IIT-JEE-Mathematics-Screening-2002

SCREENING

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1. Let $\omega = -1/2 + i\sqrt{3}/2$. Then the value of the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \text{ is:}$$

(A) 3ω
(B) $3\omega(\omega-1)$
(C) $3\omega^2$
(D) $3\omega(1-\omega)$

2. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is:
(A) 0
(B) 2
(C) 7
(D) 17

3. If $a_1\ a_2, \ldots, a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \ldots + a_{n-1} + 2a_n$ is

(A) $n(2c)^{1/n}$
(B) $(n+1)c^{1/n}$
(C) $2nc^{1/n}$
(D) $(n+1)(2c)^{1/n}$

4. Suppose *a, b, c* are I A.P. and a^2, b^2, c^2 are in G.P. If $a<b<c$ and $a+b+c=3/2$, then the value of *a* is

(A) $1/2\sqrt{2}$
(B) $1/\ 2\sqrt{3}$
(C) $½ - 1/\sqrt{3}$
(D) $½ - 1/\sqrt{3}$

5. The number of arrangements of the letters of the word BANANA in which the two N' s do not appear adjacently is

(A) 40
(B) 60
(C) 80
(D) 100

6. The sum

$$\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}, \text{ Where } \binom{p}{q}=0$$

if p > q is maximum when m is
(A) 5
(B) 10
(C) 15
(D) 20

7. The number of values of k for which the system of equations
(k+1)x + 8y=4k
kx +(k+3)y = 3k -1
has infinitely many solution is
(A) 0
(B) 1
(C) 2
(D) Infinite

8. The set of all real numbers x for which $x^2 - |x+2| + x > 0$ is
(A) (-∞, -2) ∪ (2, ∞)
(B) (-∞, -√2) ∪ (√2, ∞)
(C) (-∞, -1) ∪ (1, ∞)
(D) (√2, ∞)

9. The length of a longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is
(A) Π/3
(B) Π/2
(C) 3Π/3
(D) Π

10. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC (R being the radius of the circumcircle)?
(A) a sin A, sin B
(B) a, b, c
(C) a, sin B, R

(D) a, sin A, R

11. The number of integral values of k for which the equation 7 cos x + 5 sin x = 2k + 1 has a solution is

(A) 4

(B) 8

(C) 10

(D) 12

12. Let 0 < α < Π/2 be a fixed angle. If P = (cosθ, sinθ) and Q = (cos(α-θ), sin(α-θ)) then Q is obtained from P by

(A) Clockwise rotation around origin through an angle α

(B) Anticlockwise rotation around origin through an angle α

(C) Reflection in the line through origin with slope tan α

(D) Reflection in the line through origin with slope tan α/2

13. Let P=(-1, 0) Q=(0, 0) R=(3, 3√3) be three points. Then the equation of the bisector of the bisector of the angle PQR is

(A) √3/2 x+y=0

(B) x+√3 y=0

(C) √3 x+y=0

(D) x+√3/2 y=0

14. A straight line through the origin O meets the parallel lines 4x + 2y = 9 and 2x + y + 6 = 0 at points P and Q respectively. Then the point O divides the segment PQ in the ratio

(A) 1:2

(B) 3:4

(C) 2:1

(D) 4:3

15. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line 5x + 2y = 6 at a point Q on the y-axis, then the length of PQ is

(A) 4

(B) 2√5

(C) 5

(D) 3√5

16. If a>2b>0 then the positive value of m for which $y = mx - b\sqrt{(1 + m^2)}$ is a common tangent to $x^2+y^2=b^2$ and $(x- a)^2+y^2=b^2$ is

(A) $2b/\sqrt{(a^2 - 4b^2)}$

(B) $\sqrt{(a^2 - 4b^2)}/2b$

(C) 2b/(a-2b)
(D) b/(a-2b)

17. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2=4ax$ is another parabola with directrix
(A) x = -a
(B) x = -a/2
(C) x = 0
(D) x = a/2

18. The area bounded by the curves y = |x| - 1 and y = - |x| + 1 is
(A) 1
(B) 2
(C) 2√2
(D) 4

19. Suppose $f(x) = (x + 1)^2$ for x ≥ -1 If g(x) is the function whose graph is reflection of the graph of f(x) with respect to the line y = x then g(x) equals
(A) -√x - 1, x ≥ 0
(B) $1/(x + 1)^2$, x > -1
(C) √(x + 1), x ≥ -1
(D) √x - 1, x ≥ 0

20. Let function f : R→R be defined by f(x) = 2x + sin x for x ε R Then f is
(A) One-to-one and onto
(B) One-to-one but NOT onto
(C) Onto but NOT one-to-one
(D) Neither one-to-one nor onto

21. The domain of the derivative of the function

$$f(x) = \begin{cases} \tan^{-1} x, if\ |x| \le 1 \\ \frac{1}{2}(|x| - 1), if\ |x| > 1 \end{cases}$$ is

(A) R - {0}
(B) R - {1}
(C) R - {-1}
(D) R - {-1,1}

22. The integer n for which $\lim_{x\to 0} (\cos x-1)(\cos x-e^x) / x^n$ is a finite non-zero number is

(A) 1

(B) 2
(C) 3
(D) 4

23. Let f : R→R be such that f(1) = 3, and f'(1) = 6 Then $\lim_{x\to 0}$ (f(1+x) / f(1) $)^{1/x}$ equals

(A) 1
(B) $e^{1/2}$
(C) e^2
(D) e^3

24. The point (s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical , is (are)
(A) (± 4/√3 , -2)
(B) (± √11/3 , -0)
(C) (0, 0)
(D) (± 4/√3 , 2)

25. The equation of the common tangents to the curves $y^2 = 8x$ and xy = -1 is
(A) 3y = 9x+2
(B) y = 2x + 1
(C) 2y = x +8
(D) y = x + 2

26. Let f(x) = $\int^x_1$ √(2 - t^2) The real roots of the equation x^2 - f' (x) = 0 are
(A) ± 1
(B) ± 1/√2
(C) ± 1/2
(D) 0 and 1

27. Let T>0 be a fixed real number. Suppose f is a continuous function such that for all x ε R.f(x+T) If I = $\int_{T0\ f(x).dx}$ then the value of $\int_3^{3+3T}$ is
(A) (3/2) I
(B) I
(C) 3I
(D) 6I

28. The integral $\int_{1/2-1/2}$ ([x] + ln(1+x/1+x))dx equals
(A) -1/2
(B) 0
(C) 1
(D) 2ln(1/2)

29. If vector a and bare two vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between vector a and b is

(A) 45^0

(B) 60^0

(C) $\cos^{-1} 1/3$

(D) $\cos^{-1} 2/7$

30. Let vector $V = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{W} = \vec{i} + 3\vec{k}$. If vector U is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\vec{V}\vec{W}]$ is

(A) -1

(B) $\sqrt{10} + \sqrt{6}$

(C) $\sqrt{59}$

(D) $\sqrt{60}$