Matrices – and – Determinants

SUBJECTIVE PROBLEMS:

Q 1.

For what value of k do the following system of equations possess a non-trivial (i.e., not all zero) solution over the set of rationals Q?

x + ky + 3z = 0

3x + ky – 2z = 0

2x + 3y – 4z = 0

For that value of k, find all the solutions for the system. **(IIT JEE – 1979 – 5 Marks)**

Q 2.

Let a, b, c be positive and not all equal. Show that the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

(IIT JEE – 1981 – 4 Marks)

Q 3.

Without expanding a determinant at any stage, show that $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$ = xA +B where A and B are determinants of order 3 not involving x. **(IIT JEE - 1982 – 5 Marks)**

Q 4.

Show that the system of equations 3x – y + 4z = 3

x + 2y – 3z = -2

6x + 5y + λ z = -3

Has at least one solution for any real number $\lambda \neq -5$. Find the set of solutions if $\lambda = -5$

(IIT JEE – 1983 – 3 Marks)

Q 5.

Show that

$$\begin{vmatrix} {}^{x}C_{r} & {}^{x}C_{r+1} & {}^{x}C_{r+2} \\ {}^{y}C_{r} & {}^{y}C_{r+1} & {}^{y}C_{r+2} \\ {}^{=}C_{r} & {}^{=}C_{r+1} & {}^{=}C_{r+2} \end{vmatrix} = \begin{vmatrix} {}^{x}C_{r} & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^{y}C_{r} & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^{=}C_{r} & {}^{=+1}C_{r+1} & {}^{=}C_{r+2} \end{vmatrix}$$

(IIT JEE – 1985 – 2 Marks)

Q 6.

Consider the system of linear equations in x, y, z:

(Sin 3θ) x – y + z = 0

(Cos 2θ) x + 4y + 3z = 0

2x + 7y + 7z = 0

Find the values of θ for which this system has nontrivial solutions. **(IIT JEE – 1986 – 5 Marks)**

Q 7.

Let $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$

Show that $\sum_{a-1}^{n} \Delta a =$ c, a constant **(IIT JEE 1989 – 5 Marks)**

Q 8.

Let the three digit numbers A 28, 3 B9, and 62 C, where A, B, and C are integers between 0 and 9, be divisible by a fixed. Integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k.

(IIT JEE 1990 – 4 Marks)

Q 9.

If a ≠ p, b ≠ q, c ≠ r and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0.$ ***(IIT JEE – 1991 – 4 Marks)***

Then find the value of p/p – a + q/q – b + r/r – c

Q 10.

For a fixed positive integer n, if *(IIT JEE – 1992 – 4 Marks)*

$$D=\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$ Then show that $[D/(n!)^3 - 4]$ is divisible by n.

Q 11.

Let λ and α be real. Find the set of all values of λ for which the system of linear equations

$\lambda x + (\sin\alpha)\, y + (\cos\alpha)\, z = 0,\ x + (\cos\alpha)\, y + (\sin\alpha)\, z = 0,\ -x + (\sin\alpha)\, y - (\cos\alpha)\, z = 0$

has a non – trivial solution : for $\lambda = 1$, find all values of α. *(IIT JEE – 1993 – 5 Marks)*

Q 12.

For all values of A, B, C and P, Q, R show that *(IIT JEE – 1994 – 4 Marks)*

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

Q 13.

Let $a > 0$, $d > 0$. Find the value of the determinant *(IIT JEE 1996 – 5 Marks)*

$$\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

Q 14.

Find the value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ *(IIT JEE – 1997C – 2 Marks)*

Where a, b and c are respectively the p^{th}, q^{th} and r^{th} of a harmonic progression.

Q 15.

Prove that for all values of θ, ***(IIT JEE – 2000 – 3 Marks)***

$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right) \\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix} = 0$$

Q 16.

If matrix $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ where a, b, c are real positive numbers, abc = 1 and $A^T A = I$, then the value of $a^3 + b^3 + c^3$. ***(IIT JEE - 2003 – 2 Marks)***

Q 17.

If M is 3 x 3 matrix, where det M = 1 and $MM^T = I$, where 'I' is an identity matrix, prove that det (M – 1) = 0. ***(IIT JEE – 2004 – 2 Marks)***

Q 18.

If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix}, U = \begin{vmatrix} f \\ g \\ h \end{vmatrix}, V = \begin{vmatrix} a^2 \\ 0 \\ 0 \end{vmatrix}, X = \begin{vmatrix} x \\ y \\ z \end{vmatrix}$

And AX = U has infinitely many solutions, prove that BX = V has no unique solution. Also show that if afd ≠ 0, then BX = V has no solution. ***(IIT JEE 2004 – 4 Marks)***