IIT-JEE-Mathematics-Screening–2001

SCREENING

Time : Three hours Max. Marks : 100

Notations :
R : set of real numbers.
[x] : the greatest integer £ x.

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1. Let f:R→R be a function defined by (x)=max { x,x3 }. The set of all points where is NOT differentiable is:
(A) {-1, 1 }
(B) {-1, 0 }
(C) {0, 1 }
(D) {-1, 0, 1 }

2. Let f:(0,∞)→R and F(x)= ∫0x *f(t)*dt. If F(x2)= x2 (1+x), then f(4)equals:
(A) 5/4
(B) 7
(C) 4
(D) 2

3. The left hand derivative of f(x)=[x] sin(πx) at x=k,k an integer, is:
(A) (-1)k (k-1)π
(B) (-1)(k-1) (k-1)π
(C) (-1)k kπ
(D) (-1)(k-1) kπ

4. If f(x)=x e(x(1-x)), then f(x) is:

(A) Increasing on [-1/2 ,1]
(B) Decreasing on R
(C) Increasing on R
(D) Decreasing on [-1/2 ,1]

5. limx→0 sin(π cos2 x) / x2

(A) −π
(B) π
(C) π/2
(D) 1

6. The triangle formed by the tangent to the curve f(x)= x2+ bx-b at the point

(1, 1)and the coordinate axes, lies in the first quadrant. If its area is 2, then the value of b is:
(A) -1
(B) 3
(C) -3
(D) 1

7. Let g(x)=1+x-[x] and

$$f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}.$$

Then for all x,f[g(x)] is equal to:
(A) x
(B) 1
(C) f(x)
(D) g(x)

8. If f:[1,∞) is given by f(x)=x+1/x then f-1 (x) equals :

(A) (x+√(x2-4))/2
(B) x/1+x2
(C) (x-√(x2-4))/2
(D) 1+√(x2-4)

9. The domain of definition of f(x)= (log2 (x+3))/(x2+3x+2) is:

(A) R\{-1, -2}
(B) (-2, ∞)
(C) R/{-1,-2,-3}
(D) (-3, ∞)\{-1, -2}

10. The equation of the common tangent touching the circle (x-3)2+ y2= 9 and the parabola y2=4x above the x-axis is :

(A) √3 y=3x+1
(B) √3 y=-(x+3)
(C) √3 y=x+3
(D) √3 y=-(3x+1)

11. The value of ∫-ππ (cos2 x / 1+ax) dx, a>0, is

(A) π
(B) aπ
(C) π/2
(D) 2π

12. Let AB be a chord of the circle x2+y2 = r2 subtending a right angle at the

centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is:
(A) A parabola
(B) A circle
(C) An ellipse
(D) A pair of straight lines

13. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines 3x+4y=9 and y=mx+1 is also an integer, is :
(A) 2
(B) 0
(C) 4
(D) 1

14. The equation of the directrix of the parabola y2+4y+4x+2=0 is:
(A) x=-1
(B) x=1
(C) x=-3/2
(D) x=3/2

15. Let α and β be the roots of x2-x+p=0 and γ and δ be the roots of x2-4x+q=0.

if α,β,γ,δ are in G.P. then the integral values of P and q respectively, are:
(A) -2, -32
(B) -2, 3
(C) -6, 3
(D) -6, -32

16. In the binomial expansion of (a-b)n, n≥5, the sum of the 5th and 6th terms is zero. Then a/b equals:
(A) (n-5)/6
(B) (n-4)/5
(C) 5/(n-4)
(D) 6/(n-5)

17. Let f(x)=(1+b2) x2 + 2bx + 1 and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is:
(A) [0, 1]
(B) [0,1/2]
(C) [1/2,1]
(D) [0, 1]

18. The number of distinct roots of

$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is:

(A) 0
(B) 2
(C) 1
(D) 3

19. Let E={1,2,3,4} and F={1,2}. Then the number of onto functions from E to F is:
(A) 14
(B) 16
(C) 12
(D) 8

20. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If Tn+1 - Tn=21, then n equals:
(A) 5
(B) 7
(C) 6
(D) 4

21. The complex numbers z1,z2,and z3,satisfying (z1- z3)/ (z2 - z3) = (1-i√3)/2 are the vertices of a triangle which is :
(A) Of area zero
(B) Right-angled isosceles
(C) Equilateral
(D) Obtuse-angled isosceles

22. If the sum of the first 2n terms of the A.P. 2,5,8,……, is equal to the sum of the first n terms of the A.P.57,59,61,…..,then n equals:
(A) 10
(B) 12
(C) 11
(D) 13

23. Let z1 and z2 be nth roots of unity which subtend a right angle at the origin. Then n must be of the form:
(A) 4k+1
(B) 4k+2
(C) 4k+3
(D) 4k

24. Let the positive numbers a,b,c,d be in A.P. Then abc, abd, acd, bcd are:

(A) NOT in A.P./G.P/H.P.
(B) In A.P
(C) In G.P
(D) In H.P

25. Let f(x) = αx/(x+1), x ≠ -1. Then for what value of α is f[f(x)]=x:
(A) √2
(B) - √2
(C) 1
(D) -1

26. If $If\ \vec{a},\ \vec{b}, and\ \vec{c}\ are\ unit\ vectors, then$
$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$

does not exceed :
(A) 4
(B) 9
(C) 8
(D) 6

27. Which of the following functions is differentiable at x=0 :
(A) cos(|x|)+|x|
(B) cos(|x|)-|x|
(C) sin(|x|)+|x|
(D) sin(|x|)-|x|

28. The number of solutions of log4 (x-1)=log2 (x-3) is :
(A) 3
(B) 1
(C) 2
(D) 0

29. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1+x-y)\vec{k}$.
Then $[\vec{a},\ \vec{b},\ \vec{c}]$ depends on :

(A) Only x
(B) Only y
(C) Neither x nor y
(D) Both x and y

30. Area of the parallelogram formed by the lines y =mx, y =mx +1 , y =nx and y = nx +1 equals:
(A) |m+n|/(m-n)2
(B) 2/|m+n|
(C) 1/|m+n|
(D) 1/|m-n|

31. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals :
(A) √(PQ.RS)
(B) (PQ+RS)/2
(C) (2PQ.RS)/(PQ+RS)
(D) √((PQ2+RS2)/2)

32. A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 300. After some time, the angle of depression becomes 600. The distance (in metres) travelled by the car during this time is :
(A) 100√3
(B) (200√3)/3
(C) (100√3)/3
(D) 200√3

33. If α+β = π/2 and β+γ = α, then tan α equals:
(A) 2(tan β + tan γ)
(B) tan β + tan γ
(C) tan β + 2tan γ
(D) 2 tan β + tan γ

34. sin-1(x-x2/2 + x3/4 - …) + cos-1(x2 - x4/2 + x6/4 - …) = π/2 for 0<|x|<√(2,) then x equals :
(A) ½
(B) 1
(C) -1/2
(D) -1

35. The maximum value of (cos α1). (cos α2). ….. (cos αn), under the restrictions 0 ≤ α1,α2,……,αn ≤ π/2 and (cot α1). (cot α2). ….. (cot αn)=1 is:

(A) 1/2n/2
(B) 1/2n
(C) 1/2n
(D) 1