IIT-JEE-Mathematics-Paper2-2008

---

1. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{(-1)}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1 the particles moves $\sqrt{2}$ units in the direction of the unit vector i + j and then it moves through an angle $\Pi/2$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by

(A) $6 + 7i$

(B) $-7 + 6i$

(C) $7 + 6i$

(D) $-6 + 7i$

2. Let the function $g : (-\infty, \infty) \to (-\Pi/2 , \Pi/2)$ be given by $g(u) = 2\tan^{-1}(e^u) - \Pi/2$. Then, g is

(A) even and is strictly increasing in $(0, \infty)$

(B) odd and is strictly decreasing in $(-\infty, \infty)$

(C) odd and is strictly increasing in $(-\infty, \infty)$

(D) neither even or odd, but is strictly increasing in $(-\infty, \infty)$

3. Consider a branch of the hyperbola

$$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{(2/3)}$

(B) $\sqrt{(3/2)} - 1$

(C) $1 + \sqrt{(2/3)}$

(D) $\sqrt{3/2} + 1$

4. The area of the region between the curves $y = \sqrt{(1 + \sin x)/\cos x}$ and $y = \sqrt{(1 - \sin x)/\cos x}$ bounded by the lines $x = 0$ and $x = \Pi/4$ is

(A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

(B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

5. Consider three points $P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \Pi/4$. Then,

(A) P lies on the line segment RQ

(B) Q lies on the line segment PR

(C) R lies on the line segment QP

(D) P, Q, R are non-collinear

6. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8

(B) 3, 6 or 9

(C) 4 or 8

(D) 5 or 10

7. Let two non-collinear unit vectors a and b form an acute angle. A point P moves so that at any tiem t the position vector OP (where O is the origin) is given by a cos t + b sin t.

When P is farthest from origin O, let M be the length of vector OP and u be the unit vector along vector OP. Then,

(A) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M = $(1 + \hat{a}.\hat{b})^{\frac{1}{2}}$

(B) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M = $(1 + \hat{a}.\hat{b})^{\frac{1}{2}}$

(C) $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and M = $(1 + 2\hat{a}.\hat{b})^{\frac{1}{2}}$

(D) $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and M = $(1 + 2\hat{a}.\hat{b})^{\frac{1}{2}}$

8. Let

$$I = \int \frac{e^x}{e^{4x}+e^{2x}+1}\,dx,\ J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}\,dx.$$

Then, for an arbitrary constant C, the value of J - I equals.

(A) $\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$

(B) $\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$

(C) $\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$

(D) $\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$

9. Let g(x)&nbs Let g(x) = log f(x) where f(x) is a twice differentiable positive function on (0, ∞) such that f(x + 1) = x f(x). Then, for N = 1, 2, 3, ...,

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=$$

(A) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N-1)^2}\right\}$

(B) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N-1)^2}\right\}$

(C) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N+1)^2}\right\}$

(D) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\cdots+\frac{1}{(2N+1)^2}\right\}$

10. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1: The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P.

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

11. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and α, 1/β are the roots of the equation $ax^2 + 2bx + c = 0$, where β^2 is not belongs to {-1, 0, 1}.

STATEMENT-1: $(p^2 - q)(b^2 - ac) > 0$

and

STATEMENT-2: b ¹ pa or c ¹ qa

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

12. Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$

where p is a real number, and $C : x^2 + y^2 + 6x + 10y + 30 = 0$

STATEMENT-1: If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

and

STATEMENT-2: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

13. Let a solution y = y(x) of the differential equation

$x\sqrt{(x^2-1)}\, dy - y\sqrt{(y^2-1)}\, dx = 0$

satisfy $y(2) = 2/\sqrt{3}$.
STATEMENT-1: $y(x) = \sec(\sec^{-1} x - \pi/6)$
and
STATEMENT-2: y(x) is given by

$1/y = (2\sqrt{3})/x - \sqrt{(1-1/x^2)}$

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Paragraph

Consider the function f : (−∞, ∞) --> (−∞, ∞) defined by

$f(x) = (x^2-ax+1)/(x^2+ax+1), 0 < a < 2.$

14. Which of the following is true?

(A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$
(B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$
(C) $f'(1)f'(-1) = (2 - a)^2$
(D) $f'(1)f'(-1) = (2 + a)^2$

15. Which of the following is true?

(A) f(x) is decreasing on (−1, 1) and has a local minimum at x = 1
(B) f(x) is increasing on (−1, 1) and has a local maximum at x = 1
(C) f(x) is increasing on (−1, 1) but has neither a local maximum nor a local minimum at x = 1.
(D) f(x) is decreasing on (−1, 1) but has nether a local maximum nor a local minimum at x = 1.

16. Let

$g(x) = \int_0^{ex}(f'(t))/(1+t^2)\ dt.$

Which of the following is true?

(A) g′(x) is positive on (−∞, 0) and negative on (0, ∞)
(B) g′(x) is negative on (−∞, 0) and positive on (0, ∞)
(C) g′(x) is changes sign on both (−∞, 0) and (0, ∞)
(D) g′(x) is does not change sign on (−∞, ∞)

Paragraph

Consider the lines

L1 : (x+1)/3=(y+2)/1=(z+1)/2
L2 : (x-2)/1=(y+2)/2=(z-3)/3

17. The unit vector perpendicular to both L1 and L2 is

(A) (-i ̂+7j ̂+7k ̂)/√99
(B) (-i ̂-7j ̂+5k ̂)/(5√3)
(C) (-i ̂+7j ̂+5k ̂)/(5√3)
(D) (7i ̂-7j ̂-k ̂)/√99

18. The shortest distance between L1 and L2 is

(A) 0
(B) 17/√3
(C) 41/(5√3)
(D) 17/(5√3)

19. The distance of the point (1, 1, 1) from the plane passing through the point (−1, −2, −1) and whose normal is perpendicular to both the lines L1 and L2 is

(A) 2/√75
(B) 7/√75
(C) 13/√75
(D) 23/√75

20. Consider the lines given by

$L_1 : x + 3y - 5 = 0$

$L_2 : 3x - ky - 1 = 0$

$L_3 : 5x + 2y - 12 = 0$

Match the Statements/Expressions in Column I with the Statements/ Expressions in Column II.

Column I		**Column II**	
(A)	L_1, L_2, L_3 are concurrent if	(p)	k = -9
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q)	k = - 6/5

(C)	L_1, L_2, L_3 form a triangle if	(r)	k = 5/6
(D)	L_1, L_2, L_3 do not form a triangle if	(s)	k = 5

21. Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		**Column II**	
(A)	The minimum value of x^2+2x+4 / x+2 is	(p)	0
(B)	Let A and B be 3 × 3 matrices of real numbers, where A is symmetric, b is skew-symmetric, and (A + B) (A - B) = (A - B) (A + B). If $(AB)^t = (-1)^k$ AB, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q)	1
(C)	Let a = $\log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^\wedge-a)} <2$, , must be less than	(r)	2
(D)	If sin q = cos f, then the possible values of 1/Π(θ±Ø-Π/2) are	(s)	3

22. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		**Column II**	
(A)	The number of permutations containing the word ENDEA is	(p)	5!
(B)	The number of permutations in which the letter E occurs in the first and the last positions i	(q)	2 × 5!
(C)	The number of permutations in which one of the letters, D, L, N occurs in the last five positions is	(r)	7 × 5!
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s)	21 × 5!