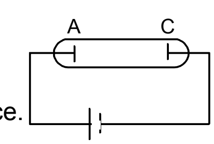
ELECTRIC CURRENT THROUGH GASES
CHAPTER 41

1. Let the two particles have charge 'q'
   Mass of electron $m_a = 9.1 \times 10^{-31}$ kg
   Mass of proton $m_p = 1.67 \times 10^{-27}$ kg
   Electric field be E
   Force experienced by Electron = qE
   accln. = qE/m_e
   For time dt
   $$S_e = \frac{1}{2} \times \frac{qE}{m_e} \times dt^2 \qquad \ldots(1)$$
   For the positive ion,
   $$\text{accln.} = \frac{qE}{4 \times m_p}$$
   $$S_p = \frac{1}{2} \times \frac{qE}{4 \times m_p} \times dt^2 \qquad \ldots(2)$$
   $$\frac{S_e}{S_p} = \frac{4m_p}{m_e} = 7340.6$$
2. $E = 5$ Kv/m $= 5 \times 10^3$ v/m ; $t = 1\ \mu s = 1 \times 10^{-6}$ s
   $F = qE = 1.6 \times 10^{-9} \times 5 \times 10^3$
   $$a = \frac{qE}{m} = \frac{1.6 \times 5 \times 10^{-16}}{9.1 \times 10^{-31}}$$
   a) S = distance travelled
   $$= \frac{1}{2}at^2 = 439.56 \text{ m} = 440 \text{ m}$$
   b) $d = 1$ mm $= 1 \times 10^{-3}$ m
   $$1 \times 10^{-3} = \frac{1}{2} \times \frac{1.6 \times 5}{9.1} 10^5 \times t^2$$
   $$\Rightarrow t^2 = \frac{9.1}{0.8 \times 5} \times 10^{-18} \Rightarrow t = 1.508 \times 10^{-9} \text{ sec} \Rightarrow 1.5 \text{ ns.}$$
3. Let the mean free path be 'L' and pressure be 'P'
   $L \propto 1/p$ for L = half of the tube length, P = 0.02 mm of Hg
   As 'P' becomes half, 'L' doubles, that is the whole tube is filled with Crook's dark space.
   Hence the required pressure = 0.02/2 = 0.01 m of Hg.



4. V = f(Pd)
   $v_s = P_s d_s$
   $v_L = P_l d_l$
   $$\Rightarrow \frac{V_s}{V_l} = \frac{P_s}{P_l} \times \frac{d_s}{d_l} \Rightarrow \frac{100}{100} = \frac{10}{20} \times \frac{1mm}{x}$$
   $\Rightarrow x = 1$ mm / 2 = 0.5 mm
5. i = ne or n = i/e
   'e' is same in all cases.
   We know,

$i = AST^2\ e^{-\phi/RT}$ ϕ = 4.52 eV, K = 1.38 × 10^{-23} J/k

$n(1000) = As \times (1000)^2 \times\ e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times1000}$

$\Rightarrow 1.7396 \times 10^{-17}$

a) T = 300 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(300)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times300}}{AS\times1.7396\times10^{-17}} = 7.05 \times 10^{-55}$$

b) T = 2000 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(2000)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times2000}}{AS\times1.7396\times10^{-17}} = 9.59 \times 10^{11}$$

c) T = 3000 K

$$\frac{n(T)}{n(1000K)} = \frac{AS\times(3000)^2\times e^{-4.52\times1.6\times10^{-19}/1.38\times10^{-23}\times3000}}{AS\times1.7396\times10^{-17}} = 1.340 \times 10^{16}$$

6. $i = AST^2\ e^{-\phi/KT}$

$i_1 = i$	i_2 = 100 mA
$A_1 = 60 \times 10^4$	$A_2 = 3 \times 10^4$
$S_1 = S$	$S_2 = S$
$T_1 = 2000$	$T_2 = 2000$
ϕ_1 = 4.5 eV	ϕ_2 = 2.6 eV

K = 1.38 × 10^{-23} J/k

$$i = (60 \times 10^4)(S) \times (2000)^2\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

$$100 = (3 \times 10^4)(S) \times (2000)^2\ e^{\frac{-2.6\times1.6\times10^{-19}}{1.38\times10^{-23}\times2000}}$$

Dividing the equation

$$\frac{i}{100} = e^{\left[\frac{-4.5\times1.6\times10}{1.38\times2}\left(\frac{-2.6\times1.6\times10}{1.38\times20}\right)\right]}$$

$$\Rightarrow \frac{i}{100} = 20\times e^{-11.014} \Rightarrow \frac{i}{100} = 20\times0.000016$$

⇒ i = 20 × 0.0016 = 0.0329 mA = 33 μA

7.

Pure tungsten	Thoriated tungsten
ϕ = 4.5 eV	ϕ = 2.6 eV
A = 60 × 10^4 A/m^2 – k^2	A = 3 × 10^4 A/m^2 – k^2

$i = AST^2 e^{-\phi/KT}$

$i_{\text{Thoriated Tungsten}} = 5000\ i_{\text{Tungsten}}$

$$\text{So, } 5000 \times S \times 60 \times 10^4 \times T^2 \times\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$$\Rightarrow S \times 3 \times 10^4 \times T^2 \times\ e^{\frac{-2.65\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}}$$

$$\Rightarrow 3 \times 10^8 \times\ e^{\frac{-4.5\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}} = e^{\frac{-2.65\times1.6\times10^{-19}}{1.38\times T\times10^{-23}}} \times 3 \times 10^4$$

Taking ‘ln’

⇒ 9.21 T = 220.29

⇒ T = 22029 / 9.21 = 2391.856 K

8. $i = AST^2 \; e^{-\phi/KT}$

$i' = AST^{12} \; e^{-\phi/KT'}$

$$\frac{i}{i'} = \frac{T^2}{T^{12}} \frac{e^{-\phi/KT}}{e^{-\phi/KT'}}$$

$$\Rightarrow \frac{i}{i'} = \left(\frac{T}{T'}\right)^2 e^{-\phi/KT + \phi KT'} = \left(\frac{T}{T'}\right)^2 e^{\phi KT' - \phi/KT}$$

$$= \frac{i}{i'} = \left(\frac{2000}{2010}\right)^2 e^{\frac{4.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}}} \left(\frac{1}{2010} - \frac{1}{2000}\right) = 0.8690$$

$$\Rightarrow \frac{i}{i'} = \frac{1}{0.8699} = 1.1495 \; = 1.14$$

9. $A = 60 \times 10^4 \text{ A/m}^2 - k^2$

$\phi = 4.5 \text{ eV}$ $\qquad \sigma = 6 \times 10^{-8} \; \omega/m^2 - k^4$

$S = 2 \times 10^{-5} \text{ m}^2$ $\qquad K = 1.38 \times 10^{-23} \text{ J/K}$

$H = 24 \; \omega'$

The Cathode acts as a black body, i.e. emissivity = 1

$\therefore E = \sigma A T^4$ (A is area)

$$\Rightarrow T^4 = \frac{E}{\sigma A} = \frac{24}{6 \times 10^{-8} \times 2 \times 10^{-5}} = 2 \times 10^{13} K = 20 \times 10^{12} K$$

$\Rightarrow T = 2.1147 \times 10^3 = 2114.7 \text{ K}$

Now, $i \;\; = AST^2 e^{-\phi/KT}$

$$= 6 \times 10^5 \times 2 \times 10^{-5} \times (2114.7)^2 \times \; e^{\frac{-4.5 \times 1.6 \times 10^{-19}}{1.38 \times T \times 10^{-23}}}$$

$= 1.03456 \times 10^{-3} \text{ A} = 1 \text{ mA}$

10. $i_p = \; CV_p^{3/2}$ …(1)

$\Rightarrow di_p = C \; 3/2 \;\; V_p^{(3/2)-1} dv_p$

$$\Rightarrow \frac{di_p}{dv_p} = \frac{3}{2} CV_p^{1/2} \quad \text{…(2)}$$

Dividing (2) and (1)

$$\frac{i}{i_p} \frac{di_p}{dv_p} = \frac{3/2CV_p^{1/2}}{CVp^{3/2}}$$

$$\Rightarrow \frac{1}{i_p} \frac{di_p}{dv_p} = \frac{3}{2V}$$

$$\Rightarrow \frac{dv_p}{di_p} = \frac{2V}{3i_p}$$

$$\Rightarrow R = \frac{2V}{3i_p} = \frac{2 \times 60}{3 \times 10 \times 10^{-3}} = 4 \times 10^3 = 4k\Omega$$

11. For plate current 20 mA, we find the voltage 50 V or 60 V.

Hence it acts as the saturation current. Therefore for the same temperature, the plate current is 20 mA for all other values of voltage.

Hence the required answer is 20 mA.

12. P = 1 W, p = ?

$V_p = 36$ V, $V_p = 49$ V, $P = I_p V_p$

$\Rightarrow I_p = \frac{P}{V_p} = \frac{1}{36}$

$I_p \propto (V_p)^{3/2}$

$I'_p \propto (V'_p)^{3/2}$

$\Rightarrow \frac{I_p}{I'_p} = \frac{(V_p)^{3/2}}{V'_p}$

$\Rightarrow \frac{1/36}{I'_p} = \left(\frac{36}{49}\right)^{3/2}$

$\Rightarrow \frac{1}{36\, I'_p} = \frac{36}{49} \times \frac{6}{7} \Rightarrow I'_p = 0.4411$

$P' = V'_p\, I'_p = 49 \times 0.4411 = 2.1613$ W = 2.2 W

13. Amplification factor for triode value

$= \mu = \frac{\text{Charge in Plate Voltage}}{\text{Charge in Grid Voltage}} = \frac{\delta V_p}{\delta V_g}$

$= \frac{250-225}{2.5-0.5} = \frac{25}{2} = 12.5$ $[\therefore\ \delta Vp = 250 - 225,\ \delta Vg = 2.5 - 0.5]$

14. $r_p = 2\ K\Omega = 2 \times 10^3\ \Omega$

$g_m = 2$ milli mho $= 2 \times 10^{-3}$ mho

$\mu = r_p \times g_m = 2 \times 10^3 \times 2 \times 10^{-3} = 4$ Amplification factor is 4.

15. Dynamic Plate Resistance $r_p = 10\ K\Omega = 10^4\ \Omega$

$\delta I_p = ?$

$\delta V_p = 220 - 220 = 20$ V

$\delta I_p = (\delta V_p / r_p) / V_g$ = constant.

$= 20/10^4 = 0.002$ A = 2 mA

16. $r_p = \left(\frac{\delta V_p}{\delta I_p}\right)$ at constant V_g

Consider the two points on $V_g = -6$ line

$r_p = \frac{(240-160)V}{(13-3)\times 10^{-3} A} = \frac{80}{10} \times 10^3 \Omega = 8K\Omega$

$g_m = \left(\frac{\delta I_p}{\delta V_g}\right) v_p$ = constant

Considering the points on 200 V line,

$g_m = \frac{(13-3)\times 10^{-3}}{[(-4)+(-8)]} A = \frac{10 \times 10^{-3}}{4} = 2.5$ milli mho

$\mu = r_p \times gm$

$= 8 \times 10^3\ \Omega \times 2.5 \times 10^{-3}\ \Omega^{-1} = 8 \times 1.5 = 20$

17. a) $r_p = 8\ K\Omega = 8000\ \Omega$

$\delta V_p = 48$ V $\qquad \delta I_p = ?$

$\delta I_p = (\delta V_p / r_p) / V_g$ = constant.

So, $\delta I_p = 48 / 8000 = 0.006$ A = 6 mA

b) Now, V_p is constant.

$\delta I_p = 6$ mA = 0.006 A

$g_m = 0.0025$ mho

$\delta V_g = (\delta I_p / g_m) / V_p$ = constant.

$= \dfrac{0.006}{0.0025} = 2.4$ V

18. $r_p = 10\ K\Omega = 10 \times 10^3\ \Omega$

$\mu = 20$ $\quad V_p = 250$ V

$V_g = -7.5$ V $\quad I_p = 10$ mA

a) $g_m = \left(\dfrac{\delta I_p}{\delta V_g}\right) V_p$ = constant

$\Rightarrow \delta V_g = \dfrac{\delta I_p}{g_m} = \dfrac{15 \times 10^{-3} - 10 \times 10^{-3}}{\mu / r_p}$

$= \dfrac{5 \times 10^{-3}}{20 / 10 \times 10^3} = \dfrac{5}{2} = 2.5$

$r'_g = +2.5 - 7.5 = -5$ V

b) $r_p = \left(\dfrac{\delta V_p}{\delta I_p}\right) V_g$ = constnant

$\Rightarrow 10^4 = \dfrac{\delta V_p}{(15 \times 10^{-3} - 10 \times 10^{-3})}$

$\Rightarrow \delta V_p = 10^4 \times 5 \times 10^{-3} = 50$ V

$V'_p - V_p = 50 \Rightarrow V'_p = -50 + V_p = 200$ V

19. $V_p = 250$ V, $V_g = -20$ V

a) $i_p = 41(V_p + 7V_g)^{1.41}$

$\Rightarrow 41(250 - 140)^{1.41} = 41 \times (110)^{1.41} = 30984\ \mu A = 30$ mA

b) $i_p = 41(V_p + 7V_g)^{1.41}$

Differentiating,

$di_p = 41 \times 1.41 \times (V_p + 7V_g)^{0.41} \times (dV_p + 7dV_g)$

Now $r_p = \dfrac{dV_p}{di_p} V_g$ = constant.

or $\dfrac{dV_p}{di_p} = \dfrac{1 \times 10^6}{41 \times 1.41 \times 110^{0.41}} = 10^6 \times 2.51 \times 10^{-3} \Rightarrow 2.5 \times 10^3\ \Omega = 2.5\ K\Omega$

c) From above,

$dI_p = 41 \times 1.41 \times 6.87 \times 7\ d\ V_g$

$g_m = \dfrac{dI_p}{dV_g} = 41 \times 1.41 \times 6.87 \times 7\ \mu$ mho

$= 2780\ \mu$ mho = 2.78 milli mho.

d) Amplification factor

$\mu = r_p \times g_m = 2.5 \times 10^3 \times 2.78 \times 10^{-3} = 6.95 = 7$

20. $i_p = K(V_g + V_p/\mu)^{3/2}$ …(1)

Diff. the equation :

$di_p = K\ 3/2\ (V_g + V_p/\mu)^{1/2}\ d\ V_g$

$\Rightarrow \dfrac{di_p}{dV_g} = \dfrac{3}{2} K \left(V_g + \dfrac{V_0}{\mu}\right)^{1/2}$

$\Rightarrow$ $g_m = 3/2\ K\ (V_g + V_p/\mu)^{1/2}$ ...(2)

From (1) $i_p = [3/2\ K\ (V_g + V_p/\mu)^{1/2}]^3 \times 8/K^2\ 27$

$\Rightarrow i_p = k'\ (g_m)^3 \Rightarrow g_m \propto 3\sqrt{i_p}$

21. $r_p = 20\ K\Omega$ = Plate Resistance

Mutual conductance = g_m = 2.0 milli mho = 2×10^{-3} mho

Amplification factor $\mu = 30$

Load Resistance = R_L = ?

We know

$A = \dfrac{\mu}{1 + \dfrac{r_p}{R_L}}$ where A = voltage amplification factor

$\Rightarrow A = \dfrac{r_p \times g_m}{1 + \dfrac{r_p}{R_L}}$ where $\mu = r_p \times g_m$

$\Rightarrow 30 = \dfrac{20 \times 10^3 \times 2 \times 10^{-3}}{1 + \dfrac{20000}{R_L}} \Rightarrow 3 = \dfrac{4R_L}{R_L + 20000}$

$\Rightarrow$ $3R_L + 60000 = 4\ R_L$

$\Rightarrow$ $R_L = 60000\ \Omega = 60\ K\Omega$

22. Voltage gain = $\dfrac{\mu}{1 + \dfrac{r_p}{R_L}}$

When A = 10, $R_L = 4\ K\Omega$

$10 = \dfrac{\mu}{1 + \dfrac{r_p}{4 \times 10^3}} \Rightarrow 10 = \dfrac{\mu \times 4 \times 10^3}{4 \times 10^3 + r_p}$

$\Rightarrow$ $40 \times 10^3 \times 10r_p = 4 \times 10^3\ \mu$ ...(1)

when A = 12, $R_L = 8\ K\Omega$

$12 = \dfrac{\mu}{1 + \dfrac{r_p}{8 \times 10^3}} \Rightarrow 12 = \dfrac{\mu \times 8 \times 10^3}{8 \times 10^3 + r_p}$

$\Rightarrow$ $96 \times 10^3 + 12\ r_p = 8 \times 10^3\ \mu$ ...(2)

Multiplying (2) in equation (1) and equating with equation (2)

$2(40 \times 10^3 + 10\ r_p) = 96 \times 10+3 + 12r_p$

$\Rightarrow$ $r_p = 2 \times 10^3\ \Omega = 2\ K\Omega$

Putting the value in equation (1)

$40 \times 10^3 + 10(2 \times 10^3) = 4 \times 10^3\ \mu$

$\Rightarrow$ $40 \times 10^3 + 20 \times 10^3) = 4 \times 10^3\ \mu$

$\Rightarrow$ $\mu = 60/4 = 15$

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