

MATHEMATICS 2005 (Delhi)

General Instructions:

The question paper consists of three Sections A, B and C. Section. In addition to Section as, every student has to attempt either Section B or Section C.

1. For Section A

Question numbers 1 to 8 are of 3 marks each.

Question numbers 9 to 15 are of 4 marks each.

Question numbers 16 to 18 are of 6 marks each.

2. For Section B/Section C

Question numbers 19 to 22 are of 3 marks each.

Question numbers 23 to 25 are of 4 marks each.

Question numbers 26 is of 6 marks.

3. All questions are compulsory.

4. Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

5. Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

SECTION - A

Q1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that $A^3 - 4A^2 + A = O$.

Q2. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.

Q3. In a single throw of three dice, determine the probability of getting (a

Q4. A class consists of 10 boys and 8 girls. Three students are selected at random. Find the probability that the selected group has

(a) All boys,

(b) All girls,

(c) 2 boys and 1 girl.

Q5. Evaluate: $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx.$

Q6. Evaluate: $\int \frac{\sqrt{16 + (\log x)^2}}{x} dx.$

Q7. Form the differential equation representing the family $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constant.

Q8. Solve the following differential equation: $(1+x^2) \frac{dx}{dy} - 2xy = (x^2+2)(x^2+1).$

Solve the following differential equation: $x \frac{dx}{dy} - y = \sqrt{x^2 + y^2}.$

Q9. Prove that: $p \leftrightarrow q = (P \rightarrow q) \wedge (q \rightarrow p)$

Or

Text the validity of the following argument:

$$S_1 : p \vee q$$

$$S_2 : \equiv p$$

$$S : q$$

Q10. Evaluate: $\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$

Q11. Differentiate $\sin^{-1} x$ from first principles.

Q12. If $x = a \left(\frac{1+t^2}{1-t^2} \right)$ and $y = \frac{2t}{1-t^2}$, find $\frac{dy}{dx}$.

Q13. The surface area of a spherical bubble is increasing at the rate of 2 cm²/sec. find the rate at which the volume of the bubble is increasing at the instant its radius is 6 cm.

Q14. Evaluate: $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$.

Q15. Evaluate: $\int_0^{\pi} \frac{dx}{5+4 \cos x}$.

Q. 16. Using matrices, solve the following system of linear equations:

$$\begin{aligned} x + y + z &= 4 \\ 2x - y + z &= -1 \\ 2x + y - 3z &= -9 \end{aligned}$$

Or

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1} .

Q17. A wire of length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum.

Q18. Find the area bounded by the curve $x^2 = 4y$ and the straight line $4y - 2 = x$.

SECTION - B

Q19. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to \vec{b} .

Q20. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ be coplanar, show that $c^2 = ab$.

Q21. A car, traveling with a uniform acceleration, has a velocity of 18 km/hour at a certain time and 54 km/hour after covering a distance of 500m. How much further will it travel to attain a velocity of 72 km/hour?

Q22. A body falls freely from the top of a tower; It covers $\frac{5}{9}$ of the whole distance in the last second. Find the height of the tower and the total by the body to fall down.

Or

A cricket ball is project with a velocity of 29.4 m/sec. Find

(a) The greatest range on the horizontal plane; and

(b) The angle of projection to give a range of 44.10 m.

Q23. Find the co-ordinates of the foot of the perpendicular drawn from the point A(1,8,4,) to the line joining the points B(0,-1,3) and C(2,-3,-1).

Q24. The resultant of two forces \vec{P} and \vec{Q} acting at a point is at right angles to force \vec{P} . while the resultant of forces \vec{P} and \vec{S} , acting at the same angle, is at right angles to force \vec{S} . Prove that $P = \sqrt{SQ}$

Q25. \vec{P} and \vec{Q} are two unlike parallel forces. When the magnitude of \vec{P} is doubled, it is found that the line of action of \vec{Q} is midway between the lines of action of the new and the original resultants, find the ratio of P and Q.

Or

Three forces acting on a particle are in equilibrium. If the angle between the first force and the second force be 120° and that between the second force and the third force be 135° , find the ratio of their magnitudes.

Q26. Find the Cartesian as well as the vector equation of the planes passing through the intersection of the planes

$\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$. Which are at unit distance from the origin?

SECTION - C

Q19. Solve the following linear programming problem graphically:

Maximize $z = 60x + 15y$

Subject to constraints

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x, y \geq 0$$

Q20. Two tailors A and B earn Rs.150 and Rs.200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Q21. A company has two plants to manufacture motor cycles. 70% motor cycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% motor cycles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motor cycle, randomly picked up, is found to be of standard quality. Find the probability that it has come out from the second plant.

Q22. The probability that student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

- None will graduate,
- Only one will graduate,
- All will graduate.

Or

In a book of 200 pages, 200 misprints are randomly distributed. Using Poisson's distribution calculate the probability that a randomly observed page of the book will be found to have at least 3 errors. [Use: $e^{-1} = 0.3679$]

Q23. A, B and C are engaged in a printing business. A being the working partner, receives 10% of the net profit as salary. The remaining profit is divided among themselves in the ratio 4:5:9. If A gets in total Rs. 3,00,000, find the total profit in the business and the shares of B and C in it.

Or

A and B are partners in a business sharing profits and losses equality. They admit a new partner C and it is agreed that now the profit and losses will be shared amongst A, B and C in the ratio 9 : 8 : 7 respectively. If C paid Rs. 2.10 lakh as premium for the good will, find the shares of A and B in the premium.

Q24. Find the present worth of an ordinary annuity of Rs. 1,200 per annum for 10 years at 12% per annum, compounded annually. [Use $(1.12)^{-10} = 0.3221$]

Q25. A calculator manufacturing company finds that the daily cost of producing

x Calculators is given by $C(x) = 200x + 7500$.

- If each calculator is sold for Rs. 350, find the minimum number of calculators that must be produced daily and sold to ensure no loss.
- If the selling price is increased by Rs. 150, what would be the break-even point?

Q26. A bill was drawn on April 4, 2004 at 8 months after date and was discounted on July 14, 2004 at 5% per annum. If the Banker's gain is Rs.200, find the face value of the bill.