7 Cubes and Cube Roots

Learn and Remember

1. Numbers like 1729, 4104, 13832 are known as Hardy Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways *i.e.*,

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

$4104 = 4096 + 8 = 16^3 + 2^3$

$4104 = 3375 + 729 = 15^3 + 9^3$

$13832 = 8000 + 5832 = 20^3 + 18^3$

$13832 = 13824 + 8 = 24^3 + 2^3$.

2. Numbers obtained when a number is multiplied by itself three times are known as cube numbers. For example, 1, 8, 27, ... are the cube of 1, 2, 3, ..., etc.
3. If in the prime factorisation of any number, each factor appears three times, then the number is a perfect cube.
4. The symbol ($\sqrt[3]{\ }$) denotes cube root. For example $\sqrt[3]{27} = 3$.
5. To be a perfect cube number, similar factors must be in form of triplets.
6. All those prime factors which appear in groups of similar prime factors of three are perfect cubes. All those which are not in the group of three are not perfect cube numbers.
7. While getting prime factors of a number, you must start with factors (2, 3, 5, 7, 11, 13) in ascending orders.
8. All those numbers whose unit's digits are 0, 2, 4, 6 and 8 are cubes of even numbers and whose unit's digits are 1, 3, 5, 7, 9 are cubes of odd numbers.
9. If n is a perfect cube and $n = m^3$, then m is the cube root of n. Cube root of n is written as $\sqrt[3]{n}$.
10. The cube root of a product of two perfect cubes is written as the product of the cube roots of the perfect cubes *i.e.*, $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.
11. The cube root of a quotient of two perfect cubes is the quotient of their cube roots *i.e.*, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, $b \neq 0$, where a and b are perfect cubes.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.1 (*Page –114*)

Q1. Which of the following numbers are not perfect cubes?
(*i*) 216 (*ii*) 128 (*iii*) 1000
(*iv*) 100 (*v*) 46656

Sol. (*i*) 216

2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factors of $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Here, all factors are in groups of 3's (in triplets).

So, 216 is a perfect cube number.

(*ii*) 128

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Here, factor 2 does not appear in group of 3's (in triplets).

So, 128 is not a perfect cube number.

(*iii*) 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

Prime factors of 1000 = 2 × 2 × 2 × 5 × 5 × 5
Here, all factors appear in a group of 3's (in triplets).
So, 1000 is a perfect cube number.

(*iv*) 100

2	100
2	50
5	25
5	5
	1

Prime factors of 100 = 2 × 2 × 5 × 5
Here, factors 2 and 5 do not appear in group of 3's (in triplets).
So, 100 is not a perfect cube number.

(*v*) 46656

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Prime factors of 46656 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3
Here, all the factors appear in a group of 3's (in triplets).
So, 46656 is a perfect cube number.

Q2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(*i*) 243 (*ii*) 256 (*iii*) 72
(*iv*) 675 (*v*) 100

Sol. (*i*) 243

3	243
3	81
3	27
3	9
3	3
	1

Prime factors of 243 = 3 × 3 × 3 × 3 × 3
Here, 3 do not appear in a group of 3's (in triplets).
So, 243 must be multiplied by 3 to make it a perfect cube number.

(*ii*) 256

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of 256 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
Here, factors 2 does not appear in a group of 3's (in triplets).
So, 256 must be multiplied by 2 to make it a perfect cube number.

(*iii*) 72

2	72
2	36
2	18
3	9
3	3
	1

Prime factors of 72 = 2 × 2 × 2 × 3 × 3
Factors of 3 are not in group of 3's (in triplets).
So, 72 must be multiplied by 3 to make it a perfect cube number.

(*iv*) 675

3	675
3	225
3	75
5	25
5	5
	1

Prime factors of $675 = 3 \times 3 \times 3 \times 5 \times 5$

Here, factor 5 does not appear in group of 3's (in triplets).

So, 675 must be multiplied by 5 to make it a perfect cube number.

(*v*) 100

2	100
2	50
5	25
5	5
	1

Prime factors of $100 = 2 \times 2 \times 5 \times 5$

Here, factors 2 and 5 do not appear in group of 3's (in triplets).

So, 100 must be multiplied by $2 \times 5 = 10$ to make it a perfect cube number.

Q3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(*i*) 81 (*ii*) 128 (*iii*) 135

(*iv*) 192 (*v*) 704

Sol. (*i*) 81

3	81
3	27
3	9
3	3
	1

Prime factors of $81 = 3 \times 3 \times 3 \times 3$

One 3's factors is not grouped in triplets.

So, this number must be divided by 3 to make it a perfect cube.

Hence, the smallest number is 3 and perfect cube number $81 \div 3 = 27$.

(*ii*) 128

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

The prime factor 2 does not appear in group of three except two triplets of 2.

So, this number must be divided by 2 to make it a perfect cube number.

Hence, the smallest number is 2.

And perfect cube number $= 128 \div 2 = 64$.

(*iii*) 135

3	135
3	45
3	15
5	5
	1

Prime factors of $135 = 3 \times 3 \times 3 \times 5$

The prime factor of 5 does not appear in group of three.

So, this number must be divided by 5 to make it a perfect cube number.

Hence, the smallest number is 5.

And perfect cube number $= 135 \div 5 = 27$.

(*iv*) 192

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Prime factors of $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

The prime factor 3 does not appear in a group of three.

So, this number must be divided by 3 to make it a perfect cube number.

Hence, the smallest number is 3.

And perfect cube number $= 192 \div 3 = 64$.

(*v*) 704

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

Prime factors of $704 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11$

The prime factor 11 does not appear in a group of three.

So, this number must be divided by 11 to make it a perfect cube number.

Hence, the smallest number is 11 and perfect cube number $= 704 \div 11 = 64$.

Q4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Sol. Given number $= 5 \times 2 \times 5$

Factors of 5 and 2 are not in group of three.

So, the number must be multiplied by $2 \times 2 \times 5 = 20$ to make it a perfect cube number.

Hence, he needs 20 cuboids.

EXERCISE 7.2 (*Page 114*)

Q1. Find the cube root of each of the following numbers by prime factorisation method.

(*i*) 64 (*ii*) 512 (*iii*) 10648
(*iv*) 27000 (*v*) 15625 (*vi*) 13824
(*vii*) 110592 (*viii*) 46656 (*ix*) 175616
(*x*) 91125

Sol. (*i*) 64

2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Therefore, $\sqrt[3]{64} = 2 \times 2 \times 2 = 4$. **Ans.**

(*ii*) 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Therefore, $\sqrt[3]{512} = 2 \times 2 \times 2 = 8$.

(*iii*) 10648

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

Prime factors of $10648 = 2 \times 2 \times 11 \times 11 \times 11$

Therefore, $\sqrt[3]{10648} = 2 \times 11 = 22$. **Ans.**

(*iv*) 27000

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Prime factors of $27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Therefore, $\sqrt[3]{27000} = 2 \times 3 \times 5 = 30$. **Ans.**

(*v*) 15625

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

Prime factors of $15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

Therefore, $\sqrt[3]{15625} = 5 \times 5 = 25$. **Ans.**

(*vi*) 13824

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factors of $13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$. **Ans.**

(*vii*) 110592

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factors of 110592 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

Therefore, $\sqrt[3]{110592}$ = 2 × 2 × 2 × 2 × 3 = 48. **Ans.**

(*viii*) 46656

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Prime factors of 46656 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3

Therefore, $\sqrt[3]{46656}$ = 2 × 2 × 3 × 3 = 36. **Ans.**

(*ix*) 175616

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Prime factors of 175616 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 7 × 7 × 7

Therefore, $\sqrt[3]{175616}$ = 2 × 2 × 2 × 7 = 56. **Ans.**

(*x*) 91125

3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Prime factors of 91125 = 3 × 3 × 3 × 3 × 3 × 3 × 5 × 5 × 5

Therefore, $\sqrt[3]{91125}$ = 3 × 3 × 5 = 45. **Ans.**

Q2. State true or false.

(*i*) Cube of any odd number is even.

(*ii*) A perfect cube does not end with two zeros.

(*iii*) If square of a number ends with 5, then its cube ends with 25.

(*iv*) There is no perfect cube which ends with 8.

(v) The cube of a two digit number may be a three digit number.

(vi) The cube of a two digit number may have seven or more digits.

(vii) The cube of a single digit number may be a single digit number.

Sol. (i) False

Since, $1^3 = 1$, $3^3 = 27$, $5^3 = 125$, ..., are all odd.

(ii) True

Since, a perfect cube ends with three zeros.

For example, $1000 = 10^3$, $8000 = 20^3$, $27000 = 30^3$, ..., so on.

(iii) False

Since, $5^2 = 25$, $5^3 = 125$, $15^2 = 225$, $15^3 = 3375$.

(Did not end with 25)

(iv) False

Since, $12^3 = 1728$ (Ends with 8)

$22^3 = 10648$ (Ends with 8)

(v) False

Since, $10^3 = 1000$ (Four digit number)

$11^3 = 1331$ (Four digit number)

(vi) False

Since, $99^3 = 970299$ (Six digit number)

(vii) True

$1^3 = 1$ (Single digit number)

$2^3 = 8$ (Single digit number)

Q3. You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768.

Sol. We know that $10^3 = 1000$.

Possible cube of $11^3 = 1331$.

Since, cube of unit's digit $1^3 = 1$.

So, cube root of 1331 is 11.

4913

We know that $7^3 = 343$

Next number comes $17^3 = 4913$

Hence, cube root of 4913 is 17.

12167

We know that $3^3 = 27$

One's digit is 7.

Now, next number comes $13^3 = 2197$

Now, next number comes $23^3 = 12167$

Hence, cube root of 12167 is 23.

32768

We know that $2^3 = 8$ which is unit digit.

Now, comes $12^3 = 1728$.

Now, next comes $22^3 = 10648$.

Now, next number comes $32^3 = 32768$.

Hence, cube root of 32768 is 32.

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