IIT-JEE-Mathematics-Screening-2003

SCREENING

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1. If *f*:[0, ¥) --> [0, ¥), and f(x)=x/(1+x) then *f* is:
(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

2. If P(B) = 3/4, P(AÇBÇC¯) = 1/3 and P(A¯ÇBÇC¯) = 1/3, then P(BÇC) is:
(a) 1/12
(b) 1/6
(c) 1/15
(d) 1/9

3. In [0, 1] Lagranges Mean Value theorem is NOT applicable to

(a) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

(b) $f(x) = \begin{cases} \frac{\sin x}{x} - x, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c) f(x)=x|x|
(d) f(x)=|x|

4. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $x^2/9+(y^2=1)/5$, is :
(a) 27/4 sq. units
(b) 9 sq. units
(c) 27/2 sq. units
(d) 27 sq. units

5. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is:
(a) 133
(b) 190
(c) 233

(d) 105

6. If

$$A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix},$$

then value of α for which $A^2 = B$, is:
(a) 1
(b) –1
(c) 4
(d) no real values

7. The value of k such that (x-4)/1=(y-2)/1=(z-k)/2 lies in the plane 2x – 4y + z = 7, is:
(a) 7
(b) –7
(c) no real value
(d) 4

8. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is:
(a) √3:(2+√3)
(b) 1 : 6
(c) 1 : 2 + √3
(d) 2 : 3

9. If $\lim_{(x\to 0)} (((a-n)nx - \tan x)\sin nx)/x^2 = 0$, where n is non zero real number, then a is equal to:
(a) 0
(b) (n+1)/n
(c) n
(d) n+1/n

10. Two numbers are selected randomly from the set S = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4 is :
(a) [1/15]
(b) [14/15]
(c) [1/5]
(d) [4/5]

11. For hyperbola $x^2/(\cos^2 \alpha)-y^2/(\sin^2 \alpha)=1$ which of the following remains constant with change in 'a' :
(a) abscissae of vertices
(b) abscissa of foci

(c) eccentricity
(d) directrix

12. Range of the function $f(x)=(x^2+x+2)/(x^2+x+1)$; x Î R is:
(a) (1, ¥)
(b) (1, 11/7)
(c) (1, 7/3)
(d) (1, 7/5)

13. $\lim_{(h\to 0)} (f(2h+2+h^2)-f(2))/(f(h-h^2+1)-f(1))$, given that $f'(2) = 6$ and $f'(1) = 4$:

(a) does not exists
(b) is equal to –3/2
(c) is equal to 3/2
(d) is equal to 3

14. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that min f(x) > max g(x), then the relation between b and c, is :

(a) no real value of b and c
(b) $0 < c < b\sqrt{2}$
(c) $|c| < |b|\sqrt{2}$
(d) $|c| > |b|\sqrt{2}$

15. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is:
(a) (4, 7)
(b) (7, 4)
(c) (9, 4)
(d) (4, 9)

16. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are :
(a) {–1, 1}
(b) {–2, 2}
(c) {–2, 1/2}
(d) {2, 1/2}

17. Domain of definition of the function $f(x) = \sqrt{(\sin^{-1}(2x) + \pi/6)}$ for real valued x, is :

(a) $\left[-\frac{1}{4},\frac{1}{2}\right]$
(b) $\left[-\frac{1}{2},\frac{1}{2}\right]$
(c) $\left(-\frac{1}{2},\frac{1}{9}\right)$
(d) $\left(-\frac{1}{4},\frac{1}{4}\right)$

18. If ½z½ = 1 and ω = z-1/z+1 (where z ≠ -1), then Re(w) is:
(a) 0
(b) $1/|z+1|^2$
(c) (|1/(z+1)|) ($1/[z+1]^2$)
(d) $\sqrt{2} / |z+1|^2$

19. If α Î (0, Π/2) then $\sqrt{(x^2 + x)}$ is always greater than or equal to :
(a) 2 tan a
(b) 1
(c) 2
(d) $\sec^2 a$

20. If $I(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $I(m, n)$ in terms of $I(m + 1, n - 1)$ is:
(a) $2^n/m+1) - n/m+1) * I(m+1,n-1)$
(b) n/m+1) * I(m+1,n-1)
(c) $2^n/(m+1) + n/(m+1) * I(m+1,n-1)$
(b) m/(m+1) * I(m+1,n-1)

21. If $f(x) = \int_x^{2(x2+1)} e^{-t2} dt$, then $f(x)$ increases in :

(a) (2, 2)
(b) no value of x
(c) (0, ¥)
(d) (-¥, 0)

22. The area of bounded by the curves $y = \sqrt{x}$, 2y + 3 = x and x-axis in the 1st quadrant is :
(a) 9
(b) 27/4
(c) 36
(d) 18

23. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is:

(a) $^{12}C_6 + 3$

(b) $^{12}C_6 + 1$

(c) $^{12}C_6$

(d) $^{12}C_6 + 2$

24. The value of 'a' so that the volume of parallelepiped formed by $\hat{i} + a\hat{j} + k$, $\hat{j} + ak$ and $a\hat{i} + k$ because minimum is:

(a) -3

(b) 3

(c) $1/\sqrt{3}$

(d) $\sqrt{3}$

25. If the system of equations $x + ay = 0$, $az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

(a) -1

(b) 1

(c) 0

(d) no real values

26. If y(t) is a solution of $(1 + t)\, dy/dt - ty = 1$ and $y(0) = -1$, then y(1) is equal to:

(a) -1/2

(b) e + ½

(c) e - ½

(d) ½

27. Tangent is drawn to ellipse $x^2/27 + y^2 = 1$ at $(3\sqrt{3} \cos \theta, \sin \theta)$ (where $\theta \hat{I} (0, \Pi/2)$. Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is

(a) p/3

(b) p/6

(c) p/8

(d) p/4

28. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is:

(a) (3, 4/5)

(b) (3, 12)

(c) (3, 3/4)

(d) (3, 9)