

IIT-JEE-Mathematics-Paper1-2007

1. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

- (A) $2/9 (p - q)(2q - p)$
- (B) $2/9 (q - p)(2p - q)$
- (C) $2/9 (q - 2p)(2q - p)$
- (D) $2/9 (2p - q)(2q - p)$

2. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$, and

$$\lim_{t \rightarrow \infty} (t^2 f(x) - x^2 f(t)) / (t - x) = 1$$

for each $x > 0$. Then $f(x)$ is

- (A) $1/3x + (2x^2)/3$
- (B) $(-1)/3x + (4x^2)/3$
- (C) $(-1)/x + 2/x^2$
- (D) $1/x$

3. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

- (A) $1/2$
- (B) $1/3$
- (C) $2/5$
- (D) $1/5$

4. The tangent to the curve $y = ex$ drawn at the point (e, e^e) intersects the line joining the points $(e - 1, e^{e-1})$ and $(e + 1, e^{e+1})$

- (A) on the left of $x = e$
- (B) on the right of $x = e$
- (C) at no points
- (D) at all points

5. $\lim_{x \rightarrow \pi/4} \int_2^{\sec^2 x} f(t) dt / (x^2 - \pi^2/16)$ equals

- (A) $8/\pi f(2)$
- (B) $2/\pi f(2)$

- (C) $2/\pi f(1/2)$
(D) $4f(2)$

6. A hyperbola, having the transverse axis of length $2 \sin \theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

- (A) $x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$
(B) $x^2 \sec^2 \theta - y^2 \operatorname{cosec}^2 \theta = 1$
(C) $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$
(D) $x^2 \cos^2 \theta - y^2 \sin^2 \theta = 1$

7. The number of distinct real values of λ , for which the vectors $-\lambda 2\hat{i} + \hat{j} + K$, $\hat{i} - \lambda 2\hat{j} + K$ and $\hat{i} + \hat{j} - \lambda 2K$ are coplanar, is

- (A) zero
(B) one
(C) two
(D) three

8. A man walks a distance of 3 units from the origin towards the north-east ($N 45^\circ E$) direction. From there, he walks a distance of 4 units towards the north-west ($N 45^\circ W$) direction to reach a point P. Then the position of P in the Argand plane is

- (A) $3e^{i\pi/4} + 4i$
(B) $(3 - 4i)e^{i\pi/4}$
(C) $(4 + 3i)e^{i\pi/4}$
(D) $(3 + 4i)e^{i\pi/4}$

9. The number of solutions of the pair of equations

$$2\sin^2 q - \cos 2q = 0$$

$$2\cos^2 q - 3 \sin q - 0$$

in the interval $[0, 2\pi]$ is

- (A) zero
(B) one
(C) two
(D) four

10. Let H_1, H_2, \dots, H_n be mutually exclusive and exhaustive events with $P(H_i) > 0, i = 1, 2, \dots, n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT-1

$$P(H_i|E) > P(E|H_i) \cdot P(H_i) \text{ for } i = 1, 2, \dots, n.$$

Because

STATEMENT-2

$$\sum_{i=1}^n P(H_i) = 1.$$

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

11. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

STATEMENT-1

The tangents are mutually perpendicular

Because

STATEMENT-2

The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

12. Let the vectors \vec{PQ} , \vec{QR} , \vec{RS} , \vec{ST} , \vec{TU} and \vec{UP} represent the sides of a regular hexagon.

STATEMENT-1

$$\vec{PQ} \times (\vec{RS} + \vec{ST}) \neq \vec{0}$$

Because

STATEMENT-2

$$\vec{PQ} \times \vec{RS} = \vec{0} \text{ and } \vec{PQ} \times \vec{ST} \neq \vec{0}$$

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

13. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

STATEMENT-1

The function $F(x)$ satisfies $F(x+p) - F(x)$ for all real x .

STATEMENT-2

$$\sin^2(x+p) = \sin^2 x \text{ for all real } x.$$

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Paragraph

Let V_r denote the sum of the first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let

$$T_r = V_{r+1} - V_r - 2 \text{ and } Q_r = T_{r+1} - T_r \text{ for } r = 1, 2, \dots$$

14. The sum of $V_1 + V_2 + \dots + V_n$ is

- (A) $\frac{1}{12} n(n+1)(3n^2 - n + 1)$
- (B) $\frac{1}{12} n(n+1)(3n^2 + n + 1)$
- (C) $\frac{1}{12} n(2n^2 - n + 1)$
- (D) $\frac{1}{12} n(2n^2 - 2n + 3)$

15. T_r is always

- (A) an odd number
- (B) an even number
- (C) a prime number
- (D) a composite number

16. Which one of the following is a correct statement?

- (A) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 5
- (B) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 6
- (C) Q_1, Q_2, Q_3, \dots are in A.P. with common difference 11
- (D) $Q_1 = Q_2 = Q_3 = \dots$

Paragraph

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

17. The ratio of the areas of the triangles PQS and PQR is

- (A) $1 : \sqrt{2}$
- (B) $1 : 2$
- (C) $1 : 4$
- (D) $1 : 8$

18. The radius of the circumcircle of the triangle PRS is

- (A) 5
- (B) $3\sqrt{3}$
- (C) $3\sqrt{2}$
- (D) $2\sqrt{3}$

19. The radius of the incircle of the triangle PQR is

- (A) 4
- (B) 3
- (C) $8/3$
- (D) 2

20. Consider the following linear equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0$$

Match the conditions/expressions in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I		Column II	
(A)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p)	the equations represent planes meeting only at a single point.
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(q)	the equations represent the line $x = y = z$.
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r)	the equations represent identical planes.
(D)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of the three dimensional space.

21. In the following $[x]$ denotes the greatest integer less than or equal to x . Match the functions in Column I with the properties in Column II.

Column I		Column II	
(A)	$x x $	(p)	continuous in $(-1,1)$
(B)	$\sqrt{ x }$	(q)	differentiable in $(-1,1)$
(C)	$x + x $	(r)	strictly increasing in $(-1,1)$
(D)	$ x - 1 + x + 1 $	(s)	not differentiable at least at one point in $(-1, 1)$

22. Match the integrals in Column I with the values in Column II.

Column I		Column II	
(A)	$\int_{-1}^1 dx/(1+x^2)$	(p)	$1/2 \log(2/3)$
(B)	$\int_0^1 dx/\sqrt{1-x^2}$	(q)	$2\log(2/3)$

(C)	$\int_2^3 dx/(1-x^2)$	(r)	$\pi/3$
(D)	$\int_1^2 dx/(x\sqrt{x^2-1})$	(s)	$-\pi/2$