HINTS & SOLUTIONS (PRACTICE PAPER-3)

ANSWER KEY

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	D	B	C	C	D	D	A	B	B	A	C	D	D	D	B
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	A	B	B	D	B	B	D	A	B	B	A	D	B	A	A
Ques.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	B	B	B	D	D	C	D	B	B	A	D	C	A		D
Ques.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	B	C	B	C	B	A	B	A	A	B	C	A	A	C	B
Ques.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	C	B	A	B	C	D	D	D	C	C	C	D	B	B	C
Ques.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	B	A	B	C	A	B	B	C	A	A	B	A	C	B	B
Ques.	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Ans.	A	D	D	A	C	D	A	A	C	D	B	D	A	B	D
Ques.	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	D	C	D	B	B	A	A	A	D	D	B	A	B	A	B

PART - I (1 Mark)

MATHEMATICS

1. $\left(x^{\frac{1}{2}} - x^{\frac{2}{3}}\right)^7$

General term = $(-1)^r\,{}^7C_r \left(x^{\frac{1}{2}}\right)^{7-r} \times \left(x^{\frac{2}{3}}\right)^r$

$T_{r+1} = (-1)^r\,{}^7C_r\; x^{\frac{7-r}{2}}\, x^{\frac{2r}{3}}$

$= (-1)^r\,{}^7C_r\; x^{\frac{7-r}{2}+\frac{2r}{3}}$

$= (-1)^r\,{}^7C_r\; x^{\frac{21-3r+4r}{6}}$

$= (-1)^r\,{}^7C_r\; x^{\frac{21+r}{6}}$

For the coefficient of x^4, $\frac{21+r}{6} = 4$

$21 + r = 24$

$r = 24 - 21$

$r = 3$

$\therefore \quad T_{3+1} = (-1)^3\,{}^7C_3$

$= -1 \times \frac{7\times 6\times 5}{3\times 2}$

$= -35.$

2. $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right| = 1$ and $|z| = 1$

Let $z = x + iy$

Then, $\bar{z} = x - iy$

$|z| = \sqrt{x^2+y^2} = 1$

or $x^2 + y^2 = 1$

$\left|\frac{x+iy}{x-iy}+\frac{x-iy}{x+iy}\right| = 1$

$\left|\frac{x^2-y^2+2ixy+x^2-y^2-2ixy}{x^2+y^2}\right| = 1$

$\left|\frac{2(x^2-y^2)}{x^2+y^2}\right| = 1$

$2x^2 - 2y^2 = x^2 + y^2$

$x^2 = 3y^2$

$x = \pm\sqrt{3}y$

Now, $x^2 + y^2 = 1$

$3y^2 + y^2 = 1$

$4y^2 = 1$

$y^2 = \frac{1}{4}$

$y = \pm\frac{1}{2}$

Then, $x = \pm\frac{\sqrt{3}}{2}$

$\therefore$ Different complex number are : $z = \frac{\sqrt{3}}{2} + i\frac{1}{2}$, $z = \frac{\sqrt{3}}{2} - i\frac{1}{2}$, $z = -\frac{\sqrt{3}}{2} + i\frac{1}{2}$ and $z = -\frac{\sqrt{3}}{2} - i\frac{1}{2}$.

3. Let $A^{-1} = B$

Therefore $AB = I$

$A\begin{bmatrix}35 & 37\\41 & 43\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$R_1 \to \frac{R_1}{35}$

$A\begin{bmatrix}1 & \frac{37}{35}\\41 & 43\end{bmatrix} = \begin{bmatrix}\frac{1}{35} & 0\\0 & 1\end{bmatrix}$

$R_2 \to R_2 - 41R_1$

$A\begin{bmatrix}1 & \frac{37}{35}\\0 & -\frac{12}{35}\end{bmatrix} = \begin{bmatrix}\frac{1}{35} & 0\\-\frac{41}{35} & 1\end{bmatrix}$

$R_2 \to -\frac{35}{12}R_2$

$$A\begin{bmatrix}1 & \frac{37}{35}\\ 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{35} & 0\\ \frac{41}{12} & -\frac{35}{12}\end{bmatrix}$$

$$R_1 \to R_1 - \frac{37}{35}R_2$$

$$A\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}-\frac{43}{12} & \frac{37}{12}\\ \frac{41}{12} & -\frac{35}{12}\end{bmatrix}$$

$$A = \begin{bmatrix}-\frac{43}{12} & \frac{37}{12}\\ \frac{41}{12} & -\frac{35}{12}\end{bmatrix}$$

$$12\,A = \begin{bmatrix}-43 & 37\\ 41 & -35\end{bmatrix}$$

det (12A) = | 43 × 35 – 41×37|

= 1505 – 1517

= – 12.

4. Given : $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

$$\sum_{n=1}^{\infty}\frac{1+3+5+...+(2n-1)}{1^3+2^3+3^3+.....+n^3}$$

$$\sum_{n=1}^{\infty}\frac{n^2}{\frac{n^2(n+1)^2}{4}}$$

$$\sum_{n=1}^{\infty}\frac{4}{(n+1)^2}$$

$$4\sum_{n=1}^{\infty}\left[\left(1+\frac{1}{2^2}+\frac{1}{3^2}+.........\right)-1\right]$$

$$4\sum_{n=1}^{\infty}\left[\frac{1}{n^2}-1\right]$$

$$4\left(\frac{\pi^2}{6}-1\right).$$

5. $4ax^2 + xy + 4y^2 = 0$

$m_1 + m_2 = \frac{-2h}{b}$

$m_1 + m_2 = \frac{-1}{4}$

$m_1 m_2 = \frac{a}{b}$

$m_1 m_2 = \frac{4a}{4} = a$

Given : $m_2 = m_1^2$

$m_1 + m_1^2 = \frac{-1}{4}$

$4m_1^2 + 4m_1 = -1$

$4m_1^2 + 4m_1 + 1 = 0$

$(2m_1 + 1)^2 = 0$

$m_1 = \frac{-1}{2}$

$m_1 m_2 = a$

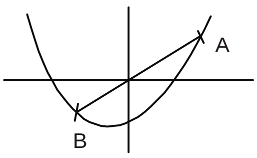
$m_1 \times m_1^2 = a$

$m_1^3 = a$

$\left(\frac{-1}{2}\right)^3 = a$

$\therefore \quad a = \frac{-1}{8}$.

6. $y = 2x^2 + x - 2$



Let point A is $(t_1, 2t_1^2 + t_1 - 2)$ and point B is $(t_2, 2t_2^2 + t_2 - 2)$

Given : Mid point of AB is (0, 0)

So, $\frac{t_1 + t_2}{2} = 0 \Rightarrow t_1 + t_2 = 0$

$\frac{2t_1^2 + t_1 - 2 + 2t_2^2 + t_2 - 2}{2} = 0$

$2(t_1^2 + t_2^2) + (t_1 + t_2) - 4 = 0$

$2[(t_1 + t_2)^2 - 2t_1t_2] + 0 - 4 = 0$

$2[0 - 2t_1t_2] = 4$

$-4t_1t_2 = 4$

$t_1 t_2 = -1$

$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$

$= 0 - 4(-1)$

$= 4$

$t_1 - t_2 = 2$.

Distance AB :

$AB = \sqrt{(t_1 - t_2)^2 + (2t_1^2 + t_1 - 2 - 2t_2^2 - t_2 + 2)^2}$

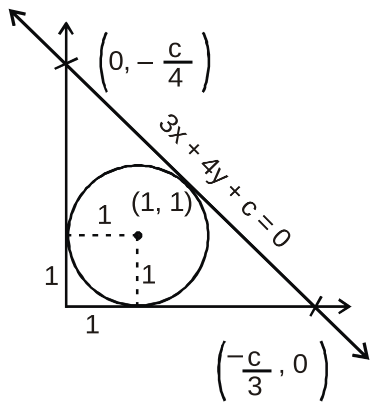
$AB = \sqrt{(t_1 - t_2)^2 + \left[2(t_1^2 - t_2^2) + (t_1 - t_2)\right]^2}$

AB = $\sqrt{(2)^2 + [2(t_1+t_2)(t_1-t_2)+(t_1-t_2)]^2}$

AB = $\sqrt{4+[2(0)+(2)]^2}$

AB = $\sqrt{4+4}$ = $\sqrt{8}$ = $2\sqrt{2}$.

7.



Perperndicular from (1, 1) on line 3x + 4y + c = 0

$\frac{|3+4+c|}{\sqrt{9+16}}$ = 1

| 7 + c | = 5

7 + c = ± 5

c = – 5 – 7

c = – 12

| c | = 12.

8. p $\cos^2\left(\frac{R}{2}\right)$ + r $\cos^2\left(\frac{P}{2}\right)$

p $\frac{s(s-r)}{pq}$ + r $\frac{s(s-p)}{qr}$

= $\frac{s}{q}$ [s – r + s – p]

= $\frac{s}{q}$ [2s – r – p]

= $\frac{s}{q}$ [p + q + r – r – p]

= s

= $\frac{p+q+r}{2}$.

9. $3\cos^2 x \sin^2 x - \sin^4 x - \cos^2 x = 0$

⇒ $3(1 - \sin^2 x)\sin^2 x - \sin^4 x - (1 - \sin^2 x) = 0$

⇒ $3\sin^2 x - 3\sin^4 x - \sin^4 x - 1 + \sin^2 x = 0$

⇒ $-4\sin^4 x + 4\sin^2 x - 1 = 0$

⇒ $4\sin^4 - 4\sin^2 x + 1 = 0$

⇒ $(2\sin^2 x - 1)^2 = 0$

⇒ $2\sin^2 x = 1$

$\Rightarrow \quad \sin^2 x = \frac{1}{2}$

$\Rightarrow \quad \sin x = \pm \frac{1}{\sqrt{2}}$

$\therefore$ sin x has 4 solution in the interval $[0, 2\pi]$.

10. $\theta = \frac{1}{2}\sin^{-1}\left(\frac{1}{4}\right)$

$\sin 2\theta = \frac{1}{4} \quad \Rightarrow \quad 2\sin\theta\cos\theta = \frac{1}{4}$

$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{8}$

Value of : $64\sin\theta + 64\cos\theta - 8\sec\theta - 8\text{cosec}\theta + \tan\theta + \cot\theta$

$= 64(\sin\theta + \cos\theta) - 8(\sec\theta + \text{cosec}\theta) + \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= 64(\sin\theta + \cos\theta) - 8\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right) + \left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$

$= 64(\sin\theta + \cos\theta) - 8\left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right) + \frac{1}{\sin\theta\cos\theta}$

$= 64(\sin\theta + \cos\theta) - 8\frac{(\sin\theta + \cos\theta)}{1/8} + 8$

$= 64(\sin\theta + \cos\theta) - 64(\sin\theta + \cos\theta) + 8$

$= 8.$

11. $L = \lim_{n\to\infty}\left\{\lim_{x\to 0}\left(x\sin\left(\frac{1}{x}\right) + x^2\sin\left(\frac{1}{x^2}\right) + ... + x^n\sin\left(\frac{1}{x^n}\right)\right)\right\}$

$L = \lim_{n\to\infty}\left\{\lim_{x\to 0} x\sin\frac{1}{x} + \lim_{x\to 0} x^2\sin\left(\frac{1}{x^2}\right) + ... + \lim_{x\to 0} x^n\sin\left(\frac{1}{x^n}\right)\right\}$

$L = \lim_{n\to\infty}\{0 + 0 + 0 + ... + 0\}$

$L = 0$

12. $f(x) = \begin{cases} x^3 & \text{for } x \geq 1 \\ ax^2 + bx + c & \text{for } x < 1 \end{cases}$

$f'(x) = 3x^2 = 2ax + b$

$[f''(x)]_{x=1} = 6x = 2a$

$\Rightarrow \quad 6 = 2a$

$a = 3.$

13. $f(x) = x^3 - 3ax^2 + b$

$f'(x) = 3x^2 - 6ax$

For the f(x) to be increasing $f'(x) > 0$.

$3x^2 - 6ax > 0$

$3x(x - 2a) > 0$

$(x - 2a) > 0 \quad [\because x > 0]$

$x > 2a$

So, $a \geq 0$.

14. $\int_1^3 [x]\cos\left(\frac{\pi}{2}(x-[x])\right)dx$

$= \int_1^2 \cos\left(\frac{\pi}{2}(x-1)\right)dx + \int_2^3 2\cos\left(\frac{\pi}{2}(x-2)\right)dx$

$= \left[\frac{\sin\left(\frac{\pi}{2}(x-1)\right)}{\frac{\pi}{2}}\right]_1^2 + 2\left[\frac{\sin\left(\frac{\pi}{2}(x-2)\right)}{\frac{\pi}{2}}\right]_2^3$

$= \frac{2}{\pi}[1-0] + \frac{4}{\pi}[1-0]$

$= \frac{6}{\pi}$.

15. Let $A = \frac{1}{n}\{(2n+1)(2n+2) \text{-------} (2n+n)\}^{1/n}$

$\log A = \frac{1}{n}\lim_{n\to\infty} \log\left[\frac{(2n+1)}{n}\frac{(2n+2)}{n}\frac{(2n+3)}{n}\cdots\cdots\frac{(2n+n)}{n}\right]$

$\log A = \frac{1}{n}\lim_{n\to\infty} \log\left[\left(2+\frac{1}{n}\right)\left(2+\frac{2}{n}\right)\left(2+\frac{2}{n}\right)\cdots\cdots\left(2+\frac{n}{n}\right)\right]$

$\log A = \int_0^1 \log(2+x)dx$

$\log A = \left|x\log(2+x) - x + 2\log(2+x)\right|_0^1$

$\log A = [\log 3 - 1 + 2\log 3 - 2\log 2]$

$\log A = [\log 3 - \log e + \log 9 - \log 4]$

$\log A = [\log(27/4e)]$

$A = (27/4e)$.

16. $\overrightarrow{V_1} + \overrightarrow{V_2} + \overrightarrow{V_3} = 0$

$\left|\overrightarrow{V_1} - \overrightarrow{V_2}\right| = ?$

$\overrightarrow{V_1} + \overrightarrow{V_2} = -\overrightarrow{V_3}$

$\overrightarrow{V_1} + \overrightarrow{V_2} \cdot \overrightarrow{V_1} + \overrightarrow{V_2} = \left(-\overrightarrow{V_3}\right) \cdot \left(-\overrightarrow{V_3}\right)$

$\left|\overrightarrow{V_1}\right|^2 + \left|\overrightarrow{V_2}\right|^2 + 2\overrightarrow{V_2}.\overrightarrow{V_2} = \left|\overrightarrow{V_3}\right|^2$

$1 + 1 + 2\overrightarrow{V_2}.\overrightarrow{V_2} = 1$

$2\overrightarrow{V_2}.\overrightarrow{V_2} = -1$

$\overrightarrow{V_2}.\overrightarrow{V_2} = \frac{-1}{2}$

$$\left|\overrightarrow{V_1}-\overrightarrow{V_2}\right|^2 = \left|\overrightarrow{V_1}\right|^2 + \left|\overrightarrow{V_2}\right|^2 - 2\overrightarrow{V_1}.\overrightarrow{V_2}$$

$= 1 + 1 - 2 \times \frac{-1}{2}$

$= 1 + 1 + 1 = 3$

$\therefore \quad \left|\overrightarrow{V_1}-\overrightarrow{V_2}\right|^2 = 3$

$\left|\overrightarrow{V_1}-\overrightarrow{V_2}\right| = \sqrt{3}$.

17. Total number of possible cases = 4 + 3 + 2 + 1 = 10
Favourable cases = {(H T T H T),(T H T H T), (T T H H T), (T T T H H)}
Total number of favourable cases = 4

$\therefore \quad p(4^{th}\text{ toss is a head}) = \frac{4}{10}$

$= \frac{2}{5}$.

18. a + b + c = even

Case 1 : All taken are even : $^5C_3 = 10$

Case 2 : One even + 2 odd : $^5C_1 \times {}^5C_2 = 50$

Total = 10 + 50 = 60.

19. 2log(x – 2y) = logx + logy
$\log(x - 2y)^2 = \log(xy)$
$(x - 2y)^2 = xy$
$x^2 + 4y^2 - 4xy = xy$
$x^2 + 4y^2 - 5xy = 0$
$x^2 - 4xy - xy + 4y^2 = 0$
$x(x - 4y) - y(x - 4y) = 0$
$(x - y)(x - 4y) = 0$
x = y (N.P.), x = 4y

$\frac{x}{y} = 4$.

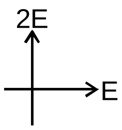
20. Let x, y and z are 2n, $n^2 - 1$ & $n^2 + 1$.
Let x = 2n, $y = n^2 - 1$ and $z = n^2 + 1$.
Option (A) : 2 does not divide x .
Clearly 2 divides x. So option (A) is not true.
Ex : 8, 15 and 17 are pythogorean triplet. 2 divides 8.
Option (B) : 2 does not divide z(x + y)
$(n^2 + 1)(2n + n^2 - 1)$ which is an odd number so, 2 does not divides z(x + y).
∴ So, option (B) is true.
Option (C) : 4 divides x + y + z
Ex : 5, 12, 13 are the pythogorean triplet.
Clearly 5 + 12 + 13 = 30, and 4 does not divides 30.
∴So, option (C) is not true.
Option (D) : 8 divides x + y + z
Ex : 8, 15, 17 are the pythogorean triplet.
Clearly 8 + 15 + 17 = 40, and 8 divides 40.
∴ So, option (B) is true.

PHYSICS

21. By work energy theorem, $W = \Delta K$
$mgh - w_f = 0$
$w_f = mgh$

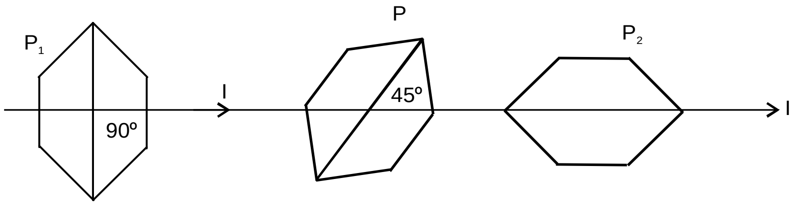
22. $E = -13.6 \times \frac{z^2}{n^2} = -13.6 \times \frac{4}{1} = 54.4\text{eV}$ Ans (D)

23.



$V \downarrow$ $F_B \uparrow$
deflected in –x direction (Ans.B)

25.



By Malus's law, $I' = I_0 \cos^2 45^\circ$
initensity after P_1

$I = I\cos^2 45 = \frac{I}{2}$

initensity after P_2

$I_2 = \frac{I_1}{2}\cos^2 45 = \frac{I}{4}$ (Ans : B)

26. Due to inertia ballon displace in the direction of motion of the bus.

27. (D) As for hydrogen like atom

$$\frac{1}{\lambda} = R.z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$ (Here z = 1, same for deuterium and hydrogen)

So, $\frac{1}{\lambda}$ will be same for deuterium as that for hydrogen.

28. Bulk modulus, $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

As in isothermal process, PV = K
On differentiating, PdV + VdP = 0

$$\frac{\Delta V}{V} = -\frac{\Delta P}{P}$$

Hence $B = \frac{-\Delta P}{\left(-\frac{\Delta P}{P}\right)} = P$

29. From brewster's law $\tan I = \frac{\mu_2}{\mu_1}$

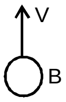
30. Cheaking dimensionally all the formula

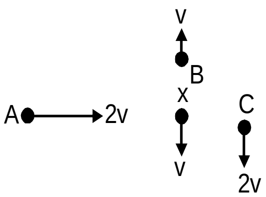
$$\frac{\lambda g}{2\pi} = [L]\left[\frac{L}{T^2}\right] = [L^2T^{-2}] \quad \text{and} \quad \sqrt{\frac{\sigma g}{\rho}} = \left[\frac{[MLT^{-2}]}{[L]} \times \frac{[L^1T^{-2}]}{[ML^{-3}]}\right]^{1/2} = [L^2T^{-2}] \qquad \text{(dimension of } v^2\text{)}$$

$$\frac{2\pi\sigma}{\lambda\rho} = \frac{[M^1L^1T^{-2}][L^{-1}]}{[L][ML^{-3}]} = [L^2T^{-2}]$$

Only (A) option is correct dimensionally

33. As v & d depends on frame of referance but change in KE and hence heat does not depend.

34. $V >> \sqrt{2gr}$ 

36. 

$$v_B = v_0\left[1 - \frac{2v}{c}\right]$$

$$v_C = v_o\left[1 - \frac{v}{c}\right]$$

$$V_A = V_0\left[1 + \frac{2v\cos\theta}{c}\right]$$

$V_A > V_C > V_B$ (Answer : C)

37. By dalton's law of partion pressure. (Ans : D)

38. $E_{in} = 0 \quad E_{out} = \frac{KQ}{r^2}$

$V_{in} = \frac{KQ}{R} = \text{const.} \qquad V_{out} = \frac{KQ}{r}$

X is for electric field versus distance graph
Y is for potential versus distance graph
Ans. is (B)

39. $E_{net} = 2\frac{kq}{(x^2 + y^2)^{3/2}}[x]$

$E_{net} = \frac{2kq}{x^2}$

CHEMISTRY

41. For weak acid $[H^+] = C\alpha$

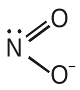
$[H^+] = 0.1 \times 0.1$

$[H^+] = 10^{-2}$

$pH = -\log[H^+]$

$pH = -\log[10^{-2}]$

$pH = 2$

42.  → it has two donor side [O & N]

43. H_3O^+

sp^3 hybridisation, due to presence of 1 lp

structure is trigonal pyramidal.

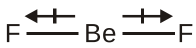
45. (D) Ba > Ca > Mg > Be

Solubility increases down the group due to increase of size of cation.

46. C > Be > B > Li

$Z^{eff} \uparrow$ I E $\uparrow$

47. Dipole moment $\mu = e \times d$



$\mu = 0$

48. $[NiCl_4]^{2-}$ is tetrahedral.

50. $mvr = \frac{nh}{2\pi}$ where n = 1, 2, 3 -------

51. (A) both valencies occupy by the same group.

52.

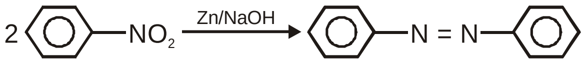
p-nitrophenol	-o-nitrophenol	m-nitrophenol
–I & –M	–I & –M (intramolecular H-bond)	–I

54. (A) $CH_2 - O - COR$ / $CH - O - COR$ / $CH - O - COR$ + 3NaOH ⟶ 3RCOONa (Salt of fatty acid) + $CH_2 - OH$ / $CH - OH$ / $CH_2 - OH$ (glycerol)

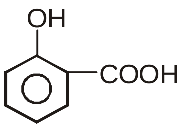
55. (B) Branching ↑ surface area ↓

vanderwaals' forces ↓ b.p. ↓

56. (C) Zn/NaOH



58. $CH_3 - CH_2 - CH(OH) - COOH$

59. (C) 

[contains phenolic group]

60. (B) stability of carbanions $\alpha \ \frac{-I \ \& \ -M}{+I \ \& \ +M}$

PART - II (2 Mark)

MATHEMATICS

81. ab = 2 (a + b)

$$\frac{a+b}{ab} = \frac{1}{2}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2} \qquad \text{...... (i)}$$

bc = 3(b + c)

$$\frac{b+c}{bc} = \frac{1}{3}$$

$$\frac{1}{b} + \frac{1}{c} = \frac{1}{3} \qquad \text{...... (ii)}$$

ca = 4(c + a)

$$\frac{c+a}{ca} = \frac{1}{4}$$

$$\frac{1}{c} + \frac{1}{a} = \frac{1}{4} \qquad \text{...... (iii)}$$

Add (i), (ii) and (iii)

$$2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{6+4+3}{24}$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{13}{24} \qquad \text{...... (iv)}$$

From (i) and (iv)

c = 24

From (ii) and (iv)

$a = \frac{24}{5}$

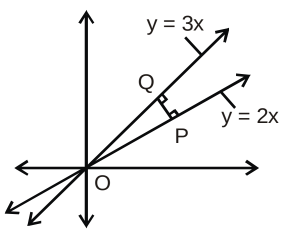
From (iii) and (iv)

$b = \frac{24}{7}$

So, value of $5a + 7b + c$
$= 24 + 24 + 24 = 72.$

82. $x^2 + ax + b = 0$
$\alpha + \beta = -a, \alpha\beta = b$
$x^4 + ax^3 + cx^2 + dx + e = 0$
Roots are $\alpha + \beta, \alpha - \beta, -\alpha + \beta$ and $-\alpha - \beta$
Sum of roots = – a
$\alpha + \beta + \alpha - \beta - \alpha + \beta - \alpha - \beta = -a$
$a = 0 \Rightarrow \alpha + \beta = 0$
Sum of roots taken two at a time = + c
$(\alpha + \beta)(\alpha - \beta) + (\alpha - \beta)(-\alpha + \beta) + (-\alpha + \beta)(-\alpha - \beta) + (-\alpha - \beta)(\alpha + \beta) + (\alpha + \beta)(-\alpha + \beta) + (\alpha - \beta)(-\alpha + \beta) = c$
$0 - (\alpha + \beta)^2 + 0 + 0 + 0 - (\alpha + \beta)^2 = c$
$-2(\alpha - \beta)^2 = c$
$-2[\alpha^2 + \beta^2 - 2\alpha\beta] = c$
$-2[(\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta] = c$
$-2[-4\alpha\beta] = c$
$c = 8\alpha\beta$
$c = 8b \neq 0$
Product of roots taken three at a time = – d
$(\alpha + \beta)(\alpha - \beta)(-\alpha + \beta) + (\alpha + \beta)(-\alpha + \beta)(-\alpha - \beta) + (-\alpha - \beta)(\alpha + \beta) + (\alpha + \beta)(-\alpha + \beta)(-\alpha + \beta)(-\alpha + \beta) = -d$
$0 + 0 + 0 + 0 = -d$
$d = 0$
Product of roots = e
$(\alpha + \beta)(\alpha - \beta)(-\alpha + \beta)(-\alpha - \beta) = e$
$e = 0$
So, c = 0 is the false statement

83.



PQ = 5
Let point Q is (x, 3x)

Given QP = 5 and QP $\perp$ line 2x – y = 0

$$\frac{|2x - 3x|}{\sqrt{4+1}} = 5$$

x = $5\sqrt{5}$

Therefore point Q is ($5\sqrt{5}$, $15\sqrt{5}$)

$\therefore$ OQ = $\sqrt{(5\sqrt{5})^2 + (15\sqrt{5})^2}$

= $\sqrt{125+1125}$

= $\sqrt{1250}$

= $25\sqrt{2}$

$\therefore$ In ΔOPQ

$OQ^2 = OP^2 + PQ^2$

$(25\sqrt{2})^2 = OP^2 + (5)^2$

$1250 - 25 = OP^2$

$OP^2 = 1225$

OP = 35 cm.

84. $\cot\frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-c)(s-c)}}$

$\cot\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-b)}}$

$\therefore$ $\cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{s^2(s-b)(s-c)}{(s-a)^2(s-b)(s-c)}}$

$\cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{s^2}{(s-a)^2}}$

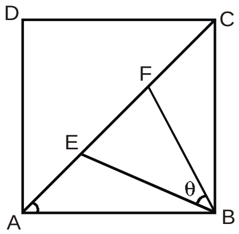
$\cot\frac{B}{2}\cot\frac{C}{2} = \frac{s}{s-a}$

Given : b + c = 3a

$$s = \frac{a+b+c}{2} = \frac{a+3a}{2} = 2a.$$

$\therefore$ $\cot\frac{B}{2}\cot\frac{C}{2} = \frac{2a}{2a-a} = \frac{2a}{a} = 2.$

85.



Given : AE = EF = FC = $\frac{AC}{3}$

Let, AB = BC = CD = DA = x

Then, AC = $\sqrt{2}$ x

Let BE = a

In ΔAEB

$$\cos 45^\circ = \frac{\frac{2x^2}{9} + x^2 - a^2}{2\frac{\sqrt{2}x}{3}.x}$$

$$\frac{1}{\sqrt{2}} = \frac{\frac{11x^2}{9} - a^2}{\frac{2\sqrt{2}x^2}{3}}$$

$$\frac{2x^2}{3} = \frac{11x^2}{9} - a^2$$

$$a^2 = \frac{11x^2}{9} - \frac{2x^2}{3}$$

$$a^2 = \frac{11x^2 - 6x^2}{9} = \frac{5x^2}{9}$$

$$a = \frac{\sqrt{5}}{3}x$$

Similarily BF = a = $\frac{\sqrt{5}x}{3}$.

In ΔBEF

$$BE = BF = \frac{\sqrt{5}x}{3}$$

and EF = $\frac{\sqrt{2}x}{3}$

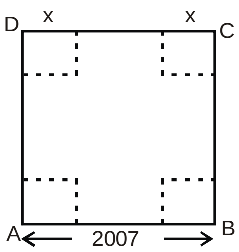
$$\cos\theta = \frac{BE^2 + BF^2 - EF^2}{2BE \times BF}$$

$$\cos\theta = \frac{\frac{5x^2}{9} + \frac{5x^2}{9} - \frac{2x^2}{9}}{2 \times \frac{\sqrt{5}x}{3} \times \frac{\sqrt{5}x}{3}}$$

$$\cos\theta = \frac{\frac{8x^2}{9}}{\frac{10x^2}{9}} = \frac{8}{10} = \frac{4}{5}.$$

$\therefore$ $\tan\theta = \frac{3}{4}$.

86.



v = Volume of box = $(2007 - 2x)^2 \times x$

$$\frac{dv}{dx} = 2(2007 - 2x)(-2x) + (2007 - 2x)^2$$

$= (2007 - 2x)[-4x + 2007 - 2x]$

$= (2007 - 2x)(2007 - 6x)$

For the maximum volume

$$\frac{dv}{dx} = 0$$

$(2007 - 2x)(2007 - 6x) = 0$

$$x = \frac{2007}{2} \text{ or } x = \frac{2007}{6}$$

$$\frac{d^2v}{dx^2} = (2007 - 6x)(-2) + (2007 - 2x)(-6)$$

$= -4014 + 12x - 12042 + 12x$

$= 24x - 16056$

$$\left(\frac{d^2v}{dx^2}\right)_{x=\frac{2007}{6}} = 24 \times \frac{2007}{6} - 16056$$

$= 8028 - 16056 < 0$

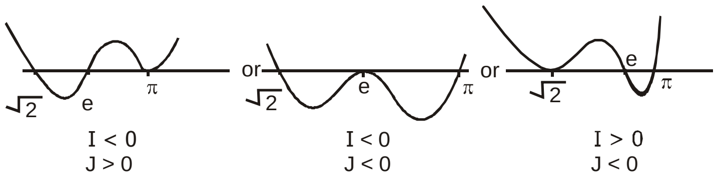
$\therefore$ Volume is maximum when $x = \frac{2007}{6} = \frac{669}{2}$ cm.

87. $S = y^2 - 4x$

S (0, 2) = 4, S (1/2, 5/2) = 25/4 – 2 = 17/4, S (5/2, 9/2) = 81/4 – 20/2 = 41/4 and S (1, 3) = 9 – 4 = 5

It is clear that (0, 2) is closest to parabola $y^2 = 4x$.

88.



So, I and J can be both negative but not both positive.

89. $|z_1 z_2 + z_2 z_3 + z_3 z_1| = |z_1 z_2 z_3| \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$ $\quad [\because |z_1| = |z_2| = |z_3|]$

$= |z_1 + z_2 + z_3|$.

90. So, the angles will be 99, 108, ..., 153, 162

$\therefore$ Sum of largest two angles = 153 + 162 = 315.

PHYSICS

91. Rotational inertia does not depend on speed so rotational inertia of the cylinder about its axis will be $\frac{1}{2} MR^2$.

92. Air column is closed at one end, so fundamental frequency

$$n = \frac{v}{4\ell} = \frac{330}{4 \times 66 \times 10^{-2}} = 125 \text{ Hz}$$

$\because n_1 : n_2 : n_3$ .... = 1 : 3 : 5 ....

So, possible frequencies are

125, 375, 675, 875, 1125 (> 1kHz)

So only possible frequenceies are 3.

93. Time period of pendulum on the surface of the earth

$$T_0 = 2p\, 2\pi\sqrt{\frac{\ell}{g}} \qquad \text{........(i)}$$

At a depth d, $\quad g' = g\left(1 - \frac{d}{R}\right)$ and $\ell' = \ell(1 + \alpha\theta)$

So new time period,

$$T = 2\pi\sqrt{\frac{\ell'}{g'}} = 2\pi\sqrt{\frac{\ell(1+\alpha\theta)}{g(1-d/R)}} = (1+\alpha\theta)^{1/2}(1-d/R)^{-1/2} = \left(1 + \frac{1}{2}\alpha\theta\right)\left(1 + \frac{d}{2R}\right)$$

94. For first darkness,

$d \sin\theta = \lambda \quad \Rightarrow \sin\theta = \frac{6\times10^{-7}}{2\times10^{-6}} = 0.3 \quad \theta = 17^{o}$

95. In LC circuit time period, $T = 2\pi\sqrt{Lc}$

$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{LC}$

96. For insulated material capacity,

$C = \frac{\varepsilon_0 A}{(d - a + a/\varepsilon_r)}$

But of metal, $\varepsilon_r = \infty$

So, $C = \frac{\varepsilon_0 A}{(d-a)}$

97. $A_{max} = g$ (block and piston remains together)

$A\omega^2 = g$

$A_{max} = \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2}$

98. $z = 0$, as $\vec{P}$ and $\vec{E}$ in same direction

$\frac{dE}{dZ} = 10^5 \quad \Rightarrow F = P\frac{dE}{dZ} = 10^{-7} \times 10^5 = 10^{-2}$ N

99. Inside the atom,

Electric field due to +v charge , $E(r) = \frac{kz_e}{r^2}$

Due to –ve charge, $E(r) = -K\frac{ze\, r}{r^3}$

So, $E(r) = \frac{ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right) \quad r < R$

For out side atom

$Q_{net} = 0$

$E(r) = 0 \quad r > R$

100. $\Delta Q = c_P \Delta T$

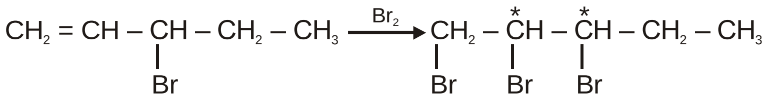
$= \left(\frac{f}{2}+1\right)R\Delta T = \left(\frac{f}{2}+1\right)P\Delta V$

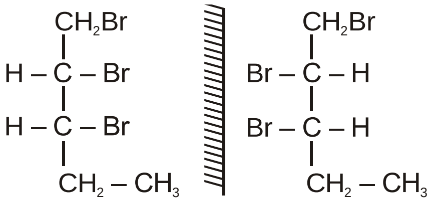
$\left(\frac{f}{2}+1\right) = \frac{\Delta Q}{PdV} = \frac{10.61}{1.01\times10^5 \times 3\times10^{-5}}$

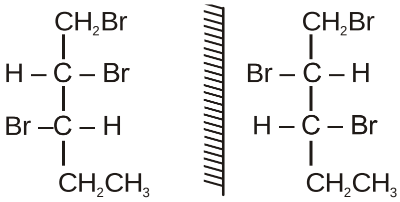
f = 4.3

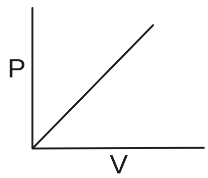
So, gas is a mixture of mono and diatomic molecules.

CHEMISTRY

101. 





102. 

according to above graph $P \alpha V$

This contradict the Boyle's law

103. $\because$ Molarity of acetic acid = 0.1 M [Suppose volume of acetic acid = x]

$\therefore$ no. of millimoles present in x ml of acetic acid = 0.1x

$\because$ Molarity of sodium acetate = 0.2

$\therefore$ no. of millimoles present in 10 ml of sodium acetate = 0.2 × 10 = 2

$$pH = pKa + \log \frac{[Salt]}{[Acid]}$$

$$4.91 = 4.76 + \log \frac{\frac{2}{10+x}}{\frac{0.1x}{10+x}} \qquad [\because \text{ total volume} = (10 + x) \text{ ml}]$$

$$0.15 = \log \frac{2}{0.1x}$$

0.15 = log20 – logx

0.15 = 1.3010 – logx

logx = 1.151

x = antilog (1.151)

x = 14.16 ml

$x \approx 14.2$ ml

104. $Ba(OH)_2 + SO_2 \longrightarrow BaSO_3 + H_2O$
(White ppt.)

$Ba(OH)_2 + SO_3 \longrightarrow BaSO_4 + H_2O$
(White ppt.)

$BaSO_3$ is soluble in dil HCl but $BaSO_4$ is not soluble in dil HCl . After filteration , filtrate contains $BaSO_3$

Br_2 + $H_2O \longrightarrow 2HBr + O$
(Bromine water)

$BaSO_3 + O \longrightarrow BaSO_4$
White ppt

105. $\lambda \propto \frac{1}{\sqrt{V}}$

according to question given potential energy = kinetic energy

$eV = \frac{1}{2}mv^2$ ----- (1)

$eV = \frac{P^2}{2m}$ $[\because P = mv]$

$eV = \frac{h^2}{\lambda^2 2m}$ $[\therefore p = \frac{h}{\lambda}]$

$\lambda^2 = \frac{h^2}{2meV}$

$\lambda = \frac{h}{\sqrt{2meV}}$

$\because$ e, m, h are constants

so $\lambda \propto \frac{1}{\sqrt{V}}$

106. $K_b = 0.52$ k Kg mol^{-1} (given)
$\Delta T_b = 0.052$ K (given)
m = given

$\therefore \Delta T_b = \frac{1000 K_b w_0}{mW}$

It means we can calculate $\frac{w_0}{W} = \frac{\Delta T_b \times m}{1000 K_b}$ ---------------- (i)

Relative lowering of vapour pressure of water

$\Rightarrow \frac{P_o - P_s}{P_o} = \frac{w_0}{W} \times \frac{M}{m}$ ------(ii)

M = molecular mass of water = 18
m = given

$\frac{w_0}{W}$ = by equation (i)

so we can calculate relative lowering of V.P. of water by equation (ii)

osmotic pressure $\pi = \frac{n}{V}ST$

But V is unknown, so we can't find out the value of osmotic pressure.

freezing point depression $\Delta T_f = \frac{1000 K_f w_o}{mW}$

But value of K_f is not given so we can't calculate the magnitude of freezing point depression.

107. **Test of Nitrogen**

Organic compound Fused with Na + $FeSO_4$ + $FeCl_3$	no ppt. observed	Nitrogen absent

Test of sulphur

Organic compound fused with Na + sodium nitroprusside	violet colour observed	S present

Thus according to question organic compound contains sulphur only.

108.