IIT-JEE-Mathematics–Screening-2005

SCREENING

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1. The area of the equilateral triangle, in which three coins of radius 1 cm are placed, as shown in the figure, is:
(a) $6 + 4\sqrt{3}$
(b) $4\sqrt{3} - 6$
(c) $7 + 4\sqrt{3}$
(d) $4\sqrt{3}$

2. The area bounded by the curves $y = (x - 1)^2$, $y = (x + 1)^2$ and $y = 1/4$ is:
(a) 1/3
(b) 2/3
(c) 1/4
(d) 1/5

3. The value of $\int_{-2}^{0}[x^3+3x^2+3x+3+(x+1)\cos(x+1)dx]$ is:
(a) 0
(b) 3
(c) 4
(d) 1

4. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at :
(a) (6, 7)
(b) (−6, 7)
(c) (6, −7)
(d) (−6, −7)

5. If $dy/dx=xy/(x^2+y^2)$, $y(1) = 1$, then one of the values of x_0 satisfying $y(x_0) = e$ is given by
(a) $e\sqrt{2}$
(b) $e\sqrt{3}$
(c) $e\sqrt{5}$
(c) $e/\sqrt{2}$

6. The locus of the centre of circle which touches $(y -1)^2 + x^2 = 1$ externally also touches x axis is:
(a) $x^2 = 4y$ È $(0, y), y < 0$
(b) $x^2 = y$

(c) $y = 4x^2$
(d) $y^2 = 4x$ È (0, y), y Î R

7. If $\int_{\sin x}^{1} t^2 f(t)\, dt = 1 - \sin x$ ∀ x Î [0, Π/2] then $f(1/\sqrt{3})$ is:

(a) 3
(b) $\sqrt{3}$
(c) 1/3
(d) none of these

8.

$$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \ldots\ldots \binom{30}{20}\binom{30}{30} =$$

(a) $^{30}C_{11}$
(b) $^{60}C_{10}$
(c) $^{30}C_{10}$
(d) $^{65}C_{55}$

9. A variable plane x/a + y/b + z/c = 1 at a unit distance from origin cuts the coordinate axes at A, B and C. Centroid (x, y, z) satisfies the equation $1/x^2 + 1/y^2 + 1/z^2 = K$. The value of K is :
(a) 9
(b) 3
(c) 1/9
(d) 1/3

10. Let $f(x) = ax^2 + bx + c$, a ¹ 0 and $D = b^2 - 4ac$. If a + b, $a^2 + b^2$ and $a^3 + b^3$ are in G.P., then :
(a) D ¹ 0
(b) bD ¹ 0
(c) cD ¹ 0
(d) bc ¹ 0

11. Tangent at a point of the ellipse $x^2/a^2 + y^2/b^2 = 1$ is drawn which cuts the coordinate axes at A and B. The minimum area of the triangle OAB is (O being the origin) :
(a) ab
(b) $(a^3 + ab + b^3)/3$
(c) $a^2 + b^2$
(d) $((a^2 + b^2))/4$

12. A fair die is rolled. The probability that the first time 1 occurs at the even throw is :
(a) 1/6

(b) 5/11
(c) 6/11
(d) 5/36

13. If xdy = y (dx + ydy), y(1) = 1 and y(x) > 0. Then y(−3) = :
(a) 3
(b) 2
(c) 1
(d) 0

14. $f(x) = \begin{cases} x, & \textit{if x is rational} \\ 0, & \textit{if x is irrational} \end{cases}$ *and*

$g(x) = \begin{cases} 0, & \textit{if x is rational} \\ x, & \textit{if x is irrational.} \end{cases}$ Then f – g is:

(a) one-one and into
(b) neither one-one nor onto
(c) many one and onto
(d) one-one and onto

15. A rectangle with sides (2n − 1) and (2m − 1) is divided into squares of unit length. The number of rectangle which can be formed with sides of odd length is :
(a) $m^2 n^2$
(b) mn(m + 1) (n + 1)
(c) $4^{m + n - 1}$
(d) none of these

16. The minimum value of $|a + bw + cw^2|$, where a, b and c are all not equal integers and $w(^{1}\ 1)$ is a cube root of unity, is:
(a) √3
(b) 1/3
(c) 1
(d) 0

17. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2005} P$ is:

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

18. The shaded region, where
P ≡ (−1, 0), Q ≡ (−1 + √2,√2)
R ≡ (−1 + √2, − √2), S ≡ (1, 0) is represented by:
(a) |z + 1| > 2, |arg (z + 1)|< π/4
(b) |z + 1| < 2, |arg (z + 1)|< π/2
(c) |z − 1| > 2, |arg (z + 1)|> π/4
(d) |z − 1| < 2, |arg (z + 1)|> π/2

19. The number of ordered pairs (α, β), where α, β Î (−Π, Π) satisfying cos (α − β) = 1 and cos (α + β) = 1/e is :
(a) 0
(b) 1
(c) 2
(d) 4

20. Let f(x) = |x|−1, then points where f(x) is not differentiable is/(are) :
(a) 0, + 1
(b) + 1
(c) 0
(d) 1

21. The second degree polynomial f(x), satisfying f(0) = 0, f(1) = 1, f′(x) > 0 for all xÎ (0, 1) :
(a) f(x) = f
(b) f(x) = ax + (1 − a) x^2; ∀ a Î (0, ¥)
(c) f(x) = ax + (1 − a) x^2; ∀ a Î (0, 2)
(d) no such polynomial

22. If f is a differentiable function satisfying f(1/n) = 0 for all n > 1, n Î I, then :
(a) f(x) = 0, x Î (0, 1]
(b) f′(0) = 0 = f(0)

(c) f(0) = 0 but f′(0) not necessarily zero
(d) |f(x)| < 1, x Î (0, 1]

23. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$,

$6A^{-1} = A^2 + cA + dI$, then (c, d) is:

(a) (−6, 11)
(b) (−11, 6)
(c) (11, 6)
(d) (6, 11)

24. In a ΔABC, among the following which one is true?
(a) (b + c) cos A/2 = a sin ((B+C)/2)
(b) (b + c) cos ((B+C)/2) = a sin A/2
(c) (b − c) cos ((B-C)/2) = a cos (A/2)
(d) (b − c) cos A/2 = a cos ((B-C)/2)

25. If $\vec{a}, \vec{b}, \vec{c}$ are three non zero, non coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, $\vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$, And $\vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}}{|\vec{b}|^2}\vec{b}$, $\vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}}{|\vec{b}_1|^2}\vec{b}_1$, $\vec{c}_3 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{c}.\vec{b}}{|\vec{b}_2|^2}\vec{b}_2$, $\vec{c}_4 = \vec{a} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}$.

Then which of the following is a set of mutually orthogonal vectors:
(a) $(\vec{a}, \vec{b}_1, \vec{c}_1)$
(b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$
(c) $(\vec{a}, \vec{b}_2, \vec{c}_3)$
(d) $(\vec{a}, \vec{b}_2, \vec{c}_4)$

26. If y = f(x) and y cos x + x cos y = Π, then the value of f′(0) is :
(a) Π
(b) − Π
(c) 0
(d) 2Π

27. Let f be twice differentiable function satisfying f(1) = 1, f(2) = 4, f(3) = 9, then :
(a) f′(x) = 2, ∀ x Î (R)
(b) f′(x) = 5 = f′′ (x), for some x Î (1, 3)
(c) There exists at least one x Î (1, 3) such that f′(x) = 2

(d) none of these

28. If X and Y are two non-empty sets where f : X --> Y is function is defined such that
f(c) = {f(x) : x Î C} for C Í X
and f^{-1} (D) = {x : f(x) Î D} for D Í y,
for any A Í X and B Í Y then :

(a) f^{-1} (f(A)) = A
(b) f^{-1} (f(A)) = A only if f(X) = Y
(c) f(f^{-1} (B)) = B only if B Í f(x)
(d) f(f^{-1} (B)) = B