IIT-JEE-Mathematics–Screening-2000

SCREENING

Time : Two hours Max. Marks : 100

PART A

1. Let f (θ) = sin θ (sin θ + sin 3 θ). Then f (θ) :
(A) ≥ 0 only when θ ≥ 0 (B) ≤ 0 for all real θ
(C) ≥ 0 for all real θ (D) ≤ 0 only when θ ≤ 0

2. If x + y = k is normal to y2 = 12 x, then k is :
(A) 3 (B) 9
(C) –9 (D) –3

3. For $2 \le r \le n$, $\binom{n}{r} \div 2\binom{n}{r-1} \div \binom{n}{r-2} =$
(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$
(C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

4. If a and β (α < β) are the roots of the equation x2 + bx + c = 0 , where c < 0 < b, then :
(A) 0 < α < β (B) α < 0 < β < |α|
(C) α < β < 0 (D) α < 0 < |α| < β

5. Let f : R → R be any function. Define g : R → R by g (x) = |f (x) |for all x.
Then g is :
Onto if f is onto
One-one is f one-one
Continuous if, f is continuous
Differentiable if f is differentiable

6. The domain of definition of the function y (x) is given by the equation 2x+ 2y = 2 is :
(A) 0< x≤1 (B) 0≤x≤1
(C) -∞ < x ≤0 (D) -∞< x<1

7. If x2+ y2=1, then :
(A) yy''' - 2(y ')2+1=0 (B) yy'' + (y ')2+1=0

(C) yy " = (y ')2-1=0 (D) yy''+2(y ')2+1=0

8. If a,b, c, d are positive real numbers such that a + b + c + d = 2, then M = (a + b) (c + d) satisfies the relation :
(A) 0 ≤ M ≤ 1 (B) 1 ≤ M ≤ 2
(C) 2 ≤ M ≤ 3 (D) 3 ≤ M ≤ 4

9. If the system of equations x − ky − z = 0, kx − y − z = 0, x + y − z = 0 has a non-zero solution, then possible values of k are :
(A) −1, 2 (B) 1, 2
(C) 0, 1 (D) −1, 1

10. The triangle PQR is inscribed in the circle x2+ y2=25. If Q and R have coordinates (3, 4) and (−4, 3) respectively, then ∠PQR is equal to:
(A) π/2 (B) π/3
(C) π/4 (D) π/6

11. In a triangle ABC, 2ac sin 1/2 (A − B + C) =
(A) a2+b2-c2 (B) c2+a2-b2
(C) b2-c2-a2 (D) c2-a2-b2

12. For x ∈ R, limn→∞((x-3)/(x+2))x =
(A) e (B) e-1
(C) e-5 (D) e5

13. Consider an infinite geometric series with first term and common ratio r. If its sum is 4 and the second term is 3/4, then :
(A) a=4/7, r=3/7 (B) a =2, r = 3/8
(C) a = 3/2, r = 1/2 (D) a = 3, r = 1/4

14. Let g (x) = ∫0x *f (t)* dt, where f is such that 1/2 ≤ *f(t)* ≤ 1 for t ∈ [0,1] and

0 ≤ *f(t)* ≤1/2 for t ∈[1,2]. Then g (2) satisfies the inequality:

(A) -3/2 ≤ *g* (2) < 1/2 (B) 0 ≤ *g* (2) <2
(C) 3/2 < *g* (2) ≤ 5/2 (D) 2 < *g* (2)< 4

15. In a triangle ABC, Let ∠ C= π/2. If r is the inradius and R is the circum-radius of the triangle, then 2 (r +R) is equal to :
(A) a + b (B) b + c
(C) c + a (D) a + b + c

16. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even position :
(A) 16 (B) 36

(C) 60 (D) 180

17. If arg (z) < 0, then arg (−z) − arg (z) =
(A) π (B) −π
(C) -π/2 (D) π/2

18. Let PS be the median of the triangle with vertices P (2, 2), Q (6-1) and R (7, 3). The equation of the line passing through (1,−1) and parallel to PS is:
(A) 2x-9 y-7=0 (B) 2 x-9 y-11=0
(C) 2 x+9 y-11=0 (D) 2 x-9 y-11=0

19. A pole stands vertically inside a triangular park Δ ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in Δ ABC the foot of the pole is at the :
(A) centroid (B) circumcentre
(C) incentre (D) orthocenter.

20. If $f(x) = \begin{cases} e^{\cos x}\sin x & for\ |x| \le 2, \\ 2 & otherwise, \end{cases}$ then $\int_{-2}^{3} f(x)\,dx =$

(A) 0 (B) 1
(C) 2 (D) 3

21. The incentre of the triangle with vertices (1,√3), (0, 0) and (2, 0) is :
(A) (1,√3/2) (B) (2/3,1/√3)
(C) (2/3,√3/2) (D) (1,1/√3)

22. Consider the following statements in S and R :
S : Both sin x and cos x are decreasing functions in the interval (π/2,π)
R : If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b).
Which of the following is true :
(A) Both S and R are wrong
(B) Both S and R are correct, but R is not the correct explanation of S.
(C) S is correct and R is correct explanation for S.
(D) S is correct and R is wrong.

23. Let f(x)= ∫ex (x-1)(x-2) dx. Then f decreases in the interval :

(A) (∞,-2) (B) (−2, −1)
(C) (1, 2) (D) (2,+∞)

24. If the circles x2+y2+2x+2ky+6=0 and x2+y2+2ky+k=0 intersect orthogonally, then k is :

(A) 2 or −3/2 (B) −2 or 3/2
(C) 2 or 3/2 (D) (2,+∞)

25. If the vectors $\vec{a},\vec{b}$ and $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then:

(A) $\vec{a}.\ \vec{b}+\vec{b}.\ \vec{c}+\vec{c}.\ \vec{a}=0$ (B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(C) $\vec{a}.\ \vec{b}=\vec{b}.\ \vec{c}=\vec{c}.\ \vec{a}$ (D) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$

26. If the normal to the curve y=f (x) at the point (3, 4) makes an angle 3 p/4 with the positive x-axis then f' (3) =

(A) –1 (B) –3/4

(C) 4/3 (D) 1

27. Let the vectors a, b, c and d be such that (a × b) × (c × d) = 0. Let P1 and P2be planes determined by the pairs of vectors a, b and c, d respectively, then the angle between P1 and P2 is :

(A) 0 (B) p/4

(C) p/3 (D) p/2

28. Let $f(x)=\begin{cases}|x| & \text{for } 0<|x|\le 2\\ 1 & \text{for } x=0.\end{cases}$

Then at x = 0, f has :

(A) A local maximum (B) no local maximum

(C) a local minimum (D) no extremum

29. If $\vec{a}.\ \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}.\ 2\vec{b}-\vec{c}.\ 2\vec{c}-\vec{a}]=$

(A) 0 (B) 1

(C) -√3 (D) √3

30. If b > a, then the equation (x-a)(x-b) –1 = 0 has :

(A) both roots in (a, b)

(B) both roots in (–¥, a)

(C) both roots in (b, + ¥)

(D) one root in (–¥, a) and the other in (b, + ¥)

31. If z1, z2 and z3 are complex numbers such that

|z1| |z2| = |z3| = |1/z1 +1/z2 +1/z3 | = 1, then |z1+z2+z3| is :

(A) equal to 1 (B) less than1

(C) greater than 3 (D) equal to 3

32. For the equation 3x2+px+3=0,p>0, if one of the root is square of the other, then p is equal to :

(A) 1/3 (B) 1

(C) 3 (D) 2/3

33. If the line x – 1 = 0 is the directrix of the parabola y2-kx+8=0 then one of the values of

k is :
(A) 1/8 (B) 8
(C) 4 (D) 1/4

34. For all ∈(0,1) :
(A) ex< 1+x (B) loge (1+x) < x

(C) sin x > x (D) loge x>x.

35. The value of the integral ∫e-1e2 | loge x / x | dx is :

(A) 3/2 (B) 5/2
(C) 3 (D) 5