Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Class XII Mathematics

CBSE Board, Set – 3

General Instructions:

(i) All questions are compulsory.

(ii) Please check that this Question Paper contents 26 Questions.

(iii) Marks for each Question are indicated against it.

(iv) Question 1 to 6 in Section-A are very short Answer Type Questions carrying one.

(v) Question 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks.

(vi) Question 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.

(vii) Please write down the serial number of the Questions before attempting it.

Section –A

Q1. Find the differential equation representing the family of curves $v = \frac{A}{r} + B$, where A and B are arbitrary constants.

Sol.1 $v = A/r + B$ {differentiating}

$\frac{dv}{dr} = \frac{-A}{r^2}$ …….. {1}

$A = -\frac{r^2 dv}{dr}$

$\frac{d^2v}{dr^2} = \frac{2}{r^3}\left(\frac{-r^2.dv}{dr}\right)$ {Upon differentiating ….1 & substituting value of A}

$\frac{d^2v}{dr^2} = \frac{-2}{r}.\frac{dv}{dr}$

Q2. Find the integrating factor of the differential equation

$\left(\frac{e-2\sqrt{x}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$

Sol.2 $\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2}\sqrt{x}}{\sqrt{x}}$

Integrating Factor $= e^{\int \frac{1}{\sqrt{x}} dx}$

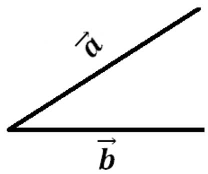
$= e^{\int x^{-\frac{1}{2}} dx}$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Integrating Factor $= e^{2\sqrt{x}}$

Q3. If $\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$.

Sol.3 Projection is|a| cos θ



$= \frac{a.b}{|b|}$

$= \frac{14+6-12}{\sqrt{36+4+9}}$

$= \frac{20-12}{7} = \frac{8}{7}$

Q4. Find λ, if the vector $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\jmath} + 3\hat{k}$ are coplanar.

Sol.4 If vector are coplanar

$x\vec{a} + y\vec{b} + z\vec{c} = 0$

$x(\hat{\imath} + 3\hat{\jmath} + \hat{k}) + y\,(2\hat{\imath} - \hat{\jmath} - \hat{k}) + z(\lambda\hat{\jmath} + 3\hat{k}) = 0$

Or

[abc] = 0

$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} \Rightarrow -3 + \lambda - 3\,(6) + 1\,(2\lambda) = 0$

$-3 + \lambda - 18 + 2\lambda = 0$

$3\lambda = 21$

$\lambda = 7$

Q5. If a line makes angles 90°, 60° and θ with x, y and z-axis respectively, where θ is acute, then find θ.

Sol.5 Sum of squares of direction cosines is 1

θ, α, γ are angles with x, y, z axis

$\cos^2\theta + \cos^2\alpha + \cos^2\gamma = 1$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$0 + \frac{1}{4} + \cos^2 \gamma = 1$

$\cos^2 \gamma = \frac{3}{4}$

$\gamma = cos^{-1}\left[\sqrt{\frac{3}{4}}\right]$

Acute angle is 30°

Q6. Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.

Sol.6 |3×3|

$a_{23} = \frac{|2-3|}{2}$

$a_{23} = \frac{1}{2}$

<u>Section – B</u>

Q7. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

Sol.7 $x = a \cos \theta + b \sin \theta$

$X^2 = a^2 \cos^2\theta + b^2\sin^2\theta + 2ab \sin\theta \cos\theta$ ..... (1)

$Y = a \sin \theta - b \cos \theta$

$Y^2 = a^2 \sin^2 \theta + b^2 \cos^2\theta - 2ab \sin\theta\cos\theta$ ..... (2)

Adding equation (1) and (2)

$X^2 + y^2 = a^2 (\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

as we know $\sin^2\theta + \cos^2\theta = 1$

$x^2 + y^2 = a^2 + b^2$ ..... (3)

Now: differentiating (3) w.r. to x

$2x + 2y .\frac{dy}{dx} = 0$ ..... (4)

$\frac{dy}{dx} = \frac{-x}{y}$..... (5)

Differentiating equation (4) we get:

$2 + 2y \cdot \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$

$1 + y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ Multiplying whole equation by Y

$y + y^2\frac{d^2y}{dx^2} + y \cdot \left(\frac{dy}{dx}\right)\left(\frac{dx}{dx}\right) = 0$

$y + y^2\frac{d^2y}{dx^2} + y \cdot \left(\frac{dy}{dx}\right) = 0$ multiplying value from equation (5)

$y^2\frac{d^2y}{dx^2} - \frac{xdy}{dx} + y = 0$

Q8. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm?

Sol.8 we know that area of equilateral triangle whose side length is "a" is given by:

$A = \frac{\sqrt{3}a^2}{4}$

Differentiating w.r. to a we get:

$\frac{dA}{da} = \frac{\sqrt{3}}{4}(2a)$

dA (rate of change of area) $= \left(\frac{\sqrt{3}a}{2}\right) \cdot$ da (rate change in side length)

putting values:

$dA = \frac{\sqrt{3}}{2}(20a) \cdot 2$ cm/sec $\quad = 20\sqrt{3}$ cm^2/sec.

Q9. Find : $\int (x + 3)\sqrt{3 - 4x - x^2}\, dx$.

Sol.9 $\int (x + 3)\sqrt{3 - 4x - x^2}\; dx$

$\frac{d}{dx}(3 - 4x - x^2) = -4 - 2x$

$= -2\,(x + 2)$

$\int (x + 2)\sqrt{3 - 4x - x^2}dx + \int \sqrt{3 - 4x - x^2} \cdot dx$

Let this be integral of two functions A & B

$A = \int (x + 2)\sqrt{3 - 4x - x^2} \cdot dx$

$B = \int \sqrt{3 - 4x - x^2} \cdot dx$

For A: $\int (x+2)\sqrt{3-4x-x^2}\cdot dx$

Let $3 - 4x - x^2 = t$

Differentiating $(-4-2x)\cdot dx = dt$

$-2(x+2)\cdot dx = dt$

$(x+2)dx = \frac{dt}{-2}$

Substituting thing in integral A

$A = \int \sqrt{t}\cdot\frac{dt}{-2} = \frac{-1}{2}\int t^{1/2}\cdot dt$

$A = \frac{-1}{2}\frac{t^{3/2}}{\frac{3}{2}} + C.$

$= \frac{-t^{3/2}}{3} + C.$

Putting value of t, we get

$= -\frac{(3-4x-x^2)^{3/2}}{3} + C$

Now $B = \int \sqrt{3-4x-x^2}\cdot dx$

$= \int \sqrt{7-4-4x-x^2}\cdot dx$

$= \int \sqrt{7-(x+2)^2}\cdot dx$

$= \int \sqrt{\left(\sqrt{7}\right)^2-(x+2)^2}\cdot dx$

Putting $x + 2 = \sqrt{7}\sin\theta$

upon differentiation $dx = \sqrt{7}\cos\theta\cdot d\theta$

putting values:

$B = \int \sqrt{\left(\sqrt{7}\right)^2-\left(\sqrt{7}\sin\theta\right)^2}\cdot\sqrt{7}\cos\theta\cdot d\theta$

$= \int \sqrt{7}\sqrt{1-\sin^2\theta}\cdot\sqrt{7}\cdot\cos\theta\cdot d\theta$

$B = \int 7\cos\theta\cdot\cos\theta\cdot d\theta \qquad = \int 7\cos^2\theta\cdot d\theta$

$= \int 7\left\{\frac{\cos 2\theta+1}{2}\right\}\cdot d\theta \qquad$ as $2\cos^2\theta - 1 = \cos 2\theta$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$= \frac{7}{2}\left\{\theta + \frac{\sin 2\theta}{2}\right\} + k.$

Putting value of θ

$$B = \frac{7}{2}\left\{\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + \frac{\sin 2\left\{\sin -\left(\frac{x+2}{\sqrt{7}}\right)\right\}}{2}\right\} + k$$

$$A + B = -\frac{(3-4x-x^2)^{3/2}}{3} + \frac{7}{2}\left[\sin - \frac{(x+2)}{\sqrt{7}} + \frac{\sin 2\left\{\sin - \frac{(x+2)}{\sqrt{7}}\right\}}{2}\right] + m.$$

Q10. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs. 100 and Rs. 50 each. The number of articles sold are given below:

Article/School	A	B	C
Hand-fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.

Write one value generated by the above situation.

Sol.10 Rate of Hand fan = 25

Rate of Mats = 100

Rate of Plates = 50

School A add 40 fans, 50 mats & 20 plates. Amount collected by

School A will be:

$A_m = (40 \times 25) + (50 \times 100) + (50 \times 20)$

$= 1000 + 5000 + 1000$

$=$ Rs. 7000

Similarly: $B_m = (25 \times 25) + (40 \times 100) + (30 \times 20)$

$= 625 + 4000 + 600$

$=$ Rs. 10625

$C_m = (35 \times 25) + (56 \times 100) + (40 \times 20)$

$= 875 + 5000 + 8000$

$=$ Rs. 6675

Total funds collected: $A_m + B_m + C_m$

$=$ Rs. $(7000 + 10625 + 6675)$

$=$ Rs. 24300

Q11. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$

Sol.11 $A = \begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$

$A^2 = \begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix} \times \begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$

$= \begin{vmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{vmatrix}$

$A^2 - 5A + 4J = \begin{vmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{vmatrix} - \begin{vmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{vmatrix} + \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$

$= \begin{vmatrix} 9 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{vmatrix} - \begin{vmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{vmatrix}$

$= \begin{vmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{vmatrix}$

So, $X = -(A^2 - 5A + 4J)$

$X = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{vmatrix}$

OR

Q.11 If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

Sol.11 $A^1 = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & -2 \\ 3 & 4 & 1 \end{vmatrix}$

$|A^1| = 1(-1-8) - 2(-8+3)$

$= -9 + 10 = 1$

Cofactor matrix $= \begin{vmatrix} -9 & +8 & -5 \\ -8 & 7 & -4 \\ -2 & +2 & -1 \end{vmatrix}$

$(\text{Cofactor})^T = \begin{vmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{vmatrix}$

$\therefore (A^1)^T = \frac{Adj(A^1)}{|A^1|} = \begin{vmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{vmatrix}$

Q12. If $t(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of f(2x) – f(x).

Sol.12 $f(2x) - f(x) = \begin{vmatrix} a & -1 & 0 \\ 2ax & a & -1 \\ 4ax^2 & 2ax & a \end{vmatrix} - \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

Q13. Find: $\int \frac{dx}{\sin x + \sin 2x}$

Sol.13 $\frac{dx}{\sin x + \sin 2x}$

Also,

$\sin x = \frac{2u}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$dx = \frac{2du}{1+u^2}$

u = tan u2

$$\frac{1}{2}\int \frac{1}{\frac{2u}{1+u^2} + 2\cdot\frac{2u}{1+u^2}\cdot\frac{1-u^2}{1-u^2}}\cdot\frac{2}{1+u^2}du$$

$$\frac{1}{2}\int\frac{u^2+1}{2u}du$$

Integration $= \frac{2\ln(u)+u^2\cdot e}{8}$

But u $= \tan\frac{x}{2}$

$\therefore \frac{2\ln\ \tan\left(\frac{x}{2}\right)+\tan^2\frac{x}{2}+c}{8}$

$= a^3\ +\ 4a^2x^2 + 4ax^2 - (a^3 + 2a^2x + ax^2)$

$= 2a^2x + 3ax^2$ (Ans.)

OR

Q.13 Integrate the following w.r.t.x

$\frac{x^2-3x+1}{\sqrt{1-x^2}}$

Sol.13 $\frac{x^2-3x+1}{\sqrt{1-x^2}}$ can be written as. . . (1)

$\frac{x^2-1-3x+2}{\sqrt{1-x^2}}$

$\frac{-(1-x^2)}{\sqrt{1-x^2}} + \frac{-3x}{\sqrt{1-x^2}} + \frac{2}{\sqrt{1-x^2}}$

$-\sqrt{1-x^2} - \frac{3x}{\sqrt{1-x^2}} + \frac{2}{\sqrt{1-x^2}}$

Now: Integration:

- $\int\sqrt{1-x^2}\cdot dx +\ \int\frac{-3x}{\sqrt{1-x^2}}\cdot dx + \int\frac{2}{\sqrt{1-x^2}}\cdot dx$

Lets divide this into 3 different integrals.

A $= \int -\sqrt{1-x^2}\cdot dx$

B $= \int\frac{-3x}{\sqrt{1-x^2}}\cdot dx$

C $= \int\frac{2}{\sqrt{1-x^2}}\cdot dx$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

A = $-\int \sqrt{1-x^2} \cdot dx$

Let x = sin θ dx = cosθ ·dθ·

= $-\int \sqrt{1-\sin^2\theta} \cdot = -\int \frac{1+\cos^2\theta}{2} \cdot d\theta$

= $(-\frac{\theta}{2} - \frac{\sin^2\theta}{4}) + c$

Putting value of θ

A = $-\left[\frac{sinx}{2} - \frac{\sin\{2(\sin^{-} x)\}}{4}\right] + c$

B = $\int \frac{-3x}{\sqrt{1-x^2}} \cdot dx$

Let $1-x^2 = z$

$-2x \cdot dx = dz$

$-x \cdot dx = \frac{dz}{2}$

B = $\int \frac{3 \cdot dz}{2\sqrt{z}}$

= $\frac{3}{2}(\sqrt{z}) \cdot 2 + l = 3\sqrt{z} + l$

Putting value of z

B = $3\sqrt{1-x^2} + l$

C = $\int \frac{2}{\sqrt{1-x^2}} \cdot dx$

This is a direct result:

C = $2\sin^{-1} x + m$

So, A + B + C ⇒

$-\frac{\sin^{-1}x}{2} - \frac{\sin(2\sin^{-1}x)}{4} + 3\sqrt{1-x^2} + 2\sin^{-1} x + C$

$= 3\sqrt{1-x^2} + \frac{3}{2}\sin^{-1} x - \sin\left(\frac{2\sin^{-1}x}{4}\right) + C$

Q14. Evaluate: $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 \, dx$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Sol.14 $\int_{-\pi}^{\pi}(\cos\ a\,x-\sin b\,x)^2\cdot dx=\int_{-\pi}^{\pi}(\cos^2 \mathrm{ax}+\sin^2 \mathrm{by}-2\sin \mathrm{bx}\cos \mathrm{ax})\,\mathrm{dx}$

$$\int_{-\pi}^{\pi}(\cos^2 ax\cdot dv+\int_{-\pi}^{\pi}\sin^2 bx-2\int_{-\pi}^{\pi}\sin bx\cos ax$$

Clearly $\cos^2$ ax is even

$\sin^2$bx is even

but sin bxcos ax is odd

so, its integral from −π to 0 will cancel the 0 to π part.

i.e. our eqn reduces to:

$$=2\int_0^{\pi}\cos^2 ax\cdot dx+2\int_0^{\pi}\sin^2 bx\cdot dx$$

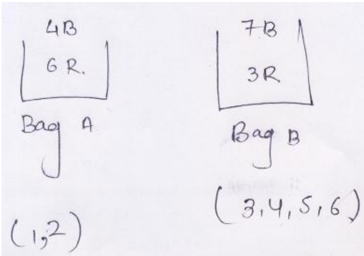
$$=2\int_0^{\pi}\frac{1+\cos 2ax}{2}\cdot dx+2\int_0^{\pi}\frac{1-\cos 2bx}{2}\cdot dx$$

$$=\left[x+\frac{\sin 2ax}{2a}\right]_0^{\pi}+\left[x-\frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$= \pi + \pi = 2\pi$

Q15. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. if two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

Sol.15



We need to draw one red & one black. This on be done via both the bags.

So, when die is thrown:

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$\frac{2}{6}$probability of bag A to be chosen

$\frac{4}{6}$ ______ bag B to be chosen

for bag A $\rightarrow \frac{2}{6} \cdot \frac{4.6}{10_{C_2}} = \frac{2}{6} \times \frac{4.6}{10.9} \times 2$

From bag B $\rightarrow \frac{4}{6} \cdot \frac{7.3}{10_{C_2}} = \frac{4}{6} - \frac{7.3}{10.9} \cdot 2$

$= \frac{14}{45}$

Total probability $= \frac{22}{45}$

OR

Q.15 An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

Sol.15 n = number of times

p = probability of success

Mean = np

$= 4 \cdot \frac{1}{2}$

Mean = 2

Variance = np (1-p)

$= 4 \cdot \frac{1}{2} \cdot \frac{1}{2}$

Variance = 1

Q16. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times j) + xy$

Sol.16 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{r} \times \hat{\imath} = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \times \hat{\imath}$

$= x\,(\hat{\imath} \times \hat{\jmath}) + y\,(\hat{\jmath} \times \hat{\imath}) + Z\,(\hat{k} \times \hat{\imath})$

$= -y\,\hat{k} + z\hat{\jmath}$ ..... (1)

Also, $\vec{r} \times \hat{\jmath} = x\,(\hat{\imath} \times \hat{\jmath}) + y\,(\hat{\jmath} \times \hat{\jmath}) + z\,(\hat{k} \times \hat{\jmath})$

$= x\hat{k} + 0 + z(-\hat{\imath})$ . . . . . (2)

So required $= (\vec{r} \times \hat{\imath}).(\vec{r} \times \hat{\jmath}) + xy$

$= (-y\,\hat{k} + z\hat{\jmath})\cdot(x\hat{k} - z\hat{\imath}) + xy$

$= -xy + xy$

$= 0$

Q17. Find the distance between the point (−1, −5, −10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x – y + z = 5.

Sol.17 Solving $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda$

So, $(3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 5$

$11\lambda + 5 = 5$

$\Lambda = 0$

∴ Point of intersection is (2, −1, 2)

So using distance formula,

Distance $= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$

$= \sqrt{9 + 16 + 144}$

$= 13$

So, Distance between then is 13.

Q18. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1}x)$, then find x.

Sol.18 $\cot^{-1}(x+1) = \tan^{-1}\left(\frac{1}{x+1}\right)$ If x +1 > 0.

$= \boldsymbol{\pi} + \tan^{-1}\left(\frac{1}{x+1}\right)$ If x+1 < 0

⇒ since sin and cos are equal both angles are complementary.

$\text{Cot}^{-1}(x+1) + \tan^{-1}x = \frac{\pi}{2}$

Case I:- x + 1 > 0

$\tan^{-1}\left(\frac{1}{x+1}\right) + \tan^{-1}x = \frac{\pi}{2}$

$\tan^{-1}\left(\frac{\frac{1}{x+1}+x}{1+\frac{x}{1+x}}\right)=\frac{\pi}{2}$

⇒ No solution.

Case II:- $x + 1 < 0 \Rightarrow x < -1$

$\pi + \tan^{-1}\left(\frac{1}{x+1}\right) + \tan^{-1} x = \frac{\pi}{2}$

$\tan^{-1} (x^2 + x + 1) = \frac{\pi}{2} - \pi$

$\tan^{-1}(x^2 + x + 1) = \frac{-\pi}{2}$

⇒ No solution

OR

Q.18 If $(\tan^{-1}x) + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then find x.

Sol.18 Let $\tan^{-1} x = t$

So, $\cot^{-1} x = \frac{\pi}{2} - t$

$\therefore t^2 + \left(\frac{\pi}{2} - t\right)^2 = \frac{5\pi^2}{8}$

$\therefore t^2 + \left(\frac{\pi^2}{4} + t^2 - \pi t\right) = \frac{5\pi^2}{8}$

$\Rightarrow 2t^2 - \pi t + \frac{\pi^2}{4} - \frac{5\pi^2}{8} = 0$

$\Rightarrow 2t^2 - \pi t \frac{-3\pi^2}{8} = 0$

Solving this quadratic equation we get

$t = \frac{-\pi}{4}$ or $-t = \frac{3\pi}{4}$

$\therefore \tan^{-1} x = \frac{-\pi}{4}$ or $\tan^{-1} x = \frac{3\pi}{4}$

$x = \tan\left(\frac{-\pi}{4}\right)$ or No solution exist $\Longrightarrow$ $x = -1$

Q19. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, $x^2 \le 1$, then find $\frac{dy}{dx}$,

Sol.19 Put $x^2 = \cos 2\theta$

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-cos 2\theta}}\right)$

$\therefore y = \tan^{-1}\left(\frac{\sqrt{2}\cos^2\theta+\sqrt{2}\sin^2\theta}{\sqrt{2}\cos^2\theta-\sqrt{2}\sin^2\theta}\right)$

$\therefore y = \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

Dividing N^r and D^r by cos θ

$\therefore y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\frac{\tan 45°+\tan\theta}{1-\tan 45°\tan\theta}\right) = \tan^{-1}(\tan(45+\ \theta))$

$y = \frac{\pi}{4} + \theta$

$\therefore \quad \frac{dy}{d\theta} = 1 \quad \ldots\ldots\ldots (1)$

Now, $\frac{dy}{d\theta} = \frac{dy}{dx} \times \frac{dx}{d\theta} = 1$

$\therefore \quad \frac{dy}{dx} = \frac{d\theta}{dx} \quad \ldots\ldots\ldots (2)$

Also,

$x^2 = \cos 2\theta$

2x dx = – 2sin 2θ dθ

$\therefore \frac{-x}{\sin 2\theta} = \frac{d\theta}{dx}$

$\therefore \frac{-x}{\sqrt{1-x^4}} = \frac{d\theta}{dx}$

$\therefore \frac{dy}{dx} = \frac{d\theta}{dx} = \frac{-x}{\sqrt{1-x^4}}$ (from eqn (1) & (2)

Section – C

Q20. If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P(B).

Sol.20 Since A and B are independent

P (A ∩ B) = P (A) P (B)

$P(\bar{A} \cap B) = P(\bar{A})\, P(B)$

$P(A \cap \bar{B}) = P(A)\, P(\bar{B}) \ldots\ldots (1)$

Let P (A) = P so P (A) = 1-p

and P(b) = q so P(B) = 1-q

Given, $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$

(1-p) a = $\frac{2}{15}$ . . . . . (2)

p(1-q) = $\frac{1}{6}$ . . . . . (3)

Solving these equation (2) and (3),

$q + \frac{1}{30} = p \rightarrow$ substitute this in one of above (2) or (3)

$p(1 - q) = \frac{1}{6}$

$p\left(q + \frac{1}{30}\right)(1 - q) = \frac{1}{6}$

$\frac{(30q+1)(1-q)}{30} = \frac{1}{6}$

$\Rightarrow \frac{30q-30q^2+1-q}{5} = 1$

$\Rightarrow -30q^2 + 2aq - 4 = 0$

$\Rightarrow 30q^{2+} - 2aq + 4 = 0$

$\Rightarrow q = \frac{1}{6}$ or $q = \frac{4}{5}$

If $q = \frac{1}{6} \Rightarrow p = \frac{1}{6} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$

If $q = \frac{4}{5} \Rightarrow p = \frac{4}{5} + \frac{1}{30} = \frac{25}{30} = \frac{5}{6}$

So , $q = = \frac{1}{6}, p = \frac{1}{5}$ Or $p \frac{5}{6}$ and $q = \frac{4}{5}$

Q21. Find the local maxima and local minima, of the function f(x) = sin x – cos x, 0 < x < 2π. Also find the local maximum and local minimum values.

Sol.21 f (x) = sin x – cos x 0 < x < 2π

f ‘ (x) = cos x + sin x = 0

cos x + sin x = 0

Or tan x = -1

$X = \frac{3\pi}{4}$ and $\frac{7\pi}{4}$

Also f''(x) = - sin x + cos x

$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}$ = negative

$f''\left(\frac{3\pi}{4}\right) = -\sin\left(\frac{3\pi}{4}\right) + \text{cosw}\left(\frac{7\pi}{4}\right)$ = positive

$\therefore x = \frac{3\pi}{4}$ is local maxima

And $x = \frac{7\pi}{4}$ is local minima

Also, $f\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = 1.414$

$f\left(\frac{3\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) - \cos\left(\frac{7\pi}{4}\right) = -1.414$

Q22. Find graphically, the maximum value of z = 2x + 5y, subject to constraints given below:

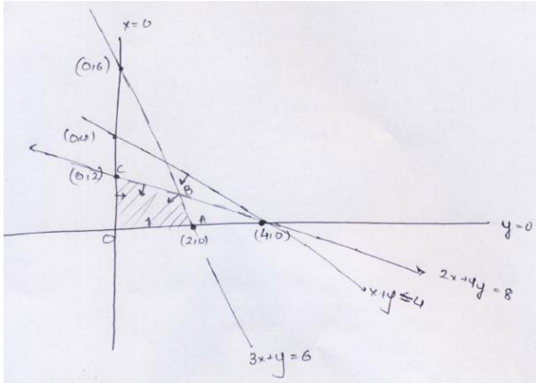
$2x + 4y \leq 8$

$3x + y \leq 6$

$x + y \leq 4$

$x \geq 0, y \geq 0$

Sol.22



Here , we have plotted all the conditional. The common area is OABC.

How value z = 2x +5y

Check 2 at C, B, A, & O

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

Z_c (0,2) = 10

Z_A (2,0) = 4

$Z_B\left(\frac{8}{8},\frac{6}{5}\right) = \frac{46}{5}$

Clearly Z_c is maximum so max of z is 10 which occurs at 0, 2

Q23. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad(b + c) = bc(a + d). Show that R is an equivalence relation.

Sol.23 ad (b + c) = bc (a + d)

adb + adc = abc + bcd

Dividing by 'abcd'

$\frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$

(i) Reflexivity: Let (a, b) be an arbitrary element of N × N

Then, (a, b) ϵ N × N ⟹ a, b ϵ N

⟹ a b (b + a) = b a (a + b)

⟹ (a, b) R (a + b)

Thus (a, b) R (a, b) for all (a, b) e N × N. So, R is Reflexive on N × N.

(ii) symmetry: Let (a, b) (c, d) ϵ N × N be such that

(a, b) ϵ (c, d). Then

(a, b) R (c, d) ⟹ ad (b + c) = bc(a + d) (by comm. Of add and multi on N)

⟹ (c, d) R (a, b)

Thus (a, b) R (c, d) ⟹ (c, d) R (a, b)

Hence it is symmetric

(iii) Transitivity: Let (a, b), (c, d), (e, f) ϵ N × n.

Such that (a, b) R (c, d) and (c, d) R (e, f). Then

(a, b) R (c, d) = ad (b + c) = bc (a + d).

$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$

$\Rightarrow \frac{1}{c}+\frac{1}{b}=\frac{1}{a}+\frac{1}{d}$ ...... (1)

and (c, d) R (e, f) $\Rightarrow \frac{1}{d}+\frac{1}{c}=\frac{1}{c}+\frac{1}{f}$ .... (2)

adding (1) and (2),

$$\frac{1}{b}+\frac{1}{e}=\frac{1}{f}+\frac{1}{a}$$

∴ (a, b) R (e, f)

and Hence

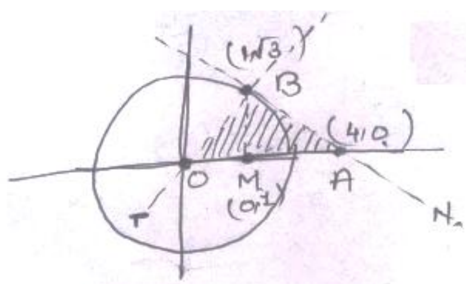
(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ (a, b) R (e, f)

Hence it is transitive also.

Q24. Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

Sol.24 Eqn of circle:



$x^2 + y^2 = 4$

Eqn of tangent BA is given by

T = O from point $(1, \sqrt{3})$

i.e. $xx_1 + yy_1 = 4$ where $x_1, y_1$is $(1, \sqrt{3})$

soeqn is $x + \sqrt{3}y = 4$

Eqn of normal OB is given by

$y = kx$ {as it passes through centre}

it also passes through $1, \sqrt{3}$

i.e. $k = \sqrt{3}$

so normal is $\sqrt{3}x = y$

So Area of triangle OBA is

= Area OBM + Area MBA

$$= \int_0^1 \sqrt{3}x \cdot dx + \int_1^4 \left(\frac{4-x}{3}\right) \cdot dx \qquad \{\text{see figure}\}$$

Integrating:

$$= \sqrt{3}\left(\frac{x^2}{2}\right)_0^1 + \frac{1}{\sqrt{3}}\left(4x - \frac{x^2}{2}\right)_1^4$$

$$= \frac{\sqrt{3}}{2}(1-0) + \frac{1}{\sqrt{3}}\left[(16-8) - \left(4 - \frac{1}{2}\right)\right]$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[8 - 4 + \frac{1}{2}\right]$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left\{\frac{9}{2}\right\}$$

$$= \frac{1}{2}\left(\sqrt{3} + \frac{9}{\sqrt{3}}\right) = \frac{1}{2}(4\sqrt{3}) = 2\sqrt{3} \text{ sq units}$$

OR

Q24. Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1)\,dx$ as a limit of a sum.

Sol.24 $\int_1^3 (e^{2-3x} + x^2 + 1)\,dx \qquad x = at - b$

$$\begin{array}{c|c} x = 1 & x = 3 \\ t = 0 & t = 1 \end{array}$$

$1 = -b$

$3 = a - b$

$a = 3 + b$

$a = 2$

$\therefore x = 2t + 1$

$dx = 2dt$

$$\int_1^3 (e^{(2-(2t+1)3)} + (2t+1)^2 + 1$$

$$= \int_0^1 \left(e^{2-6t-3} + 4t^2 + 4t + 3\right) 2dt$$

$$= 2\int_0^1 \left(e.^{\perp} (e^6)^t + 4t^2 + 4t + 3\right)$$

$$= 2\int_0^1 \frac{1}{e} \cdot e^{-6t} dt + \int_0^1 4t^2 dt + \int_0^1 4tdt + \int_0^1 3dt$$

$$\lim_{x\to\infty} \frac{2}{e} \sum_{r=1}^{n} e^{-6\frac{r}{n}} \cdot \frac{1}{n} + \sum_{r=1}^{n} 4\left(\frac{r}{n}\right)^2 \cdot \frac{1}{n} + \sum_{r=1}^{n} 4\left(\frac{r}{n}\right)\frac{1}{n} + 3\sum_{r=1}^{n} \frac{3}{n}$$

$$= \frac{2}{e} \cdot \frac{1}{n}\left[\cdot e^{\frac{-6}{n}} + e\frac{-12}{n}\ldots\ldots\right] + \frac{4}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{4}{n^2}\frac{n(n+1)}{2} + \frac{3}{n} \cdot n$$

$$\lim_{x\to\infty} \frac{2}{e} \cdot \frac{1}{n} \cdot \frac{e^{\frac{-6}{n}}}{1 - e^{\frac{-6}{n}}} + 2 + \left\{\frac{1}{6} \cdot \frac{4}{n^2}\left\{2n^2 + 3n + 1\right\} + \frac{4}{n}\frac{(n+1)}{2} + \frac{3}{n} \cdot n\right\}$$

$$= 0 + \frac{4 \cdot 2}{6} + \frac{4}{2} + 3 = \frac{4}{3} + 2 + 3 = \frac{10}{3} + 3 = \frac{19 \cdot 2}{3} = \frac{38}{3}$$

Q25. Solve the differential equation:

$(\tan^{-1}y - x)dy = (1 + y^2)dx.$

Sol.25 $(\tan^{-1} y - x)dy = (1 + y^2)dx$

$$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$

$$\frac{dx}{dy} = \frac{x}{1 + y^2} + \frac{\tan^{-1} y}{1 + y^2}$$

$$\frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{\tan^{-1} y}{1 + y^2}$$

$$\frac{dx}{dy} + p\,x = Q.$$

This is a linear differential of 1st order

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

The solution is

$x \cdot (\text{If}) = \int Q \cdot (If) \cdot dy$

where $Q = \frac{\tan^{-1} y}{1+y^2}$

$\text{If} = e^{\int \frac{1}{1+y^2} \cdot dy}$

$= e^{\tan^{-1} y}$

Putting values in soln we get

$x \cdot e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} \cdot e^{\tan^{-1} y} \cdot dy$

Let $\tan^{-1} y = t$

Differentiating: $\frac{1}{1+y^2} \cdot dy = dt$

$x \cdot e^{\tan^{-1} y} = \int t.e^t \cdot dt$

solving by parts 1

letting t as first function e^t as second:

$= \left[t \cdot \int e^t \cdot dt - \int \left(\frac{d}{dt}(t) \cdot \int e^t \cdot dt\right)\right] \cdot dt$

$= t \cdot e^t + c - \int 1 \cdot e^t \cdot dt$

$= te^t - e^t + k.$

$xe^{\tan^{-1} y} e^t(t-1) + k$

putting value of t:

$xe^{\tan^{-1} y} = e^{\tan^{-1} y}\{\tan^{-1} y - 1\} + k.$

OR

Q25. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that $y = 1$, when $x = 0$.

Sol.25 $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

$(x^2 + y^2) \cdot dy = xy\, dx \quad \ldots\ldots (1)$ Homogeneous differential eqn.

Put $y = vx$ (where v is a variable)

$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

Substituting in eqn (1)

$(x^2 + v^2x^2)\left\{v + x \cdot \frac{dv}{dx}\right\} = x \cdot vx$

$x^2(1 + v^2)\left(v + x \cdot \frac{dv}{dx}\right) = x^2 v$

$v + x \cdot \frac{dv}{dx} = \frac{v}{1+v^2}$

$$\frac{xdv}{dx} = \frac{v}{1+v^2} - v$$

$$\frac{xdv}{dx} = \frac{-v3}{1+v^2}$$

$$\frac{(1+v^2)}{v3} dv = \frac{-dx}{x}$$

Integrating both sides: $\int \left(\frac{1}{v3} + \frac{1}{v}\right) \cdot dv = \int \frac{-dx}{x}$

$\Longrightarrow \frac{1}{-2v^2} + in\ v + k. = -in\ x + c$

$\text{in } v\,x = \frac{1}{2v^2} + m.$

Putting value of v $\Longrightarrow$ in $y = \frac{x^2}{2y^2} + m.$

Q26. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find, the value of k and hence find the equation of the plane containing these lines.

Sol.26 The Lines

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}, \frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

The intersect, so they must have a common point:

Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = k$

$x = 2k + 1, y = 3k - 1, z = 4k + 1$ ... (1)

Let $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = l\ (assume)$

$x = l + 3, y = 2l + k, z = l$ ... (2)

Solving Eq. (1) and (2)

From x we get $2k + 1 = l + 3$

From z we get $l = -4k + 1$

Solving $2k + 1 = 4k + 1 + 3$

$2k = -3$

$k = -3/2 l = -5$

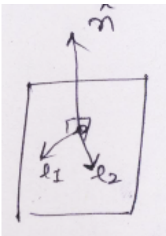
Equating y, we get

$2l + k = 3k - 1$

Putting values of l & k $\quad -10 + k = \frac{-9}{2} - 1$

$k = 10 - \frac{11}{2}$

$l = \frac{9}{2}$



So the lines are:

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{y}$ & $\frac{x-3}{1} = \frac{y-\frac{9}{2}}{2} = \frac{z-1}{1}$

If these two lines lies in a plane, than normal of plane will be perpendicular to both the direction ratios:

So, $\hat{n} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}$

$\hat{n} = [\therefore -5i + 2j + k]$

Now eqn of plane is

– 5 × 2y + z = p ...... (3)

To find p we need any point on the plane

From (1) we know

x = 2k + 1 y = 3k – 1 z = 4k + 1

put k = 0

Transweb Educational Services Pvt Ltd
B -147, Sector -6 Noida, UP - 201301,
www.askiitians.com
Email: info@askiitians.com
Tel: 1800-2000-838

$x = 1 \qquad y = -1 \qquad z = 1$

put this in eqn (3) we get

$-5(1) + 2(-1) + 1 = p$

$P = -5 - 2 + 1 = -6$

Eqn of plane will be

$-5x + 2y + z + 6 = 0$