

Exercise 6.1**Question 1:**

Solve $24x < 100$, when (i) x is a natural number (ii) x is an integer

Answer

The given inequality is $24x < 100$.

$$24x < 100$$

$$\Rightarrow \frac{24x}{24} < \frac{100}{24} \quad \text{[Dividing both sides by same positive number]}$$

$$\Rightarrow x < \frac{25}{6}$$

(i) It is evident that 1, 2, 3, and 4 are the only natural numbers less than $\frac{25}{6}$.

Thus, when x is a natural number, the solutions of the given inequality are 1, 2, 3, and 4.

Hence, in this case, the solution set is $\{1, 2, 3, 4\}$.

(ii) The integers less than $\frac{25}{6}$ are $\dots-3, -2, -1, 0, 1, 2, 3, 4$.

Thus, when x is an integer, the solutions of the given inequality are $\dots-3, -2, -1, 0, 1, 2, 3, 4$.

Hence, in this case, the solution set is $\{\dots-3, -2, -1, 0, 1, 2, 3, 4\}$.

Question 2:

Solve $-12x > 30$, when

(i) x is a natural number (ii) x is an integer

Answer

The given inequality is $-12x > 30$.

$$-12x > 30$$

$$\Rightarrow \frac{-12x}{-12} < \frac{30}{-12} \quad \text{[Dividing both sides by same negative number]}$$

$$\Rightarrow x < -\frac{5}{2}$$

(i) There is no natural number less than $\left(-\frac{5}{2}\right)$.

Thus, when x is a natural number, there is no solution of the given inequality.

(ii) The integers less than $\left(-\frac{5}{2}\right)$ are ..., -5, -4, -3.

Thus, when x is an integer, the solutions of the given inequality are ..., -5, -4, -3.

Hence, in this case, the solution set is {..., -5, -4, -3}.

Question 3:

Solve $5x - 3 < 7$, when

(i) x is an integer (ii) x is a real number

Answer

The given inequality is $5x - 3 < 7$.

$$5x - 3 < 7$$

$$\Rightarrow 5x - 3 + 3 < 7 + 3$$

$$\Rightarrow 5x < 10$$

$$\Rightarrow \frac{5x}{5} < \frac{10}{5}$$

$$\Rightarrow x < 2$$

(i) The integers less than 2 are ..., -4, -3, -2, -1, 0, 1.

Thus, when x is an integer, the solutions of the given inequality are ..., -4, -3, -2, -1, 0, 1.

Hence, in this case, the solution set is {..., -4, -3, -2, -1, 0, 1}.

(ii) When x is a real number, the solutions of the given inequality are given by $x < 2$, that is, all real numbers x which are less than 2.

Thus, the solution set of the given inequality is $x \in (-\infty, 2)$.

Question 4:

Solve $3x + 8 > 2$, when

(i) x is an integer (ii) x is a real number

Answer

The given inequality is $3x + 8 > 2$.

$$\begin{aligned}3x + 8 &> 2 \\ \Rightarrow 3x + 8 - 8 &> 2 - 8 \\ \Rightarrow 3x &> -6 \\ \Rightarrow \frac{3x}{3} &> \frac{-6}{3} \\ \Rightarrow x &> -2\end{aligned}$$

(i) The integers greater than -2 are $-1, 0, 1, 2, \dots$

Thus, when x is an integer, the solutions of the given inequality are $-1, 0, 1, 2, \dots$

Hence, in this case, the solution set is $\{-1, 0, 1, 2, \dots\}$.

(ii) When x is a real number, the solutions of the given inequality are all the real numbers, which are greater than -2 .

Thus, in this case, the solution set is $(-2, \infty)$.

Question 5:

Solve the given inequality for real x : $4x + 3 < 5x + 7$

Answer

$$\begin{aligned}4x + 3 &< 5x + 7 \\ \Rightarrow 4x + 3 - 7 &< 5x + 7 - 7 \\ \Rightarrow 4x - 4 &< 5x \\ \Rightarrow 4x - 4 - 4x &< 5x - 4x \\ \Rightarrow -4 &< x\end{aligned}$$

Thus, all real numbers x , which are greater than -4 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-4, \infty)$.

Question 6:

Solve the given inequality for real x : $3x - 7 > 5x - 1$

Answer

$$\begin{aligned}3x - 7 &> 5x - 1 \\ \Rightarrow 3x - 7 + 7 &> 5x - 1 + 7 \\ \Rightarrow 3x &> 5x + 6 \\ \Rightarrow 3x - 5x &> 5x + 6 - 5x\end{aligned}$$

$$\Rightarrow -2x > 6$$

$$\Rightarrow \frac{-2x}{-2} < \frac{6}{-2}$$

$$\Rightarrow x < -3$$

Thus, all real numbers x , which are less than -3 , are the solutions of the given inequality. Hence, the solution set of the given inequality is $(-\infty, -3)$.

Question 7:

Solve the given inequality for real x : $3(x - 1) \leq 2(x - 3)$

Answer

$$3(x - 1) \leq 2(x - 3)$$

$$\Rightarrow 3x - 3 \leq 2x - 6$$

$$\Rightarrow 3x - 3 + 3 \leq 2x - 6 + 3$$

$$\Rightarrow 3x \leq 2x - 3$$

$$\Rightarrow 3x - 2x \leq 2x - 3 - 2x$$

$$\Rightarrow x \leq -3$$

Thus, all real numbers x , which are less than or equal to -3 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, -3]$.

Question 8:

Solve the given inequality for real x : $3(2 - x) \geq 2(1 - x)$

Answer

$$3(2 - x) \geq 2(1 - x)$$

$$\Rightarrow 6 - 3x \geq 2 - 2x$$

$$\Rightarrow 6 - 3x + 2x \geq 2 - 2x + 2x$$

$$\Rightarrow 6 - x \geq 2$$

$$\Rightarrow 6 - x - 6 \geq 2 - 6$$

$$\Rightarrow -x \geq -4$$

$$\Rightarrow x \leq 4$$

Thus, all real numbers x , which are less than or equal to 4 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 4]$.

Question 9:

Solve the given inequality for real x : $x + \frac{x}{2} + \frac{x}{3} < 11$

Answer

$$\begin{aligned}x + \frac{x}{2} + \frac{x}{3} &< 11 \\ \Rightarrow x \left(1 + \frac{1}{2} + \frac{1}{3} \right) &< 11 \\ \Rightarrow x \left(\frac{6+3+2}{6} \right) &< 11 \\ \Rightarrow \frac{11x}{6} &< 11 \\ \Rightarrow \frac{11x}{6 \times 11} &< \frac{11}{11} \\ \Rightarrow \frac{x}{6} &< 1 \\ \Rightarrow x &< 6\end{aligned}$$

Thus, all real numbers x , which are less than 6, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 6)$.

Question 10:

Solve the given inequality for real x : $\frac{x}{3} > \frac{x}{2} + 1$

Answer

$$\begin{aligned}\frac{x}{3} &> \frac{x}{2} + 1 \\ \Rightarrow \frac{x}{3} - \frac{x}{2} &> 1 \\ \Rightarrow \frac{2x - 3x}{6} &> 1 \\ \Rightarrow -\frac{x}{6} &> 1 \\ \Rightarrow -x &> 6 \\ \Rightarrow x &< -6\end{aligned}$$

Thus, all real numbers x , which are less than -6 , are the solutions of the given inequality. Hence, the solution set of the given inequality is $(-\infty, -6)$.

Question 11:

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

Solve the given inequality for real x :

Answer

$$\begin{aligned}\frac{3(x-2)}{5} &\leq \frac{5(2-x)}{3} \\ \Rightarrow 9(x-2) &\leq 25(2-x) \\ \Rightarrow 9x - 18 &\leq 50 - 25x \\ \Rightarrow 9x - 18 + 25x &\leq 50 \\ \Rightarrow 34x - 18 &\leq 50 \\ \Rightarrow 34x &\leq 50 + 18 \\ \Rightarrow 34x &\leq 68 \\ \Rightarrow \frac{34x}{34} &\leq \frac{68}{34} \\ \Rightarrow x &\leq 2\end{aligned}$$

Thus, all real numbers x , which are less than or equal to 2 , are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Question 12:

Solve the given inequality for real x : $\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$

Answer

$$\begin{aligned}\frac{1}{2}\left(\frac{3x}{5}+4\right) &\geq \frac{1}{3}(x-6) \\ \Rightarrow 3\left(\frac{3x}{5}+4\right) &\geq 2(x-6) \\ \Rightarrow \frac{9x}{5}+12 &\geq 2x-12 \\ \Rightarrow 12+12 &\geq 2x-\frac{9x}{5} \\ \Rightarrow 24 &\geq \frac{10x-9x}{5} \\ \Rightarrow 24 &\geq \frac{x}{5} \\ \Rightarrow 120 &\geq x\end{aligned}$$

Thus, all real numbers x , which are less than or equal to 120, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 120]$.

Question 13:

Solve the given inequality for real x : $2(2x + 3) - 10 < 6(x - 2)$

Answer

$$\begin{aligned}2(2x+3)-10 &< 6(x-2) \\ \Rightarrow 4x+6-10 &< 6x-12 \\ \Rightarrow 4x-4 &< 6x-12 \\ \Rightarrow -4+12 &< 6x-4x \\ \Rightarrow 8 &< 2x \\ \Rightarrow 4 &< x\end{aligned}$$

Thus, all real numbers x , which are greater than or equal to 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $[4, \infty)$.

Question 14:

Solve the given inequality for real x : $37 - (3x + 5) \geq 9x - 8(x - 3)$

Answer

$$\begin{aligned}
37 - (3x + 5) &\geq 9x - 8(x - 3) \\
\Rightarrow 37 - 3x - 5 &\geq 9x - 8x + 24 \\
\Rightarrow 32 - 3x &\geq x + 24 \\
\Rightarrow 32 - 24 &\geq x + 3x \\
\Rightarrow 8 &\geq 4x \\
\Rightarrow 2 &\geq x
\end{aligned}$$

Thus, all real numbers x , which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

Question 15:

Solve the given inequality for real x : $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Answer

$$\begin{aligned}
\frac{x}{4} &< \frac{(5x-2)}{3} - \frac{(7x-3)}{5} \\
\Rightarrow \frac{x}{4} &< \frac{5(5x-2) - 3(7x-3)}{15} \\
\Rightarrow \frac{x}{4} &< \frac{25x - 10 - 21x + 9}{15} \\
\Rightarrow \frac{x}{4} &< \frac{4x - 1}{15} \\
\Rightarrow 15x &< 4(4x - 1) \\
\Rightarrow 15x &< 16x - 4 \\
\Rightarrow 4 &< 16x - 15x \\
\Rightarrow 4 &< x
\end{aligned}$$

Thus, all real numbers x , which are greater than 4, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(4, \infty)$.

Question 16:

Solve the given inequality for real x : $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Answer

$$\begin{aligned} \frac{(2x-1)}{3} &\geq \frac{(3x-2)}{4} - \frac{(2-x)}{5} \\ \Rightarrow \frac{(2x-1)}{3} &\geq \frac{5(3x-2)-4(2-x)}{20} \\ \Rightarrow \frac{(2x-1)}{3} &\geq \frac{15x-10-8+4x}{20} \\ \Rightarrow \frac{(2x-1)}{3} &\geq \frac{19x-18}{20} \\ \Rightarrow 20(2x-1) &\geq 3(19x-18) \\ \Rightarrow 40x-20 &\geq 57x-54 \\ \Rightarrow -20+54 &\geq 57x-40x \\ \Rightarrow 34 &\geq 17x \\ \Rightarrow 2 &\geq x \end{aligned}$$

Thus, all real numbers x , which are less than or equal to 2, are the solutions of the given inequality.

Hence, the solution set of the given inequality is $(-\infty, 2]$.

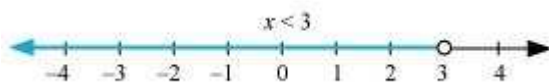
Question 17:

Solve the given inequality and show the graph of the solution on number line: $3x - 2 < 2x + 1$

Answer

$$\begin{aligned} 3x - 2 &< 2x + 1 \\ \Rightarrow 3x - 2x &< 1 + 2 \\ \Rightarrow x &< 3 \end{aligned}$$

The graphical representation of the solutions of the given inequality is as follows.



Question 18:

Solve the given inequality and show the graph of the solution on number line: $5x - 3 \geq 3x - 5$

Answer

