HINTS & SOLUTIONS (PRACTICE PAPER-3)

ANSWER KEY

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	B	C	B	D	B	D	A	A	C	B	B	D	C	A
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	B	C	D	B	B	D	D	C	C	D	C	A	D
Ques.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	D	C	A	B	B	C	A	A	A	B	C	D	C	C	D
Ques.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	C	B	A	B	A	D	C	D	B	D	A	D	B	B
Ques.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	A	C	C	C	B	B	C	D	A	D	D	D	B	A	B
Ques.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	B	B	D	C	A	C	B	C	B	B	D	C	D	B	C
Ques.	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Ans.	D	A	B	C	D	A	C	C	B	B	B	D	A	A	B
Ques.	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	C	A	C	D	B	C	B	A	B	A	D	B	A	C	C

PART-I (1 Mark)

MATHEMATICS

1. Given : a_1, a_2, a_3 ....AP and a_1, a_2, a_4, a_8 ......GP.

Let common difference of A.P. = d

$a_2 = a_1 + d$

$a_4 = a_1 + 3d$

$a_8 = a_1 + 7d$

$$\therefore \quad \frac{a_2}{a_1} = \frac{a_4}{a_2} = \frac{a_8}{a_4} = r$$

$$\frac{a_1 + d}{a_1} = \frac{a_1 + 3d}{a_1 + d} = \frac{a_1 + 7d}{a_1 + 3d} = r$$

$(a_1 + d)^2 = a_1(a_1 + 3d)$

$a_1^2 + d^2 + 2\,a_1 d = a_1^2 + 3a_1\,d$

$d^2 = a_1\,d \qquad (d \neq 0)$

$d = a_1 \qquad$ ....(i)

Hence, $\dfrac{a_2}{a_1} = r$; $\dfrac{a_1 + d}{a_1} = r$

$$\frac{a_1 + a_1}{a_1} = r \qquad \text{(using (i))}$$

r = 2.

2. $\frac{T_k}{T_{k-1}} = \frac{\sqrt{101}}{k}$ till k = 10

$T_k > T_{k-1}$
Let k = 11
$T_{11} < T_{10} \Rightarrow T_{10}$ is maximum at k = 10.

3. $x = \sqrt{2} + \sqrt{3} + \sqrt{6}$

$\left(x - \sqrt{2}\right)^2 = \left(\sqrt{3} + \sqrt{6}\right)^2$

$x^2 + 2 - 2\sqrt{2} = 9 + 6\sqrt{2}$

$x^2 - 7 = 8\sqrt{2}$

$(x^2 - 7)^2 = 64 \times 2$

So, smallest possible value of n is 4.

4. Let the three players are A, B, C.
Now, each player get 0 score after playing 9 games. It happened only when each player wins 3 games and loss 6.
So,

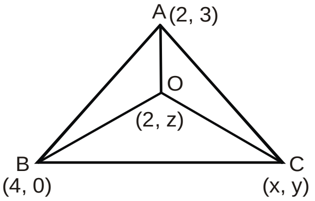
A win 3 games out of 9 $\rightarrow {}^9C_3$
B win 3 games out of remaining 6 $\rightarrow {}^6C_3$
C win 3 games out of remaining 3 $\rightarrow {}^3C_3$

So, required way = ${}^9C_3 \times {}^6C_3 \times {}^3C_3$

$= \frac{9 \times 8 \times 7 \times 6!}{3 \times 2 \times 1 \times 6!} \times \frac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \times 3!} \times 1$

= 1680.

5.



O is circumcenter

$\therefore$ OA = OB = OC = circumradius

$(2 - 2)^2 + (z - 3)^2 = (4 - 2)^2 + (0 - z)^2$

$z^2 + 9 - 6z = 4 + z^2$

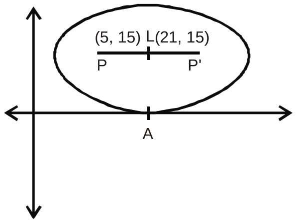
$9 - 6z = 4$

$5 = 6z$

$\frac{5}{6} = z$

$\therefore$ Circumcenter = $\sqrt{(z-3)^2 + (2-2)^2} = | z - 3 | = \left|\frac{5}{6} - 3\right| = \frac{13}{6}$.

6.



Mid point of PP' $= \left(\frac{5+21}{2}, \frac{15+15}{2}\right)$

L = (13, 15)

∴ Point A will be (13, 0)

By property PA + PA' = 2a

PA = $\sqrt{(13-5)^2 + (0-15)^2}$

$= \sqrt{64+225}$

$= \sqrt{289}$ = 17 cm

PA' = $\sqrt{(13-21)^2 + (0-15)^2}$

$= \sqrt{64+225}$

$= \sqrt{289}$ = 17 cm

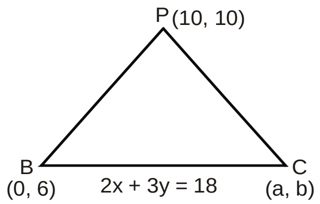
∴ 2a = PA + PA'

2a = 17 + 17

2a = 34 cm

So, length of major axis = 2a = 34 cm.

7.



PB = PC

$(10-0)^2 + (10-6)^2 = (a-10)^2 + (b-10)^2$

$100 + 16 = a^2 + 100 - 20a + b^2 + 100 - 20b$

$a^2 + b^2 - 20a - 20b + 84 = 0$ ....(i)

Also (a, b) i.e. on 2x + 3y = 18

2a + 3b = 18

$a = 9 - \frac{3b}{2}$

Using equation (i)

$\left(9 - \frac{3b}{2}\right)^2 + b^2 - 20\left(9 - \frac{3b}{2}\right) - 20b + 84 = 0$

$81 + \frac{9b^2}{4} - 27b + b^2 - 180 + 30b - 20b + 84 = 0$

$\frac{13b^2}{4} - 17b - 15 = 0$

$13b^2 - 68b - 60 = 0$

$13b^2 - 78b + 10b - 60 = 0$
$13b(b - 6) + 10(b - 6) = 0$

$b = 6$ or $b = \frac{-10}{13}$

$\therefore$ When $b = 6$, then $a = 9 - \frac{3 \times 6}{2} = 0$

When $b = \frac{-10}{13}$, then $a = 9 + \frac{3 \times 10}{2 \times 13} = 9 + \frac{30}{26} = \frac{132}{13}$

$\therefore 8a + 2b = 8 \times \frac{132}{13} + 2 \times \frac{-10}{13}$

$= \frac{1056 - 20}{13} \approx 79$.

8. $\operatorname{cosec}^2(\alpha + \beta) - \sin^2(\beta - \alpha) + \sin^2(2\alpha - \beta) = \cos^2(\alpha - \beta)$

$\operatorname{cosec}^2(\alpha + \beta) + \sin^2(2\alpha - \beta) = \frac{\cos^2(\alpha - \beta) + \sin^2(\beta - \alpha)}{1}$

$\operatorname{cosec}^2(\alpha + \beta) = 1 - \sin^2(2\alpha - \beta)$
$\operatorname{cosec}^2(\alpha + \beta) = \cos^2(2\alpha - \beta)$
Minimum value of $\operatorname{cosec}^2(\alpha + \beta)$ is 1 and maximum value of $\cos^2(2\alpha - \beta)$ is 1.
$\therefore$ They will be equal for the value 1.

$\alpha + \beta = \frac{\pi}{2}$ .....(i)

$2\alpha - \beta = 0$ .....(ii)

By adding (i) & (ii)

$3\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{6}$

$\beta = \frac{\pi}{3}$

$\therefore$ $\sin(\alpha - \beta) = \sin\left(\frac{\pi}{6} - \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{6}\right) = \frac{-1}{2}$.

9. $\sin x + \sin y = \frac{7}{5}$ ....(1)

$\cos x + \cos y = \frac{1}{5}$ ....(2)

By $(1)^2 + (2)^2$ we get
$2 + 2\sin x \sin y + 2\cos x \cos y = 2$

sinx siny + cosx cosy = 0
cos(x – y) = 0
∴ x – y = 90º
By (1) × (2) we get

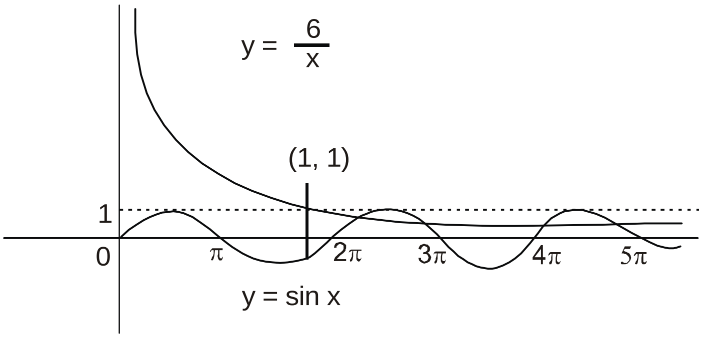
$$\sin x \cos x + \sin x \cos y + \sin y \cos x + \sin y \cos y = \frac{7}{25}$$

$$\sin(90 + y)\cos x + \sin(x + y) + \sin(x - 90) \cos y = \frac{7}{25}$$

$$\cos y \cos x + \sin(x + y) - \cos x \cos y = \frac{7}{25}$$

$$\sin(x + y) = \frac{7}{25}.$$

10.



Clearly, curve meet each other twice in 2π – 3π
4π – 5π
6π – 7π
8π – 9π
10π – 11π

∴ Total 10 Times.

11. f(x) is differentiable on R.
So, it will be contincous on R.
Continuity at x = 0
LHL

$$\lim_{x\to 0^-} \frac{\sin x^2}{x}$$

Put x = 0 – h, then h → 0

$$\lim_{h\to 0} \frac{\sin(0-h)^2}{-h}$$

$$\lim_{h\to 0} \frac{\sin h}{-h} \times \frac{h}{h} = 0$$

RHL

$$\lim_{x\to 0^+} x^2 + ax + b$$

Put x = 0 + h, then h → 0

$$\lim_{h\to 0} h^2 + ah + b = b$$

Value of f(x) at x = 0
f(0) = b.

$\because$ f(x) is contineous at x = 0

$\therefore$ LHL = RHL = f(0)

0 = b = b

b = 0

Differentiability at x = 0

LHD

$$\lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$$

$$\lim_{h\to 0} \frac{\frac{\sin h^2}{-h} - b}{-h}$$

$$\lim_{h\to 0} \frac{\sin h^2}{h^2} = 1$$

RHD

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$\lim_{h\to 0} \frac{h^2+ah+b-b}{h}$$

$$\lim_{h\to 0} \frac{h(h+a)}{h} = a.$$

$\because$ f(x)is differentiable at x = 0, LHD = RHD

a = 1.

12. Let point $p(x_1, y_1)$ is on the curve $y^2 = 4x$.

$\therefore$ $y_1^2 = 4x_1 \Rightarrow x_1 = \frac{y_1^2}{4}$

$$PA = \sqrt{(x_1-0)^2+(y_1-3)^2}$$

$AP^2 = x_1^2 + y_1^2 - 6y_1 + 9$

$AP^2 = x_1^2 + y_1^2 - 6y_1 + 9$

Let AP = z

$z^2 = x_1^2 + y_1^2 - 6y_1 + 9$

$$z^2 = \left(\frac{y_1^2}{4}\right)^2 + y_1^2 - 6y_1 + 9$$

$$z^2 = \frac{y_1^4}{16} + y_1^2 - 6y_1 + 9$$

Diff. w.r.t. y_1

$$2z\frac{dz}{dy_1} = \frac{4y_1^3}{16} + 2y_1 - 6$$

$$2z\frac{dz}{dy_1} = \frac{y_1^3}{4} + 2y_1 - 6$$

$$= \frac{y_1^3+8y_1-24}{4}$$

$$2z\frac{dz}{dy_1} = (y_1-2)(y_1^2+2y_1+12)$$

For the critical points

$$\frac{dz}{dy_1} = 0$$

$(y_1 - 2)(y_1^2 + 2y_1 + 12) = 0$

$y_1 = 2$

$\therefore \quad y_1^2 = 4x_1$

$\Rightarrow \quad (2)^2 = 4x_1$

$\Rightarrow \quad x_1 = 1.$

$$2\left(\frac{dz}{dy_1}\right)^2 + 2z\frac{d^2z}{dy_1^2} = (y_1^2 + 2y_1 + 12) + (y_1 - 2)(2y_1 + 2)$$

$$= y_1^2 + 2y_1 + 12 + 2y_1^2 - 4y_1 + 2y_1 - 4$$

$$= 3y_1^2 + 8.$$

when $y_1 = 2$ and $\frac{dz}{dy_1} = 0$

$$\frac{d^2z}{dy_1^2} > 0$$

$\therefore \quad$ z is min at (1, 2)

Minimum distance $= \sqrt{(1-0)^2 + (2-3)^2} = \sqrt{1+1} = \sqrt{2}$.

13. We can find the answer through option as the sum of weight of packet taken from trucks is 1022870 gm and its unit digit is 0. The truck that have heavier bags have unit digit 0. So, the truck have lighter bags in which the sum of weight of bags must have unit digit 0.
So, according to option D. i.e. truck no. 2, 8
Track 2 have 2^1 bags and total weight = $2^1 \times 999$ gm = ...8 gm
Truck have 2^7 bags and total weight = $2^7 \times 999 = 128 \times 999$ gm = ...2 gm
So, the unit digit of the weight contain by truck 2, 8 together is 0.

14. $\int_0^1 \cos(\pi x)\cos([2x]\pi)\,dx$

$$= \int_0^{1/2} \cos(\pi x)\cos 0\,dx + \int_{1/2}^1 \cos(\pi x)\cos \pi\,dx$$

$$= \int_0^{1/2} \cos(\pi x)\,dx - \int_{1/2}^1 \cos(\pi x)\,dx$$

$$= \left[\frac{\sin \pi x}{\pi}\right]_0^{1/2} - \left[\frac{\sin \pi x}{\pi}\right]_{1/2}^1$$

$$= \left[\frac{1}{\pi}\right] - \left[0 - \frac{1}{\pi}\right]$$

$$= \frac{2}{\pi}.$$

15. $I_N = \left[\frac{-\cos(nx)x^{10}}{n}\right]_0^1 + 10\int_0^1 \frac{\cos(nx)x^9 dx}{n}$

$$= 0 + \frac{10}{n}\left[\left[\frac{\sin(nx)x^9}{n}\right]_0^1 - \frac{9}{n}\int_0^1 \frac{\sin(nx)x^8 dx}{n}\right]$$

$$= -\frac{10 \times 9}{n^2}\left[\int_0^1 \sin(nx)x^8 dx\right]$$

$$= \frac{10!}{n^{10}}\left[\int_0^1 \sin(nx)dx\right]$$

= 0 as Denom $\to \infty$

16. $y = x^2$ & $y = 1 - x^2$

Point of intersections of graphs $x^2 = 1 - x^2$

$2x^2 = 1$

$x = \pm \frac{1}{\sqrt{2}}$

$\therefore$ Point of intersections = $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ and $\left(\frac{-1}{\sqrt{2}}, \frac{1}{2}\right)$.

Area under graph :

$$= \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left|x^2 - (1-x)^2\right|$$

$$= \int_{-1/\sqrt{2}}^{1/\sqrt{2}} 2x^2 - 1 = \left[\frac{2x^3}{3} - x\right]_{-1/\sqrt{2}}^{1/\sqrt{2}}$$

$$= 2 \times \left|\frac{2}{6\sqrt{2}} - \frac{1}{\sqrt{2}}\right|$$

$$= 2\left|\frac{2-6}{6\sqrt{2}}\right| = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}.$$

17. $\vec{a} = 3\vec{i} - 4\vec{k}$ and $\vec{b} = 5\vec{j} + 12\vec{k}$

$|\vec{a}| = \sqrt{(3)^2 + (4)^2} = 5$ and $|\vec{b}| = \sqrt{(5)^2 + (12)^2} = 13$

Therefore, a vector which bisects the angle is $13\left(3\vec{i} - 4\vec{k}\right) + 5\left(5\vec{j} + 12\vec{k}\right) = 39\vec{i} + 25\vec{j} + 8\vec{k}$.

19. Let $M = 2^{x_1}.3^{x_2}.5^{x_3}......$, $N = 2^{y_1}.3^{y_2}.5^{y_3}......$ x_i & $y_i \in$ w

$$\sum_{d/m} d = \sum_{d/N} d \Rightarrow \left(\frac{2^{x_1}-1}{2-1}\right)\left(\frac{3^{x_2}-1}{3-1}\right)\left(\frac{5^{x_3}-1}{5-1}\right)....... = \left(\frac{2^{y_1}-1}{2-1}\right)\left(\frac{3^{y_2}-1}{3-1}\right).........$$

$$\frac{\sum_{d/M} 1/d}{\sum_{d/N} 1/d} = \frac{\left(\frac{\left(\frac{1}{2}\right)^{x_1}-1}{1/2-1}\right)\left(\frac{\left(\frac{1}{3}\right)^{x_2}-1}{1/3-1}\right)....}{\left(\frac{\left(\frac{1}{2}\right)^{y_1}-1}{1/2-1}\right)\left(\frac{\left(\frac{1}{3}\right)^{y_2}-1}{1/3-1}\right)....}$$

$$\Rightarrow \frac{(2^{x_1}-1)(3^{x_2}-1).....}{2^{x_1}3^{x_2}.....\frac{(2^{y_1}-1)(3^{y_2}-1)......}{2^{y_1}3^{y_2}}} = \frac{N}{M} > 1.$$

20. $$m_{C_0} + \underbrace{m_{C_1} n}_{\text{only one element from A}} + m_{C_2} n^2 + ---mC_m n^m \Rightarrow (1+n)^m$$

PHYSICS

27. Sphere is hollow so potential inside sphere will be same as that on surface.

28. Heat supplied $Q = du + \Delta W$ (at contat pressure)

$PV = RT$

$PdV = RdT$

$$dT = \frac{PdV}{R}$$

$Q = C_V dT + PdV$

$$Q = C_V \frac{PdV}{R} + PdV$$

Work done at constant pressue, W

$W = PdV$

$$\frac{Q}{W} = \frac{C_V \frac{PdV}{R} + PdV}{PdV}$$

$$\frac{Q}{W} = \frac{CV}{R} + 1$$ (For diatomic gos, $CV = \frac{5}{2}R$)

$$\frac{Q}{W} = \frac{5R}{2R} + 1$$

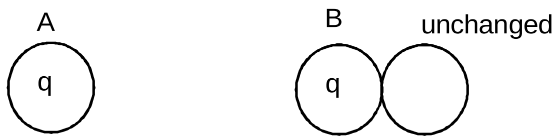
$$\frac{Q}{W} = \frac{7}{2} \quad \Rightarrow \quad \frac{W}{Q} = \frac{2}{7}$$

29. $$\frac{\frac{1}{\lambda_\alpha} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \text{ For lyman series}}{\frac{1}{\lambda_\beta} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \text{ For balmer series}} \qquad \left(\frac{1}{4} - \frac{1}{9} = \frac{9-4}{36}\right)$$

$$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{3/4}{5/36} = \frac{3}{1} \times \frac{9}{5} = \frac{27}{5}$$

$$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{5}{27}$$

30.



charge divides $\frac{q}{2}$ and $\frac{q}{2}$

Than, on touching $\frac{q}{2}$ sphere to q

Charge divides $\frac{q/2 + q}{2} = \frac{3q}{4}$

force between $\frac{3q}{4} \xrightarrow{R} \frac{q}{2}$

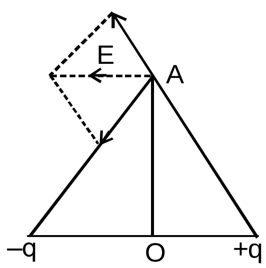
$$f' = \frac{K3q^2}{8R^2}$$

$$f' = \frac{3}{8} \times \frac{kq^2}{R^2} = \frac{3}{8}F$$

31. Intially block enters in the magnetic filed rate of change in flux will be constant so costant current will produce, when it mass in side the magnetic field there is no change in magnetic flux, current I = 0, when it use the filed the rate of the change in flux will be again constant between in decrecaing order so contant current will induced on opposite.

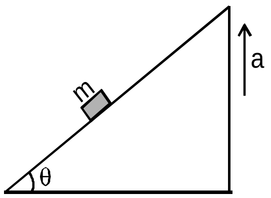
32. No change in moment of inertia

34.



Electric field at each point of OA obtained $\perp$ to it and opposite to direction of dipole moment.

38.



Total force in upward direction m × (g + a) because mass m is stationary on inclined plane and whole system is accelerated with acceleatration a in upward.

39. Force of positve charge = Electric force + Magnetic force

F = (qE + qVB)

This force is in upward direction so no any particle will pass through the hole.

40. Potential energy at H height = Kinetic energy at the lowest point of circular path.

$$mgH = \frac{1}{2}mv^2$$

To complete the circular motion minmum velocity at lowest point will be $V = \sqrt{5gR}$

$$mgH = \frac{1}{2}m\,(5gR)$$

$$H = \frac{5}{2}R$$

CHEMISTRY

41. According to Graham's law

$$\text{Rate of diffusion} \propto \frac{1}{\sqrt{\text{Molar mass}}}$$

due to highest molar mass of CO_2 rate of diffusion is slowest.

42. Moles of $H_2 = \frac{3}{2}$, Moles of $O_2 = \frac{4}{32}$

Kinetic energy of n moles of gas = $\frac{3}{2}$ nRT

$$\text{so, } \frac{\text{Kinetic energy of hydrogen}}{\text{Kinetic energy of oxygen}} = \frac{\frac{3}{2}n_1RT}{\frac{3}{2}n_2RT}$$

$$= \frac{n_1}{n_2}$$

$$= \frac{3/2}{4/32}$$

= 12 : 1

44. $ClF_3 \longrightarrow sp^3d$ hybridisation,
but due to presence of two lp on central atom Cl, according to VSEPR theory shape is 'T'

45. $HCO_3^- + H^+ \longrightarrow H_2CO_3$
Bronsted base

$HCO_3^- \longrightarrow H^+ + CO_3^{2-}$
Bronsted acid

47. Isoelectronic means same no. of electrons
CO has 6 + 8 = 14 electrons
CN^- has 6 + 7 + 1 = 14 electrons

48. CO_2, due to sp hybridisation bond angle = 180º

49. Diethyl ether, because it is inert towards the Grignard reagent

50. $CH_3-CH_2-CH_2-CHO + CH_3-CH_2-CH_2-MgBr$

$$\downarrow$$

$$CH_3-CH_2-CH_2-\underset{\large OMgBr}{\overset{\large H}{\underset{|}{\overset{|}{C}}}}-CH_2-CH_2-CH_3 \xrightarrow{H_3O^+} CH_3-CH_2-CH_2-\underset{\large OH}{\overset{\large H}{\underset{|}{\overset{|}{C}}}}-CH_2-CH_2-CH_3$$

Achiral Secondary alcohol

51. $[Ni(PPh_3)_2Cl_2]$ ⇒ dsp^2 hybridisation, because PPh_3 is strong ligand hence pairing of electrons takes place
$[NiCl_4]^{2-}$ ⇒ sp^3 hybridisation, because Cl^- is weak ligand hence pairing of electrons is not takes place

52. $$16H^+ + 2MnO_4^- + 5\underset{\large COO^-}{\underset{|}{COO^-}} \longrightarrow 2Mn^{2+} + 8H_2O + 10CO_2$$

53. Suppose equilibrium constant for the following reaction is K_1

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 \quad ; \quad K_1 = \frac{[NH_3]^2}{[N_2][H_2]^3} \qquad \text{-------- (i)}$$

and equilibrium constant for the following reaction is K_2

$$\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftharpoons NH_3 \ ; \ K_2 = \frac{[NH_3]}{[N_2]^{1/2}[H_2]^{3/2}} \qquad \text{-------- (ii)}$$

square the both side of equation (ii)

$$K_2^2 = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$K_2^2 = k_1$ [by equation (i)]

$K_2 = \sqrt{k_1}$

$K_2 = \sqrt{41}$ [$\because K_1 = 41$]

$K_2 = 6.4$

54. Suppose reaction is $2A \longrightarrow$ Product
according to rate law
Rate $R = k\,[A]^2$

or $\dfrac{R_1}{R_2} = \dfrac{[A]_1^2}{[A]_2^2}$

according to question $\dfrac{R_1}{R_2} = \dfrac{[A]_1^2}{\left(\dfrac{[A]_1}{2}\right)^2}$ $\left\{\because [A]_2 = \dfrac{[A]_1}{2}\right\}$

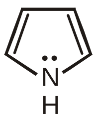
$\Rightarrow \dfrac{R_1}{R_2} = 4$

$\Rightarrow R_2 = \dfrac{R_1}{4}$

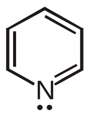
55. $HCO_3^- \longrightarrow H^+ + CO_3^{2-}$
Conjugate base

$NH_3 \longrightarrow H^+ + NH_2^-$
Conjugate base

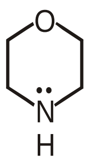
56. (II) & (IV)
Because both have close system of conjugated double bond and follow Huckel's (4n+2) π e^- rule.

57. 

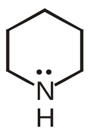
ℓp of N takes part in resonance with conjugated double bonds, so it is not easily available on N for the protonation.



ℓp is not taken part in resonance so easily available for the protonation.



due to high E.N. of O availability of ℓp on N decreases.



No extra effect, so availability of ℓp on N increases.

58. Gauche conformer.

because angle between same groups is 60º

59. Suppose initial quantity = N_o

after 75% completion of the reaction

remaining quantity N = $N_o \times \frac{25}{100} = \frac{N_o}{4}$

$$T = \frac{2.303}{K} \log\left(\frac{N_o}{N}\right)$$

$$T = \frac{2.303}{K} \log\left(\frac{N_o}{N_o/4}\right)$$

$$T = \frac{1.386}{K} \text{ -------- (i)}$$

$$\because T_{1/2} = \frac{0.693}{K}$$

$$K = \frac{0.693}{30} \text{ -------- (ii)}$$

so by equation (i) and (ii)

$$T = \frac{1.386}{0.693/30}$$

T = 60 min.

60. Concentration of H^+ ions in H_2SO_4 solution = 2 × 0.1 = 0.2 M

So no. of moles of H^+ ions in 10 ml H_2SO_4

solution = $\frac{0.2 \times 10}{1000}$ = 0.002

concentration of OH^- ions in 0.1 KOH solution

= 1 × 0.1

= 0.1 M

So no. of moles of OH^- ions in 10 ml KOH

solution = $\frac{0.1 \times 10}{1000}$

= 0.001

after mixing remaining moles of H^+ ions = 0.002 – 0.001

= 0.001

so concentration of H^+ ions in mixture of

solutions = $\frac{0.001}{10+10} \times 1000$ = 0.05 M

PART-II (2 Mark)

MATHEMATICS

81. $p(x) = a_0 + a_1x + ...+ a_nx^n$

$p(0) = 7$

$a_0 = 7$

$p(1) = a_0 + a_1 + a_2 + ...+ a_n = 9$

$p(-1) = a_0 - a_1 + a_2 .... = 1$

$p(2) = a_0 + 2a_1 + 4a_2 + ... = 13$

$p(-2) = a_0 - 2a_1 + 4a_2 .... = -15$

$p(1) + p(-1) = 2[a_0 + a_2 + ...] = 10$

$a_0 + a_2 + a_4 = 5$ .....(1)

$7 + a_2 + a_4 = 5$

$a_2 + a_4 = -2$ .....(2)

$p(2) + p(-2) = 13 - 15$

$2(a_0 + 4a_2 + ...) = -2$

$a_0 + 4a_2 + 16a_4 = -1$

$4a_2 + 16a_4 = -8$ .....(2)

$p(3) = 25$

$a_0 + 3a_1 + 9a_2 + ... = 25$

$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 + 243a_5 = 25$ .....(3)

From (1) and (2)

$$\begin{array}{r} 4a_2 + 4a_4 = -8 \\ 4a_2 + 16a_4 = -8 \\ - \quad - \qquad + \\ \hline a_4 = 0 \text{ and } a_2 = -2 \end{array}$$

$\therefore$ Smallest possible value of n is 3.

82. $$\sum_{abc}(a-b)^2 \ge 0 \Rightarrow \frac{\sum a^2}{\sum ab} \ge 1$$

$|a - b| < c$ ...... (1) [Triangle inequalities]

$|b - c| < a$ ...... (2)

$|c - a| < b$ ...... (3)

Squaring and adding

$a^2 + b^2 + c^2 < 2ab + 2bc + 2ca$

$$\frac{\sum a^2}{\sum ab} < 2$$

So, $b \in [1,2)$.

83. $y = \left||x-3|-4\right| - 5$

When, $x < -1$

$y = |3 - x - 4| - 5$

$y = -x - 1 - 5$

$y = -x - 6$

When $-1 \le x < 3$

$y = |3 - x - 4| - 5$

$= |-x - 1| - 5$

$= x + 1 - 5$

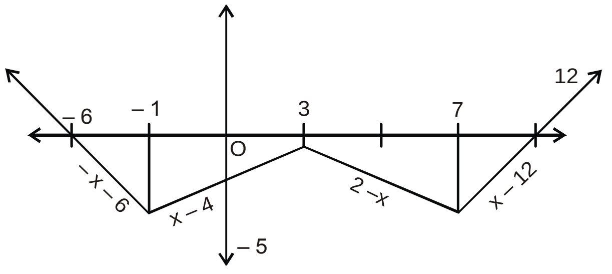
$= x - 4$

When, $3 \le x < 7$

$y = |x - 7| - 5$

$y = 7 - x - 5$

$y = 2 - x$



When, $x \geq 7$

$y = |x - 7| - 5$

$= x - 7 - 5$

$= x - 12.$

Area bounded region

$$= \int_{-6}^{-1}(-x-6)dx + \int_{-1}^{3}(x-4)dx + \int_{3}^{7}(2-x)dx + \int_{7}^{12}(x-12)dx$$

$$= \left[\frac{-x^2}{2} - 6x\right]_{-6}^{-1} + \left[\frac{x^2}{2} - 4x\right]_{-1}^{3} + \left[2x - \frac{x^2}{2}\right]_{3}^{7} + \left[\frac{x^2}{2} - 12x\right]_{7}^{12}$$

$$= \left(\frac{-1}{2} + 6\right) - \left(\frac{-36}{2} + 36\right) + \left(\frac{9}{2} - 12\right) - \left(\frac{1}{2} + 4\right) + \left(14 - \frac{49}{2}\right) - \left(6 - \frac{9}{2}\right) + \left(\frac{144}{2} - 144\right) - \left(\frac{49}{2} - 84\right)$$

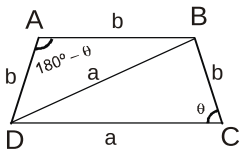
$$= \frac{11}{2} - 18 - \frac{15}{2} - \frac{9}{2} - \frac{21}{2} - \frac{3}{2} - 72 + \frac{119}{2}$$

$$= \frac{130}{2} - \frac{48}{2} - 90$$

$= 65 - 24 - 90$

$= 49$ sq. unit.

84.



$$\cos\theta = \frac{a^2 + b^2 - a^2}{2ab} = \frac{b}{2a} \quad \text{....(i)}$$

$$\cos(180^\circ - \theta) = \frac{b^2 + b^2 - a^2}{2b^2}$$

$$-\cos\theta = \frac{2b^2 - a^2}{2b^2}$$

$$\cos\theta = \frac{a^2 - 2b^2}{2b^2} \quad \text{....(ii)}$$

From (i) & (ii)

$$\frac{a^2 - 2b^2}{2b^2} = \frac{b}{2a}$$

$$\left(\frac{a}{b}\right)^3 - 2\left(\frac{a}{b}\right) - 1 = 0$$

Let $\frac{a}{b} = x$

$x^3 - 2x - 1 = 0$

$(x + 1)(x^2 - x - 1) = 0$

$x = -1$ or $x = \frac{1 \pm \sqrt{1 - 4(-1)(1)}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

$\because$ x cannot be negative

$\therefore$ $x = \frac{1}{2}(\sqrt{5} + 1)$

85. $a_n = \sqrt{\frac{1 + a_{n-1}}{2}}$

$a_1 = \sqrt{\frac{1 + a_0}{2}}$

$a_1 = \sqrt{\frac{1 + \cos\theta}{2}}$

$a_1 = \sqrt{\frac{2\cos^2\frac{\theta}{2}}{2}}$

$a_1 = \cos\frac{\theta}{2}$

$a_2 = \sqrt{\frac{1 + a_1}{2}}$

$= \sqrt{\frac{1 + \cos\frac{\theta}{2}}{2}}$

$= \sqrt{\frac{2\cos^2\frac{\theta}{4}}{2}}$

$= \cos\frac{\theta}{2^2}$

-

-

$a_n = \cos\frac{\theta}{2^n}$

$\lim_{n\to\infty} 4^n(1 - a_n)$

$\lim_{n\to\infty} 4^n\left(1 - \cos\frac{\theta}{2^n}\right)$

$$\lim_{n\to\infty} 4^n \times 2\sin^2 \frac{\theta}{2^{n+1}}$$

$$\lim_{n\to\infty} \frac{2^{2n} \times 2\sin^2 \frac{\theta}{2^{n+1}}}{\frac{\theta}{2^{n+1}} \times \frac{\theta}{2^{n+1}}} \times \frac{\theta}{2^{n+1}} \times \frac{\theta}{2^{n+1}}$$

$$\lim_{n\to\infty} \frac{\theta^2}{2} \times \frac{\sin^2 \frac{\theta}{2^{n+1}}}{\frac{\theta}{2^{n+1}} \times \frac{\theta}{2^{n+1}}}$$

$$= \frac{\theta^2}{2}.$$

86. $f(x) = (\sin x)^{\sin x}$

$f(x) = e^{\sin x \log \sin x}$

Minimum value of sinxlog(sinx) is 0.

$\therefore$ Maximum value of $(\sin x)^{\sin x}$ is $e^0 = 1$.

Maximum value of sinxlog(sinx) is $-\frac{1}{e}$.

$\therefore$ Minimum value of $(\sin x)^{\sin x}$ is $e^{\frac{-1}{e}}$.

87. $\int_1^{10} \frac{1}{x} dx$

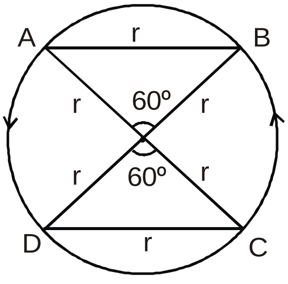
$= [\log x]_1^{10}$ = log10 = 2.303

$B = 1 + \frac{1}{2} + ... + \frac{1}{9}$

$= 1 + 0.5 + 0.33 + 0.25 + 0.20 + 0.16 + 0.14 + 0.12 + 0.11 \approx 2.81$

$C = \frac{1}{2} + ... + \frac{1}{9} + \frac{1}{10} \approx 2.81 - 1 + 0.1 = 1.91$

So, C < A < B and B – A $\approx$ 0.51, A – C = 2.303 – 1.91 $\approx$ 0.40. So, B – A > A – C.

88. 

As we want the distance between two point is at least r. Now when the point A, B are at distance r.Then the angle made arc BA is 60º.

Now as chord AB come closer to centre the length of chord AB is increased that is it is greater than r and the angle is also increases i.e. from 60º to 180º and now when chord AB move way from centre then the length of chord AB decreases , when chord AB reach CD the length of AB equal to r and the angle chang from 180º to 60º

So, the angle required for desired conditions = 2(180 – 60) = 240
Total angle for all around the circle = 360º

So, required propability = $\frac{240}{360} = \frac{2}{3}$

89. Let a_N be p_{th} digit no.

So $\frac{4(5^{P-1})}{5-1} < N \le \frac{4(5^P-1)}{5-1}$ $\quad\quad$ $2 \times 10^{P-1} < a_N \le \frac{8}{9}(10^P-1)$

$5^{P-1} - 1 < N \le 5^P - 1$

$$\left(\frac{\log a_N}{\log N}\right)_{min} = \frac{\ln(2\times10^{P-1})}{\ln(5^P-1)} = \frac{10^{P-1}(\ln 10)\times(5^P-1)}{10^{P-1}-5^P \ln 5}$$

$$= \frac{\ln 10}{\ln 5} = \log_5^{10}$$

$$\left(\frac{\log a_N}{\log N}\right)_{max} = \frac{\ln\frac{8}{9}(10^P-1)}{\ln(5^{P-1}-1)} = \frac{\frac{8}{9}10^P \ln 10\times(5^{P-1}-1)}{\frac{8}{9}(10^P-1)\,5^{P-1}\ln 5}$$

$$= \log_5^{10}$$

By sandwitch therom limit is $\log_5^{10}$.

90. $S = \sum_{1\le i\le j\le n} |i-j|$

$= |1-1| + |1-2| + \text{--------} |1-n| = \frac{(n-1)n}{2}$

$|2-2| + \text{--------} |2-n| = \frac{(n-2)(n-1)}{2}$ ....

$$S = \sum_{r=1}^{N} \frac{(n-r)(n-r+1)}{2}$$

$= {}^{n+1}C_3$

PHYSICS

91. Mass of sphere of radius r

$m = \frac{Mr^3}{R^3}$ $\quad\quad$ $\left(\frac{M}{\frac{4}{3}\pi R^3} = \rho\right)$

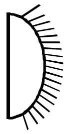
$X_{cm} = \frac{M\times 0 - m(R-r)}{M-m}$ $\quad\quad$ $\left(\frac{\frac{4}{3}\pi r^3 M}{4/3\pi R^3} = m\right)$

$X_{cm} = \frac{Mr^3}{R^3}(R-r) \Big/ \left(M - \frac{R^3 M}{R^3}\right)$

$$= \frac{r^3(R-r)}{R^3\left(\frac{R^3-r^3}{R^3}\right)} = \frac{r^3(R-r)}{(R-r)(R^2+rR+r^2)} \qquad \Rightarrow X_{cm} = \frac{r^3}{R^2+rR+r^2}$$

92. For planeo convex lens

$$\frac{1}{F_L} = (\mu-1)\frac{1}{R}$$



$$F_L = \frac{R}{\mu-1}$$

Refraction through less

$$\frac{1}{V} = \frac{\mu-1}{R} + \frac{1}{u}$$

This v must be centre of mirror

$$-\frac{1}{R} = \frac{1}{\left(\frac{R}{r-1}\right)} + \frac{1}{u} \qquad \Rightarrow u = -\frac{R}{\mu}$$

93. In cyclic process $\Delta u = 0$

$\Delta u = \Delta W$ = Area of loop

$= (P_1 - P_2)(V_2 - V_1)$

94. On comparing both the figures

$$x = \frac{(R+x)(6R)}{(R+x)+6R}$$

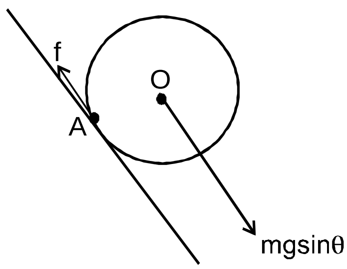
$x^2 + xR - 6R^2 = 0$

$$x = \frac{-x \pm \sqrt{R^2+(4\times 6R^2)}}{2}$$

$$x = \frac{-R+5R}{2} = 2R$$

95. Torque about point A

$$\left(\frac{2}{5}MR^2 + MR^2\right)\alpha = mg\sin\theta R$$



$$\alpha = \frac{5g\sin\theta}{7R}$$

Applying Newton's law,

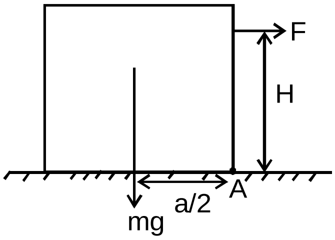
$Ma = Mg\sin\theta - \mu Mg\cos\theta$

$$\mu g\cos\theta \geq g\sin\theta - \frac{5g\sin\theta}{7} \qquad \Rightarrow \mu g\cos\theta \geq \frac{2g\sin\theta}{7}$$

$$\theta = \tan^{-1}\frac{7\mu}{2}$$

Velocity of sound $= \sqrt{\frac{\gamma RT}{M}}$ $\qquad$ T $\uparrow$ Velocity of sound $\downarrow$

96.



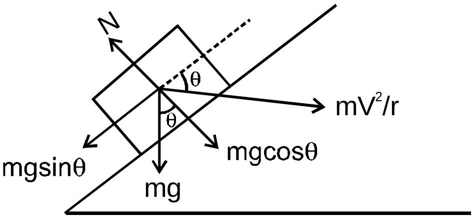
For height greater than H
Balancing torque about point A

$F \times H = mg\frac{a}{2}$ ....(1)

For height less than H
$F = \mu gh$ ....(2)
From (1) and (2)

$\mu = \frac{a}{2H}$

97.

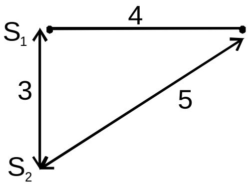


$$\frac{MV^2}{r}\cos\theta = mg\sin\theta + \mu\left[mg\cos\theta + \frac{mv^2}{r}\sin\theta\right]$$

$$\sin\theta\left[g + \frac{\mu V^2}{r}\right] = \cos\theta\left[\mu g - \frac{V^2}{r}\right]$$

$$\tan\theta = \frac{V^2 - \mu rg}{\mu V^2 + rg}$$

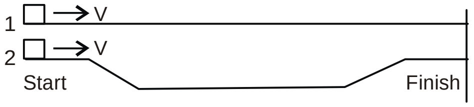
98.



Path difference = 5 – 4 = 1 m
For constructive interference
$n\lambda_1 = 1m$
For destructive interference
For n = 1, $\lambda_1 = 1m$, $\lambda_2 = 2m$

$$\frac{(2n-1)\lambda_2}{2} = 1m$$

99. Two small blocks slide without losing contact with the surface along two frictionless tracks 1 and 2, starting at the same time with same initial speed v. Track 1 is perfectly horizontal, while track 2 has a dip in the middle, as shown.



Which block reaches the finish line first ?
[Hint : Use velocity-time graph to solve]
(A) Block on track 1 reaches the finish line first
(B*) Block on track to reaches the finish line first
(C) Both blocks reach the finish line at the same time
(D) It depends on the length of the dip in the second track, relative to the total length of the tracks.

100. $\Delta Q = \Delta u + P\Delta V$ (1^{st} law of thermodynamics)

$$m\ell_v = \Delta u + 1.01 \times 10^5 \left(\frac{1}{1/1.8} - \frac{1}{10^3}\right)$$

$\Delta u \cong 20.8 \times 10^5 \text{ J kg}^{-1}$

CHEMISTRY

101. Millimoles of NH_4OH in 10 ml of 0.1 M NH_4OH solution = 0.1 × 10 = 1
millimoles of NH_4Cl in 10 ml of 1M NH_4Cl solution = 1 × 10 = 10

$$pOH = pK_b + \log \frac{[Salt]}{[Base]}$$

$$6 = pK_b + \log \frac{10/10+10}{1/10+10} \qquad [\because pOH = 14 - pH = 14 - 8 = 6]$$

$6 = pK_b + \log 10$
$pK_b = 6 - 1$
$pK_b = 5$

102. $2C_4H_{10} + 13O_2 \longrightarrow 8CO_2 + 10H_2O \qquad \Delta H = -2658 \text{ kJ/mol}$

Butane present in cylinder = 11.6 kg
= 11600 g

$$= \frac{11600}{58} \text{ mol}$$

$\because$ Combustion of 1 mol of C_4H_{10} gives = 2658 kJ energy

$\therefore$ Combustion of 11600/58 mol of C_4H_{10} gives $= \frac{2658 \times 11600}{58}$ kJ
= 531600 kJ energy

energy consumes in 1 day = 15000 kJ

so 531600 kJ energy will be consumed in = $\frac{531600}{15000} \approx 35$ days

103. $W = d \times V = 0.879 \times 50 = 43.95$, $K_f = 5.12 \text{ Kg kg mol}^{-1}$
$= 5120 \text{K g mol}^{-1}$

$$\Delta T_f = \frac{K_f w_o}{mW}$$

$$5.51 - 5.03 = \frac{5120 \times 0.643}{m \times 43.95} \qquad \Rightarrow m = \frac{329216}{21.096}$$

$m = 156 \text{ g mol}^{-1}$

104. $Zn + 2Ag^{+} \longrightarrow Zn^{2+} + 2Ag$

(0.04M) (0.28M)

$$E_{cell} = E^{o} - \frac{0.059}{n} \log \frac{[Zn^{2+}][Ag]^2}{[Ag^{+}]^2[Zn]}$$

$$E_{cell} = 2.57 - \frac{0.059}{2} \log \frac{0.28 \times 1^2}{(0.04)^2 \times 1} \qquad \{\because [Ag] = [Zn] = 1\}$$

by solving the equation we get

$$E_{cell} \approx 2.50\ V$$

105. $Co^{2+} + \text{Con. HCl} \longrightarrow CoCl_4^{2-}$

$$CoCl_4^{2-} \xrightarrow{HOH_{(excess)}} [Co(H_2O)_6]^{2+}$$

Pink

106. $$\ln k = \frac{-11067}{T} + 31.33$$

$$2.303 \log k = \frac{-11067}{T} + 31.33$$

suppose k_1 and T_1 are rate constant and temperature in case-I and k_2 and T_2 are rate constant and temperature in case-II

So, $$2.303 \log k_1 = -\frac{11067}{T_1} + 31.33 \text{ --------- (1)}$$

$$2.303 \log k_2 = -\frac{11067}{T_2} + 31.33 \text{ --------- (2)}$$

by subracting equation (2) from equation (1)

$$2.303 \log \frac{k_1}{k_2} = -11067 \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$2.303 \log \frac{k_1}{2k_1} = -11067 \left(\frac{1}{298} - \frac{1}{T_2}\right) \qquad [\because k_2 = 2k_1]$$

$$2.303 \times (-0.3010) = \frac{-11067}{298} + \frac{11067}{T_2}$$

by solving the above equation we get

$T_2 \approx 303.7$ K

$T_2 \approx 31$ºC

107. $K_1 = \dfrac{[CuCl_3Br^{2-}][Cl^-]}{[CuCl_4^{2-}][Br^-]}$ ------- (i)

$K_2 = \dfrac{[CuCl_2Br_2^{2-}][Cl^-]}{[CuCl_3Br^{2-}][Br^-]}$ ------- (ii)

$K_3 = \dfrac{[CuClBr_3^{2-}][Cl^-]}{[CuCl_2Br_2^{2-}][Br^-]}$ ------- (iii)

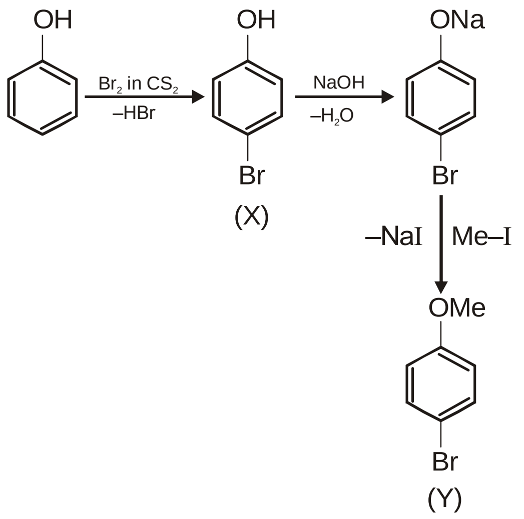
$K_4 = \dfrac{[CuBr_4^{2-}][Cl^-]}{[CuClBr_3^{2-}][Br^-]}$ ------- (iv)

equilibrium constant for given equation

$K = \dfrac{[CuClBr_3^{2-}][Cl^-]^3}{[CuCl_4^{2-}][Br^-]^3}$ ------- (v)

by multiplying the right hand side of equation (i), (ii) and (iii) we get right hand side of equation (v)
it means $K = K_1K_2K_3$

108.



109. $^{234}_{90}Th \longrightarrow ^{206}_{82}Pb + x\,^{4}_{2}He + y\,^{0}_{-1}e$

By comparing mass no.

234 = 206 + 4x + oy

x = 7

By comparing nuclear charge

90 = 82 + 2x 1y

y = 82 + 2 × 7 – 90

y = 6

110.