CONTENTS

1 Rational Numbers

Learn and Remember

1. A **rational number** can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example, $-\frac{4}{7}, \frac{3}{8}, \frac{-11}{7}$ are all rational numbers.
2. Rational numbers and integers are *closed* under the operations of addition, subtraction and multiplication, but not closed under division.
3. Whole numbers are *closed* under addition and multiplication, but not *closed* for subtraction and division.
4. Rational numbers are *commutative* and *associative* for addition and multiplication, but not for subtraction and division.

   If a, b and c are any three rational numbers, then

   (*a*) $a + b = b + a$ (Commutative for addition)

   (*b*) $(a + b) + c = a + (b + c)$ (Associative for addition)

   (*c*) $a \times b = b \times a$ (Commutative for multiplication)

   (*d*) $(a \times b) \times c = a \times (b \times c)$ (Associative for multiplication)
5. *Distributivity* of rational numbers over addition and subtraction for all rational numbers a, b and c is $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
6. Rational number zero (0) is the **additive identity** and 1 is the **multiplicative identity** for rational numbers, integers and whole numbers.
7. The **additive inverse** of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa *i.e.*, $\frac{a}{b} + \left(-\frac{a}{b}\right) = 0$.

8. The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{c}{d}$, if $\frac{a}{b} \times \frac{c}{d} = 1$.
9. Rational numbers can be represented on a number line.
10. Between any two given rational numbers, there are countless rational numbers. The idea of mean helps us to find rational numbers between two rational numbers.

    If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.
11. Zero (0) is also a rational number, but zero (0) has no reciprocal rational number.
12. Having equal denominators by taking L.C.M. and making the numbers in larger forms of both rational numbers, you can get countless rational numbers.

    If $\frac{1}{4}$ and $\frac{29}{32}$ are two rational numbers, then L.C.M. of 4 and 32 is 32. So, $\frac{1}{4}$ and $\frac{29}{32}$ can be written as $\frac{8}{32}$ and $\frac{29}{32}$. Rational numbers between them are $\frac{9}{32}, \frac{10}{32}, \ldots\ldots, \frac{28}{32}$.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 1.1 *(Page –14-15)*

Q1. Using appropriate properties find.

(i) $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$ **(ii)** $\frac{2}{5} \times \left(\frac{3}{-7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$

Sol. *(i)* $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$

$$= \frac{-2}{3} \times \frac{3}{5} - \frac{3}{5} \times \frac{1}{6} + \frac{5}{2}$$

$$= \frac{3}{5}\left(\frac{-2}{3} - \frac{1}{6}\right) + \frac{5}{2} \quad \text{(By distributivity property)}$$

$$= \frac{3}{5}\left(\frac{-4-1}{6}\right) + \frac{5}{2}$$

$$= \frac{3}{5} \times \frac{-5}{6} + \frac{5}{2}$$

$$= -\frac{1}{2} + \frac{5}{2} = \frac{-1+5}{2} = \frac{4}{2} = 2$$

(ii) $\frac{2}{5} \times \left(\frac{3}{-7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$

$$= \frac{2}{5} \times \left(\frac{-3}{7}\right) + \frac{1}{14} \times \frac{2}{5} - \frac{1}{6} \times \frac{3}{2}$$

(By associativity property)

$$= \frac{2}{5} \times \left(\frac{-3}{7} + \frac{1}{14}\right) - \frac{1}{4} \quad \text{(By distributivity property)}$$

$$= \frac{2}{5} \times \left(\frac{-6+1}{14}\right) - \frac{1}{4}$$

$$= \frac{2}{5} \times \frac{-5}{14} - \frac{1}{4} = -\frac{1}{7} - \frac{1}{4}$$

$$= \frac{-4-7}{28} = \frac{-11}{28}.$$

Q2. Write the additive inverse of each of the following.

(i) $\frac{2}{8}$ **(ii)** $\frac{-5}{9}$ **(iii)** $\frac{-6}{-5}$ **(iv)** $\frac{2}{-9}$ **(v)** $\frac{19}{-6}$.

Sol. We know that additive inverse of a rational number $\frac{a}{b}$ is $\left(\frac{-a}{b}\right)$, such that $\frac{a}{b} + \frac{(-a)}{b} = 0$.

(i) $\frac{2}{8}$

$\frac{-2}{8}$ is the additive inverse of $\frac{2}{8}$.

$$\left[\because \frac{2}{8} + \left(-\frac{2}{8}\right) = \frac{2-2}{8} = \frac{0}{8} = 0\right]$$

(ii) $\frac{-5}{9}$

$\frac{5}{9}$ is the additive inverse of $\frac{-5}{9}$.

$$\left[\because \frac{-5}{9} + \frac{5}{9} = \frac{-5+5}{9} + \frac{0}{9} = 0\right]$$

(*iii*) $\frac{-6}{-5}$

$\frac{-6}{5}$ is the additive inverse of $\frac{-6}{-5}$.

$$\left[\because \frac{-6}{-5}+\left\{-\left(\frac{-6}{-5}\right)\right\}=\frac{6}{5}-\frac{6}{5}=\frac{0}{5}=0\right]$$

(*iv*) $\frac{2}{-9}$

$\frac{2}{9}$ is the additive inverse of $\frac{2}{-9}$.

$$\left[\because \frac{2}{-9}+\left\{-\left(\frac{2}{-9}\right)\right\}=\frac{2}{-9}+\frac{2}{9}=\frac{-2+2}{9}+\frac{0}{9}=0\right]$$

(*v*) $\frac{19}{-6}$

$\frac{19}{6}$ is the additive inverse of $\frac{19}{-6}$.

$$\left[\because \frac{19}{-6}+\left\{-\left(\frac{19}{-6}\right)\right\}=\frac{-19}{6}+\frac{19}{6}=\frac{-19+19}{6}=\frac{0}{6}=0\right]$$

Q3. Verify that $-(-x) = x$ for.

(*i*) $x = \frac{11}{15}$ (*ii*) $x = -\frac{13}{17}$

Sol. According to the given equation,

$$-(-x) = x \qquad ...(i)$$

(*i*) On putting $x = \frac{11}{15}$, in given equation (*i*)

L.H.S. $= -\left(-\frac{11}{15}\right) = \frac{11}{15} =$ R.H.S.

$\Rightarrow$ L.H.S. = R.H.S.

Hence, verified.

(*ii*) On putting $x = -\frac{13}{17}$, in given equation (*i*)

L.H.S. $= -\left\{-\left(-\frac{13}{17}\right)\right\} = -\frac{13}{17} =$ R.H.S.

$\Rightarrow$ L.H.S. = R.H.S.

Hence, verified.

Q4. Find the multiplicative inverse of the following.

(*i*) -13 (*ii*) $\frac{-13}{19}$ (*iii*) $\frac{1}{5}$

(*iv*) $\frac{-5}{8} \times \frac{-3}{7}$ (*v*) $-1 \times \frac{-2}{5}$ (*vi*) -1

Sol. We know that multiplicative inverse of a rational number a is $\frac{1}{a}$, such that $a \times \frac{1}{a} = 1$.

(*i*) -13

$\frac{-1}{13}$ is multiplicative inverse of -13. $\left[\because -13\times\frac{1}{-13}=1\right]$

(*ii*) $\frac{-13}{19}$

$\frac{-19}{13}$ is multiplicative inverse of $\frac{-13}{19}$.

$$\left[\because \frac{-13}{19}\times\frac{19}{-13}=1\right]$$

(*iii*) $\frac{1}{5}$

5 is multiplicative inverse of $\frac{1}{5}$. $\left[\because \frac{1}{5}\times 5=1\right]$

(*iv*) $\frac{-5}{8} \times \frac{-3}{7} = \frac{15}{56}$

$\frac{56}{15}$ is multiplicative inverse of $\frac{15}{56}$. $\left[\because \frac{15}{56}\times\frac{56}{15}=1\right]$

(*v*) $-1 \times \frac{-2}{5} = \frac{+2}{5}$

$\frac{5}{2}$ is multiplicative inverse of $\frac{2}{5}$. $\left[\because \frac{2}{5}\times\frac{5}{2}=1\right]$

(*vi*) -1

$\frac{1}{-1}$ is multiplicative inverse of (-1).

$$\left[\because (-1)\times\left(\frac{1}{-1}\right)=1\right]$$

Q5. Name the property under multiplication used in each of the following.

(*i*) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ (*ii*) $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{17}$

(*iii*) $\frac{-19}{29} \times \frac{29}{-19} = 1.$

Sol. (*i*) 1 is the multiplicative identity. (*ii*) Commutativity.

(*iii*) Multiplicative inverse.

Q6. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

Sol. The reciprocal of $\frac{-7}{16}$ is $\frac{-16}{7}$.

According to the condition, $\frac{6}{13} \times \left(\frac{-16}{7}\right) = \frac{-96}{91}$.

Q7. Tell what property allows you to compute $\frac{1}{3} \times \left(6 \times \frac{4}{3}\right)$ as $\left(\frac{1}{3} \times 6\right) \times \frac{4}{3}$.

Sol. By associativity property of multiplication, as we know that $a \times (b \times c) = (a \times b) \times c$.

Q8. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

Sol. We have, $\frac{8}{9} \times \left(-1\frac{1}{8}\right) = \frac{8}{9} \times \frac{-9}{8} = -1$

Its product must be positive 1.

So, $\frac{8}{9}$ is not multiplicative inverse of $-1\frac{1}{8}$.

Q9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

Sol. We have, $0.3 \times 3\frac{1}{3} = \frac{3}{10} \times \frac{10}{3} = 1$

Yes, its product is 1, so 0.3 is the multiplicative inverse of $3\frac{1}{3}$.

Q10. Write

(*i*) The rational number that does not have a reciprocal.

(*ii*) The rational numbers that are equal to their reciprocals.

(*iii*) The rational number that is equal to its negative.

Sol. (*i*) 0 (*ii*) 1 and -1 (*iii*) 0.

Q11. Fill in the blanks.

(*i*) Zero has ______ reciprocal.

(*ii*) The numbers ______ and ______ are their own reciprocals.

(*iii*) The reciprocal of -5 is ______.

(*iv*) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ______.

(*v*) The product of two rational numbers is always a ______.

(*vi*) The reciprocal of a positive rational number is ______.

Sol. (*i*) No (*ii*) 1, -1 (*iii*) $\frac{-1}{5}$

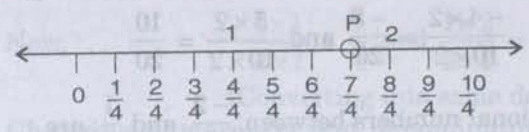
(*iv*) x (*v*) Rational number (*vi*) Positive.

EXERCISE 1.2 *(Page –20)*

Q1. Represent these numbers on the number line.

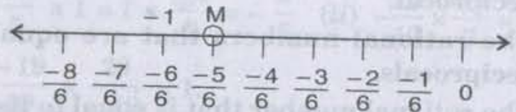
(*i*) $\frac{7}{4}$ (*ii*) $\frac{-5}{6}$

Sol. (*i*) $\frac{7}{4} = 1\frac{3}{4}$



$P = 1\frac{3}{4} = \frac{7}{4}$.

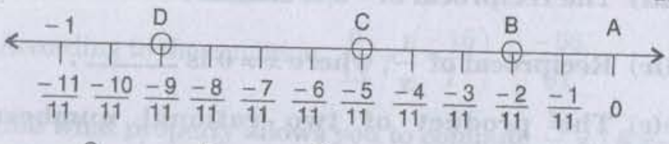
(ii) $\frac{-5}{6}$



$M = \frac{-5}{6}$.

Q2. Represent $\frac{-2}{11}$, $\frac{-5}{11}$, $\frac{-9}{11}$ on the number line.

Sol. We draw a number line to represent, $\frac{-2}{11}$, $\frac{-5}{11}$ and $\frac{-9}{11}$.



$B = \frac{-2}{11}$, $C = \frac{-5}{11}$, $D = \frac{-9}{11}$.

***Q3. Write five rational numbers which are smaller than 2.**

Sol. $\frac{1}{3}, \frac{1}{4}, \frac{1}{2}, \frac{-1}{2}, \frac{-1}{5}$ and so on or $1, \frac{1}{2}, 0, -1, \frac{-1}{2}$.

Q4. Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Sol. $\frac{-2}{5}$ and $\frac{1}{2}$

L.C.M. of 5 and 2 is 10.

Now, $\frac{-2\times2}{5\times2} = \frac{-4}{10}$ and $\frac{1}{2}\times\frac{5}{5} = \frac{5}{10}$

(Converting with same denominators)

Changing numerators and denominators in larger numbers.

$$\frac{-4\times2}{10\times2} = \frac{-8}{20} \text{ and } \frac{5\times2}{10\times2} = \frac{10}{20}$$

Ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$ are

$$\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{2}{20}.$$

*Answer may be different.

***Q5. Find five rational numbers between**

(i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$

Sol. (i) $\frac{2}{3}$ and $\frac{4}{5}$

L.C.M. of 3 and 5 is 15,

Now, $\frac{2\times5}{3\times5} = \frac{10}{15}$ and $\frac{4\times3}{5\times3} = \frac{12}{15}$

Changing numerators and denominators in larger numbers.

$$\frac{10\times4}{15\times4} = \frac{40}{60} \text{ and } \frac{12\times4}{15\times4} = \frac{48}{60}$$

Hence, five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$ are

$$\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$$

(ii) $\frac{-3}{2}$ and $\frac{5}{3}$

L.C.M. of 2 and 3 is 6.

Now, $\frac{-3\times3}{2\times3} = \frac{-9}{6}$ and $\frac{5\times2}{3\times2} = \frac{10}{6}$

(Converting into same denominator.)

Hence, five rational numbers between $\frac{-3}{2}$ and $\frac{5}{3}$ are

$$\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}.$$

(iii) $\frac{1}{4}$ and $\frac{1}{2}$

L.C.M. of 4 and 2 is 4.

Now, $\frac{1\times1}{4\times1} = \frac{1}{4}$ and $\frac{1\times2}{2\times2} = \frac{2}{4}$

(Converting into same denominator.)

Changing numerators and denominators in larger numbers,

*Answer may be different.

$$\frac{1\times8}{4\times8}=\frac{8}{32} \text{ and } \frac{2\times8}{4\times8}=\frac{16}{32}$$

Hence, five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$ are

$$\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}.$$

Q6. Write five rational numbers greater than – 2.

Sol. Five rational numbers greater than – 2 are

$\frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$. (Other numbers may also be possible.)

Q7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

Sol. L.C.M. of 5 and 4 is 20.

$$\frac{3\times4}{5\times4}=\frac{12}{20} \text{ and } \frac{3\times5}{4\times5}=\frac{15}{20}$$

(Converting into same denominator.)

Changing numerators and denominators in larger numbers,

$$\frac{12\times8}{20\times8}=\frac{96}{160} \text{ and } \frac{15\times8}{20\times8}=\frac{120}{160}$$

Hence, rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$ are

$$\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}.$$

□□