{a^2-b^2}$

Answer

The equation of a circle with centre (*h*, *k*) and radius *r* is given as

$(x - h)^2 + (y - k)^2 = r^2$

It is given that centre (*h*, *k*) = (–*a*, –*b*) and radius (*r*) = $\sqrt{a^2-b^2}$ .

Therefore, the equation of the circle is

$$(x+a)^2+(y+b)^2=\left(\sqrt{a^2-b^2}\right)^2$$
$$x^2+2ax+a^2+y^2+2by+b^2=a^2-b^2$$
$$x^2+y^2+2ax+2by+2b^2=0$$

**Question 6:**

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Answer

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$(x + 5)^2 + (y - 3)^2 = 36$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is (–5, 3), while its radius is 6.

**Question 7:**

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Answer

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$x^2 + y^2 - 4x - 8y - 45 = 0$

$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$

$\Rightarrow \{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4)+ 4^2\} - 4 - 16 = 45$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 65$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = \left(\sqrt{65}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$, and $r=\sqrt{65}$.

Thus, the centre of the given circle is (2, 4), while its radius is $\sqrt{65}$.

**Question 8:**

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Answer

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$x^2 + y^2 - 8x + 10y - 12 = 0$

$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$

$\Rightarrow \{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$

$\Rightarrow (x-4)^2 + \{y-(-5)\}^2 = \left(\sqrt{53}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is (4, −5), while its radius is $\sqrt{53}$.

**Question 9:**

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Answer

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2 + 2y^2 - x = 0$$

$$\Rightarrow \left(2x^2 - x\right) + 2y^2 = 0$$

$$\Rightarrow 2\left[\left(x^2 - \frac{x}{2}\right) + y^2\right] = 0$$

$$\Rightarrow \left\{x^2 - 2.x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2\right\} + y^2 - \left(\frac{1}{4}\right)^2 = 0$$

$\Rightarrow \left(x - \frac{1}{4}\right)^2 + (y-0)^2 = \left(\frac{1}{4}\right)^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$, and $r = \frac{1}{4}$.

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0\right)$, while its radius is $\frac{1}{4}$.

**Question 10:**

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$(4 - h)^2 + (1 - k)^2 = r^2$ … (1)

$(6 - h)^2 + (5 - k)^2 = r^2$ … (2)

Since the centre ($h$, k) of the circle lies on line $4x + y = 16$,

$4h + k = 16$ … (3)

From equations (1) and (2), we obtain

$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$

$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$

$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$

$\Rightarrow 4h + 8k = 44$

$\Rightarrow h + 2k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of $h$ and $k$ in equation (1), we obtain

$(4 - 3)^2 + (1 - 4)^2 = r^2$

$\Rightarrow (1)^2 + (-3)^2 = r^2$

$\Rightarrow 1 + 9 = r^2$

$\Rightarrow r^2 = 10$

$\Rightarrow r = \sqrt{10}$

Thus, the equation of the required circle is

$(x - 3)^2 + (y - 4)^2 = \left(\sqrt{10}\right)^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$x^2 + y^2 - 6x - 8y + 15 = 0$

**Question 11:**

Find the equation of the circle passing through the points (2, 3) and (−1, 1) and whose centre is on the line $x - 3y - 11 = 0$.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (2, 3) and (−1, 1),

$(2 - h)^2 + (3 - k)^2 = r^2$ … (1)

$(-1 - h)^2 + (1 - k)^2 = r^2$ … (2)

Since the centre ($h$, k) of the circle lies on line $x - 3y - 11 = 0$,

$h - 3k = 11$ … (3)

From equations (1) and (2), we obtain

$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$

$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$

$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$

$\Rightarrow 6h + 4k = 11$ … (4)

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of $h$ and $k$ in equation (1), we obtain

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2 + \left(\frac{2y+5}{2}\right)^2 = \frac{130}{4}$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

**Question 12:**

Find the equation of the circle with radius 5 whose centre lies on $x$-axis and passes through the point (2, 3).

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the $x$-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2-h)^2 + 3^2 = 25$$
$$\Rightarrow (2-h)^2 = 25-9$$
$$\Rightarrow (2-h)^2 = 16$$
$$\Rightarrow 2-h = \pm\sqrt{16} = \pm 4$$

If $2-h=4$, then $h=-2$.

If $2-h=-4$, then $h=6$.

When $h = -2$, the equation of the circle becomes

$(x + 2)^2 + y^2 = 25$

$x^2 + 4x + 4 + y^2 = 25$

$x^2 + y^2 + 4x - 21 = 0$

When $h = 6$, the equation of the circle becomes

$(x - 6)^2 + y^2 = 25$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 + y^2 - 12x + 11 = 0$

**Question 13:**

Find the equation of the circle passing through (0, 0) and making intercepts $a$ and $b$ on the coordinate axes.

Answer

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$(0 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts $a$ and $b$ on the coordinate axes. This means that the circle passes through points ($a$, 0) and (0, $b$). Therefore,

$(a - h)^2 + (0 - k)^2 = h^2 + k^2$ … (1)

$(0 - h)^2 + (b - k)^2 = h^2 + k^2$ … (2)

From equation (1), we obtain

$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$\Rightarrow a^2 - 2ah = 0$

$\Rightarrow a(a - 2h) = 0$

$\Rightarrow a = 0$ or $(a - 2h) = 0$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain

$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$\Rightarrow b^2 - 2bk = 0$

$\Rightarrow b(b - 2k) = 0$

$\Rightarrow b = 0$ or$(b - 2k) = 0$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \frac{b}{2}$.

Thus, the equation of the required circle is

$$\left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2$$

$$\Rightarrow\left(\frac{2x-a}{2}\right)^2+\left(\frac{2y-b}{2}\right)^2=\frac{a^2+b^2}{4}$$

$$\Rightarrow 4x^2-4ax+a^2+4y^2-4by+b^2=a^2+b^2$$

$$\Rightarrow 4x^2+4y^2-4ax-4by=0$$

$$\Rightarrow x^2+y^2-ax-by=0$$

**Question 14:**

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Answer

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius ($r$) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r=\sqrt{(2-4)^2+(2-5)^2}=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-2)^2=\left(\sqrt{13}\right)^2$$
$$x^2-4x+4+y^2-4y+4=13$$
$$x^2+y^2-4x-4y-5=0$$

**Question 15:**

Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Answer

The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 5^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

∴Centre = (0, 0) and radius = 5

Distance between point (–2.5, 3.5) and centre (0, 0)

$$=\sqrt{(-2.5-0)^2+(3.5-0)^2}$$
$$=\sqrt{6.25+12.25}$$
$$=\sqrt{18.5}$$
$$=4.3\ (\text{approx.})<5$$

Since the distance between point (–2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (–2.5, 3.5) lies inside the circle.

## Exercise 11.2

**Question 1:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$

Answer

The given equation is $y^2 = 12x$.

Here, the coefficient of $x$ is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 12 \Rightarrow a = 3$

$\therefore$Coordinates of the focus = $(a, 0) = (3, 0)$

Since the given equation involves $y^2$, the axis of the parabola is the $x$-axis.

Equation of direcctrix, $x = -a$ i.e., $x = -3$ i.e., $x + 3 = 0$

Length of latus rectum = $4a = 4 \times 3 = 12$

**Question 2:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$

Answer

The given equation is $x^2 = 6y$.

Here, the coefficient of $y$ is positive. Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

$\therefore$Coordinates of the focus = $(0, a)$ = $\left(0, \frac{3}{2}\right)$

Since the given equation involves $x^2$, the axis of the parabola is the $y$-axis.

Equation of directrix, $y = -a \text{ i.e., } y = -\frac{3}{2}$

Length of latus rectum = $4a = 6$

**Question 3:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = -8x$

Answer

The given equation is $y^2 = -8x$.

Here, the coefficient of $x$ is negative. Hence, the parabola opens towards the left.

On comparing this equation with $y^2 = -4ax$, we obtain

$-4a = -8 \Rightarrow a = 2$

$\therefore$Coordinates of the focus = $(-a, 0) = (-2, 0)$

Since the given equation involves $y^2$, the axis of the parabola is the $x$-axis.

Equation of directrix, $x = a$ i.e., $x = 2$

Length of latus rectum = $4a = 8$

**Question 4:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -16y$

Answer

The given equation is $x^2 = -16y$.

Here, the coefficient of $y$ is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$-4a = -16 \Rightarrow a = 4$

$\therefore$Coordinates of the focus = $(0, -a) = (0, -4)$

Since the given equation involves $x^2$, the axis of the parabola is the $y$-axis.

Equation of directrix, $y = a$ i.e., $y = 4$

Length of latus rectum = $4a = 16$

**Question 5:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$

Answer

The given equation is $y^2 = 10x$.

Here, the coefficient of $x$ is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 10 \Rightarrow a = \frac{5}{2}$

∴Coordinates of the focus = ($a$, 0) $= \left(\frac{5}{2}, 0\right)$

Since the given equation involves $y^2$, the axis of the parabola is the $x$-axis.

Equation of directrix, $x = -a$, i.e., $x = -\frac{5}{2}$

Length of latus rectum = $4a$ = 10

**Question 6:**

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -9y$

Answer

The given equation is $x^2 = -9y$.

Here, the coefficient of $y$ is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$-4a = -9 \Rightarrow b = \frac{9}{4}$

∴Coordinates of the focus = $(0, -a) = \left(0, -\frac{9}{4}\right)$

Since the given equation involves $x^2$, the axis of the parabola is the $y$-axis.

Equation of directrix, $y = a$, i.e., $y = \frac{9}{4}$

Length of latus rectum = $4a$ = 9

**Question 7:**

Find the equation of the parabola that satisfies the following conditions: Focus (6, 0); directrix $x = -6$

Answer

Focus (6, 0); directrix, $x = -6$

Since the focus lies on the $x$-axis, the $x$-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y^2 = 4ax$ or

$y^2 = -4ax$.

It is also seen that the directrix, $x = -6$ is to the left of the $y$-axis, while the focus (6, 0) is to the right of the $y$-axis. Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

Thus, the equation of the parabola is $y^2 = 24x$.

**Question 8:**

Find the equation of the parabola that satisfies the following conditions: Focus (0, –3); directrix $y = 3$

Answer

Focus = (0, –3); directrix $y = 3$

Since the focus lies on the $y$-axis, the $y$-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

It is also seen that the directrix, $y = 3$ is above the $x$-axis, while the focus (0, –3) is below the $x$-axis. Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

Thus, the equation of the parabola is $x^2 = -12y$.

**Question 9:**

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0); focus (3, 0)

Answer

Vertex (0, 0); focus (3, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the positive $x$-axis, $x$-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = 4ax$.

Since the focus is (3, 0), $a = 3$.

Thus, the equation of the parabola is $y^2 = 4 \times 3 \times x$, i.e., $y^2 = 12x$

**Question 10:**

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) focus (–2, 0)

Answer

Vertex (0, 0) focus (–2, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the negative $x$-axis, $x$-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$.

Since the focus is (–2, 0), $a = 2$.

Thus, the equation of the parabola is $y^2 = -4(2)x$, i.e., $y^2 = -8x$

**Question 11:**

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) passing through (2, 3) and axis is along $x$-axis

Answer

Since the vertex is (0, 0) and the axis of the parabola is the $x$-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

The parabola passes through point (2, 3), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point (2, 3) must satisfy the equation $y^2 = 4ax$.

$$\therefore 3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$y^2 = \frac{9}{2}x$$

$$2y^2 = 9x$$

**Question 12:**

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to $y$-axis

Answer

Since the vertex is (0, 0) and the parabola is symmetric about the $y$-axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point

(5, 2) must satisfy the equation $x^2 = 4ay$.

$$\therefore (5)^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$$x^2 = 4\left(\frac{25}{8}\right)y$$

$$2x^2 = 25y$$

## Exercise 11.3

**Question 1:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Answer

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 6 and *b* = 4.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 8

Eccentricity, $e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 16}{6}=\frac{16}{3}$

**Question 2:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Answer

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b$ = 2 and $a$ = 5.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = 2$a$ = 10

Length of minor axis = 2$b$ = 4

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{\sqrt{21}}{5}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

**Question 3:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a$ = 4 and $b$ = 3.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$.

The coordinates of the vertices are $\left(\pm 4,0\right)$.

Length of major axis = 2*a* = 8

Length of minor axis = 2*b* = 6

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

**Question 4:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

Answer

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain *b* = 5 and *a* = 10.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = 2*a* = 20

Length of minor axis = 2*b* = 10

Eccentricity, $e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$

**Question 5:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49} + \frac{y^2}{36} = 1$

Answer

The given equation is $\frac{x^2}{49} + \frac{y^2}{36} = 1$ or $\frac{x^2}{7^2} + \frac{y^2}{6^2} = 1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain *a* = 7 and *b* = 6.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{49 - 36} = \sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13}, 0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = 2*a* = 14

Length of minor axis = 2*b* = 12

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{7}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 36}{7} = \frac{72}{7}$

**Question 6:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

Answer

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 10$ and $a = 20$.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2*a* = 40

Length of minor axis = 2*b* = 20

Eccentricity, $e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 100}{20}=10$

**Question 7:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Answer

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$

$$\text{Or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\text{Or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$ .

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ , we obtain *b* = 2 and *a* = 6.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$.

The coordinates of the vertices are (0, ±6).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 4

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$

**Question 8:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Answer

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$

$$\text{Or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\text{Or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are (0, ±4).

Length of major axis = 2*a* = 8

Length of minor axis = 2*b* = 2

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$

**Question 9:**

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Answer

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$4x^2 + 9y^2 = 36$

Or, $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Or, $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ ...(1)

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain *a* = 3 and *b* = 2.

$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5}, 0\right)$.

The coordinates of the vertices are (±3, 0).

Length of major axis = 2*a* = 6

Length of minor axis = 2*b* = 4

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

**Question 10:**

Find the equation for the ellipse that satisfies the given conditions: Vertices (±5, 0), foci (±4, 0)

Answer

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is the semi-major axis.

Accordingly, *a* = 5 and *c* = 4.

It is known that $a^2 = b^2 + c^2$.

$\therefore 5^2 = b^2 + 4^2$

$\Rightarrow 25 = b^2 + 16$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b = \sqrt{9} = 3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

**Question 11:**

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Answer

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where *a* is the semi-major axis.

Accordingly, *a* = 13 and *c* = 5.

It is known that $a^2 = b^2 + c^2$.

$\therefore 13^2 = b^2 + 5^2$

$\Rightarrow 169 = b^2 + 25$

$\Rightarrow b^2 = 169 - 25$

$\Rightarrow b = \sqrt{144} = 12$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$.

**Question 12:**

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Answer

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 6, *c* = 4.

It is known that $a^2=b^2+c^2$.

$\therefore 6^2=b^2+4^2$

$\Rightarrow 36=b^2+16$

$\Rightarrow b^2=36-16$

$\Rightarrow b=\sqrt{20}$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ or $\frac{x^2}{36}+\frac{y^2}{20}=1$.

**Question 13:**

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Answer

Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 3 and *b* = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ i.e., $\frac{x^2}{9}+\frac{y^2}{4}=1$.

**Question 14:**

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Answer

Hyperbola 11.4

$$\frac{y^2}{5} - \frac{x^2}{5} = 1$$