

Mechanical Properties of Solids & Fluids - Solutions

Subjective Problems

Sol 1.

M is the mid-point of tube AB.

At equilibrium

$$p_1 \times A + mg = p_2 \times A$$

$$p_1 \times A + 10 \times A \times d_{Hg} = p_2 \times A$$

$$\Rightarrow p_1 + 10d_{Hg} \times g = p_2$$

For air present in column AP

$$p \times 45 \times A = p_1 \times (45 + x) \times A$$

$$\Rightarrow p_1 = 45 / (45 + x) \times 76 d_{Hg} \times g \quad \dots(ii)$$

For air present in column QB

$$p \times 45 \times A = p_2 \times (45 - x) \times A$$

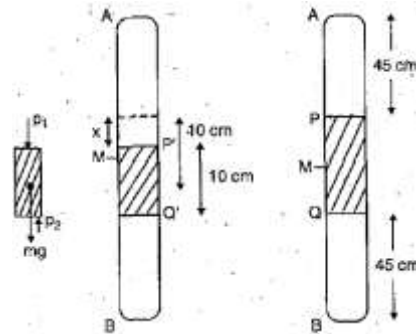
$$\Rightarrow p_2 = 45 / (45 - x) \times 76 d_{Hg} \times g \quad (iii)$$

From (i), (ii) and (iii)

$$45 \times 76 \times d_{Hg} \times g / (45 + x) + 10 d_{Hg} \times g = 45 / (45 - x) \times 76 \times d_{Hg} \times g$$

$$\Rightarrow 45 \times 76 / (45 + x) + 10 = 45 \times 76 / (45 - x)$$

$$x = 2.95 \text{ cm.}$$



Sol 2.

From fig. (b), due to equilibrium

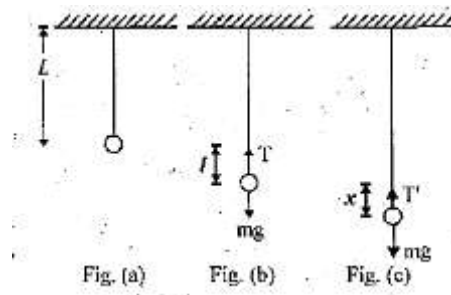
$$T = mg \quad \dots(i)$$

$$\text{But } Y = T / A \ell / L$$

$$\Rightarrow T = YA \ell / L$$

From (i) and (ii)

$$mg = YA \ell / L \quad (iii)$$



From fig. (c)

Restoring force

$$= - [T - mg] = - [YA (\ell + x) / L - YA \ell / L] \text{ [from (iii)]}$$

$$= - YA x / L$$

On comparing this equation with $F = - m\omega^2 x$, we get

$$m\omega^2 = YA / L$$

$$\Rightarrow \omega = \sqrt{YA / mL} \Rightarrow 2\pi / T = \sqrt{YA / mL}$$

$$\text{Frequency } f = 1 / T = 1 / 2\pi \sqrt{YA / mL}$$

Sol 3.

Let the edge of cube be ℓ . When mass is on the cube of wood

$$200 \text{ g} + \ell^3 d_{\text{wood}} g = \ell^3 d_{\text{water}} g$$

$$\Rightarrow \ell^3 d_{\text{wood}} = \ell^3 d_{\text{water}} - 200 \quad \dots \text{ (i)}$$

When the mass is removed

$$\ell^3 d_{\text{wood}} = (\ell - 2) \ell^2 d_{\text{water}} \quad \dots \text{ (ii)}$$

From (i) and (ii)

$$\ell^3 d_{\text{water}} - 200 = (\ell - 2) \ell^2 d_{\text{water}}$$

But $d_{\text{water}} = 1$

$$\therefore \ell^3 - 200 = \ell^2 (\ell - 2)$$

$$\Rightarrow \ell = 10 \text{ cm}$$

Sol 4. When the lump of sugar is just immersed

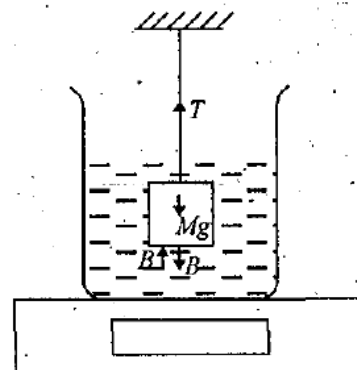
$$T = Mg - B$$

(For equilibrium of lump of sugar)

The reading on the pan balance

$$= Mg + Vd_w g$$

Where V = Volume of lump of sugar



d_w = density of water

When the lump is half dissolved,

The reading on the pan balance = $Mg + v / 2 d_s g + V / 2 d_w g$

When d_s = density of sugar

Since $d_s > d_w$

\therefore the reading will increase.

Thus, we can conclude that as the time passes the reading will keep increasing.

Sol 5.

When the stones were in the boat, the weight of stones were balanced by the buoyant force.

$$V_s d_s = V_\ell d_\ell$$

V_ℓ, V_s = volue of liquid and stone respectively

V_ℓ, V_s = density of liquid and stone respectively

Since, $d_s > d_\ell \quad \therefore V_s < V_\ell$

Therefore when stones are put in water, the level of water falls.

Sol 6.

P.E. of liquid in cylinder 1

$$U_1 = (m) g h_1 / 2 = (\rho \times A \times h_1) g h_1 / 2 = \rho A g h_1^2 / 2$$

Note :

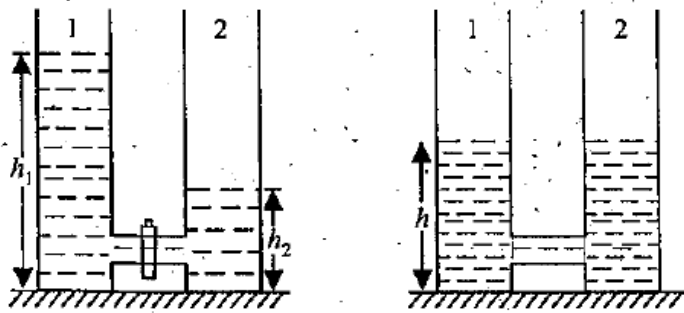
[The total mass can be supposed to be concentrated at the center of the filled part which will be at height $h_1 / 2$]

Similarly P.E. of liquid in cylinder 2 $U_2 = \rho A g h_2^2 / 2$

\therefore Total P.E. initially $U = U_1 + U_2 (h_1^2 + h_2^2)$

After the equalising of levels.

P.E. of liquid in cylinder 1 $U_1 = m g h / 2 = \rho A g / 2 h^2$



P.E. of liquid in cylinder 2 $U_2' = mg h / 2 = \rho Ag / 2 h^2$

\therefore Total P.E. finally $U' = U_1' + U_2' = \rho Agh^2$

The change in P.E.

$$\Delta U = U - U' = \rho Ag [h^2_1 / 2 + h^2_2 / 2 - h^2]$$

Total volume remains the same.

$$Ah_1 + Ah_2 = Ah + Ah$$

$$\Rightarrow h = h_1 + h_2 / 2$$

Therefore,

$$\Delta U = \rho Ag [h^2_1 / 2 + h^2_2 / 2 - (h_1 + h_2 / 2)^2]$$

$$= \rho Ag / 4 (h_1 - h_2)^2$$

This change in P.E. is the work done by gravity

Sol 7. (a) For equilibrium $F_{net} = 0$ and $\tau_{net} = 0$

Taking moment about O

$$mg \times \ell / 2 \sin \theta = F_T (\ell - x / 2) \sin \theta \quad \dots (i)$$

$$\text{Also } F_T = \text{wt. of fluid displaced} = [(\ell - x)] \times \rho_w g \quad \dots (ii)$$

$$\text{And } m = (\ell A) 0.5 \rho_w \quad \dots (iii)$$

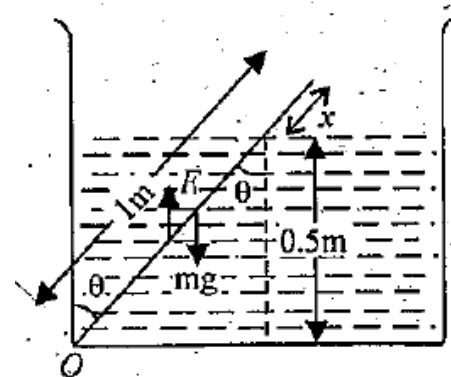
Where A is the area of cross section of the rod

From (i), (ii) and (iii)

$$(\ell A) 0.5 \rho_w g \times \ell / 2 \sin \theta = [(\ell - x) A] \rho_w g \times (\ell - x / 2) \sin \theta$$

$$\text{Here, } \ell = 1 \text{ m } \therefore (1 - x)^2 = 0.5 \Rightarrow x = 0.293 \text{ m}$$

$$\text{From the diagram } \cos \theta = 0.5 / 1 - x = 0.5 / 0.707 \Rightarrow \theta = 45^\circ$$



Sol 8.

(a) Let the ball be dropped from a height h . During fall

$$V = ut + at = 0 + g t_1 / 2 \Rightarrow t_1 = 2v / g$$

In the second case the ball is made to fall through the same height and then the ball strikes the surface of liquid of density d_L . When the ball reaches inside the liquid, it is under the influence of two force (i) Vdg , the weight of ball in downward direction (ii) Vd_Lg , the upthrust in upward direction.

Note :

The viscous forces are absent (given)

Since, $d_L > d$

The upward force is greater and the ball starts retarding.

For motion B to C

$$u = V, v = 0, t = t, a = -a$$

$$v = u + at \Rightarrow 0 = v + (-a) t$$

$$\Rightarrow t = v / a$$

$$\text{Now, } a = F_{\text{net}} / m = Vd_Lg - Vdg / Vd = (d_L - d)g / d$$

$$\Rightarrow t = vd / (d_L - d)g \quad \dots(\text{iii})$$

Therefore,

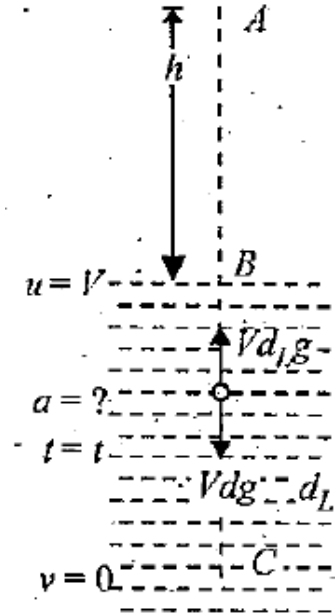
$$t_2 = t_1 + 2t = t_1 + 2dv / (d_L - d)g$$

$$= t_1 + 2d / (d_L - d)gt_1g / 2 = t_1 [1 + d / d_L - d]$$

$$\Rightarrow t_2 = d_L t_1 / d_L - d$$

(b) Since the retardation is not proportional to displacement, the motion of the ball is not simple harmonic

(c) If $d = d_L$ then the retardation $\alpha = 0$. Since the ball strikes the water surface with some velocity, it will continue with the same velocity in downward direction (until it is interrupted by some other force).



Sol 9.

(a) (i)

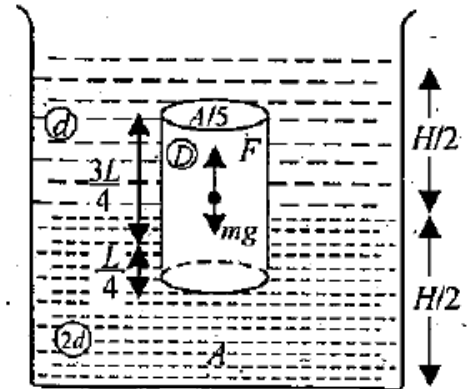
Since the cylinder is in equilibrium in the liquid therefore Weight of cylinder = upthrust

$$mg = F_{T_1} + F_{T_2} \text{ where}$$

F_{T_1} and F_{T_2} = upthrust due to lower and upper liquid respectively

$$A / 5 \times L \times D \times g = A / 5 \times L / 4 \times 2d \times g + A / 5 \times 3L / 4 \times d \times g$$

$$\Rightarrow D = 2d / 4 + 3d / 4 = 5d / 4$$



(ii) Total pressure at the bottom of the cylinder = Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density 2d + Pressure due to cylinder [Weight / Area]

$$P = P_0 + H / 2 \, dg + H / 2 \times 2d \times g + A/5 \times L \times D \times g / A$$

$$P = P_0 + (3H / 2 + L / 4)dg \quad [\because D = 5d/4]$$

(b)

Applying Bernoulli's theorem

$$P_0 + [H / 2 \times d \times g + (H / 2 - h) 2d \times g]$$

$$= P_0 + \frac{1}{2} (2d)v^2$$

$$\Rightarrow v = \sqrt{(3H - 4h) / 4g}$$

Horizontal Distance x

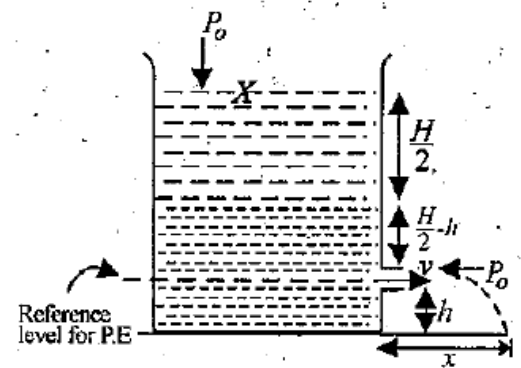
$$U_x = v; t = t; \quad x = vt \quad \dots(i)$$

For vertical motion of liquid falling from hole

$$u_y = 0, S_y = h, a_y = g, t_y = t$$

$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow h = \frac{1}{2} gt^2 \Rightarrow t = \sqrt{2h / g} \dots (ii)$$



From (i) and (ii)

$$x = v_y \times \sqrt{2h / g} = \sqrt{(3H - 4H) g / 2} \times \sqrt{2h / g}$$

$$= \sqrt{(3H - 4h)h} \quad \dots(\text{iii})$$

For finding the value of h for which x is maximum, we differentiate equation (iii) w.r.t. t

$$\frac{dx}{dt} = \frac{1}{2} [3H - 4h]^{-1/2} \{3H - 8h\}$$

Putting $\frac{dx}{dt} = 0$ for finding values h for maxima / minima

$$\frac{1}{2} [3H - 4h]^{-1/2} [3H - 8h] = 0$$

$$\Rightarrow h = 3H / 8$$

$$\therefore x_m = \sqrt{[3H - 4(3H / 8)] 3H / 8}$$

$$= \sqrt{12H / 8 \times 3H / 8} = 6H / 8 = 3H / 4$$

Sol 10.

(i) Let at any instant of time during the flow, the height of liquid in the container is x.

The velocity of flow of liquid through small hole in the orifice by Toricelli's theorem is

$$v = \sqrt{2gx} \quad \dots(\text{i})$$

The mass of liquid flowing per second through the orifice

= p x volume of liquid flowing per second

$$\frac{dm}{dt} = \rho \times \sqrt{2gx} \times A / 100 \quad \dots(\text{ii})$$

Therefore, the rate of change of momentum of the system in forward direction

$$= \frac{dm}{dt} \times v = 2gx \times A \times \rho / 100 \quad (\text{from (i) and (ii)})$$

(Alternatively you may use $F = \rho av^2$)

The rate of change of momentum of the system in the backward direction

= Force on backward direction = m x a

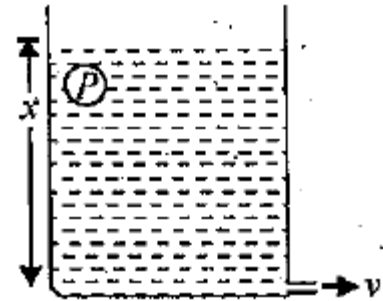
Where m is mass of liquid in the container at the instant t $m = \text{volume} \times \text{density} = A \times x \times \rho$

\therefore The rate of change of momentum of the system in the backward direction

$A \times \rho \times x \times a$

By conservation of linear momentum

$$A \times \rho \times x \times a = 2gx \times A \times \rho / 100 \Rightarrow a = g / 50$$



(ii) By Toricelli's theorem

$$v' = \sqrt{2g \times (0.25 h)}$$

Where h is the initial height of the liquid in the container. m_0 . the initial mass is

$$m_0 = Ah \times \rho \Rightarrow h = m_0 / A\rho$$

$$\therefore v' = \sqrt{2g \times 0.25 \times m_0 / A\rho} = \sqrt{gm_0 / 2A\rho}$$

Sol 11.

Given that

$$\rho = 1000 \text{ kg/m}^3, h_1 = 2\text{m}, h_2 = 5 \text{ m}$$

$$A_1 = 4 \times 10^{-3} \text{ m}^2, A_2 = 8 \times 10^{-3} \text{ m}^2, v_1 = 1 \text{ m/s}$$

Equation of continuity

$$A_1 v_1 = A_2 v_2 \therefore v_2 = A_1 v_1 / A_2 = 0.5 \text{ m/s}$$

According to Bernoulli's theorem,

$$(p_1 - p_2) = \rho g (h_2 - h_1) - \frac{1}{2} \rho (v_2^2) - v_1^2$$

Where $(p_1 - p_2) = \text{work done / vol. [by the pressure]}$

$$\rho g (h_2 - h_1) = \text{work don/vol. [by gravity forces]}$$

Now, work done / vol. by gravity forces

$$= \rho g (h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3 \text{ J/m}^3$$

$$\text{And } \frac{1}{2} \rho (v_2^2 - v_1^2) = \frac{1}{2} \times 10^3 [1/4 - 1] = - 3/8 \times 10^3 \text{ J/m}^3$$

$$= - 0.375 \times 10^3 \text{ J/m}^3$$

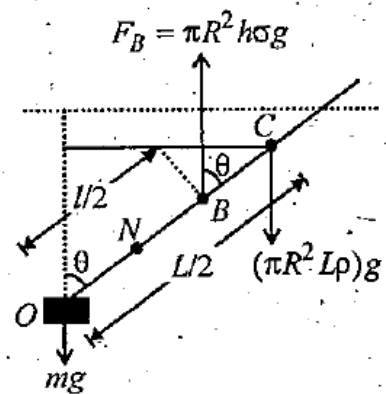
\therefore Work done / vol. by pressure

$$= 29.4 \times 10^3 - 0.375 \times 10^3 \text{ J / m}^3 = 29.025 \times 10^3 \text{ j/m}^3$$

Sol 12.

Note :

For the wooden stick-mass system to be in stable equilibrium the center of gravity of stick-mass system should be lower than the



center of buoyancy. Also in equilibrium the centre of gravity (N) and the centre of buoyancy (B) lie on the same vertical axis.

The above condition 1 will be satisfied if the mass is towards the lower side of the stick as shown in the figure. The two forces will create a torque which will bring the stick-mass system in the vertical position of the stable equilibrium.

Let ℓ be the length of the stick immersed in the liquid.

Then $OB = \ell / 2$

For vertical equilibrium

$$F_G = F_B \Rightarrow (M + m)g = F_B$$

$$\Rightarrow \pi R^2 L \rho g + mg = \pi R^2 \ell \sigma g$$

$$\ell = \pi R^2 L \rho + m / \pi R^2 \sigma \quad \dots(1)$$

Let the distance of the center of mass N of the (rod + mass) system from the origin O be $ON = y$. Then

$$y = My_1 + my_2 / M + m$$

Since mass m is at O, the origin, therefore $y_2 = 0$

$$\therefore y = M(L/2) + m \times 0 / M + m = ML / 2 (M + m)$$

$$= (\pi R^2 L \rho) L / 2 (\pi R^2 L \rho + m) \quad \dots(2)$$

Therefore for stable equilibrium

$$\ell / 2 > y$$

$$\therefore \pi R^2 L \rho + m / 2(\pi R^2 L \sigma) > (\pi R^2 L \rho) L / 2 (\pi R^2 L \rho + m)$$

$$\Rightarrow m \geq \pi R^2 L (\sqrt{\rho \sigma} - \rho)$$

$$\therefore \text{Minimum value of } m \text{ is } \pi R^2 L (\sqrt{\rho \sigma} - \rho)$$

Sol 13.

(a) As the pressure exerted by liquid A on the cylinder is radial and symmetric, the force due to this pressure cancels out and the net value is zero.

(b) For equilibrium, Buoyant force = weight of the body

$$\Rightarrow h_A \rho_A Ag + h_B \rho_B Ag = (h_A + h + h_B) A \rho_C g$$

(where ρ_C = density of cylinder)

$$h = (h_A \rho_A + h_B \rho_B / \rho_C) - (h_A + h_B) = 0.25 \text{ cm}$$

$$(c) a = F_{\text{Buoyant}} - Mg / M$$

$$= [h_A \rho_A + \rho_B (h + h_B) - (h + h_A + h_B) \rho_C / \rho_C (h + h_A + h_C)] g$$

$$= g / 6 \text{ upwards}$$

Sol 14.

When the force due to excess pressure in the bubble equals the force of air striking at the bubble, the bubble will detach from the ring.

$$\therefore \rho A v^2 = 4T / R \times A \Rightarrow R = 4T / \rho v^2$$

Sol 15.

When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2} \rho v_1^2 + \rho g H = \frac{1}{2} \rho v_0^2 + P_0$$

$$\Rightarrow P + \rho g H = \frac{1}{2} \rho (v_0^2 - v_1^2)$$

But according to equation of continuity

$$Y_1 = A_2 v_0 / A_1$$

$$\therefore P + \rho g H = \frac{1}{2} \rho [v_0^2 - (A_2 / A_1 v_0)^2]$$

$$P + \rho g H = \frac{1}{2} \rho v_0^2 [1 - (A_2 / A_1)^2]$$

$$\text{Here, } P + \rho g H = \Delta P$$

According to Poiseuille's equation

$$Q = \pi (\Delta P) a^4 / 8 \eta l \Rightarrow \eta = \pi (\Delta P) a^4 / 8 Q l$$

$$\therefore \eta = \pi (P + \rho g H) a^4 / 8 Q l = \pi / 8 Q l \times \frac{1}{2} \rho v_0^2 [1 - (A_2 / A_1)^2] \times a^4$$

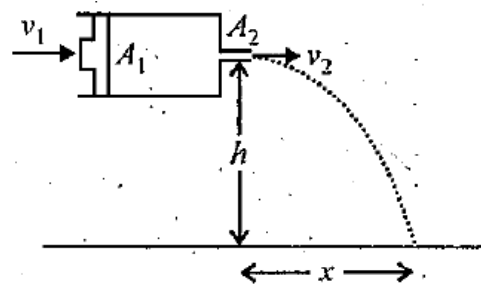
$$\text{Where } A_2 / A_1 = d^2 / D^2$$

$$H = \pi / 8 Q l \times \frac{1}{2} \rho v_0^2 [1 - d^4 / D^4] \times a^4$$

Sol 16.

From law of continuity

$$A_1 v_1 = A_2 v_2 \text{ Given } A_1 = \pi \times (4 \times 10^{-3} \text{ m})^2, A_2 = \pi \times (1 \times 10^{-3} \text{ m})^2$$



$$v_1 = 0.25 \text{ m/s}$$

$$\therefore v_2 = \pi \times (4 \times 10^{-3})^2 \times 0.25 / \pi \times (1 \times 10^{-3})^2$$

$$\text{Also, } h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{2h / g}$$

$$\text{Horizontal range } x = v^2 \sqrt{2h / g} = 4 \times \sqrt{2 \times 1.25 / 10} = 2 \text{ m}$$

Sol 17.

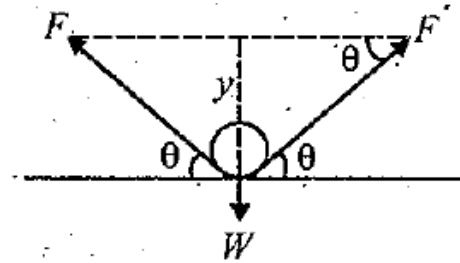
The free body diagram of wire is given below. If ℓ is the length of wire, then for equilibrium $2E \sin \theta = W$.

$$F = S \times \ell$$

$$\text{or, } 2S \times \ell \times \sin \theta = \lambda \times \ell \times g$$

$$\text{or, } S = \lambda g / 2 \sin \theta$$

$$\therefore S = \lambda g / 2y / a = a \lambda g / 2y \quad [\because \sin \theta = y / a]$$



Sol 18

Weight of liquid of height H

$$= \pi d^2 / 4 \times H \times \rho \times g \quad \dots(i)$$

Let us consider a mass dm situated at a distance x from A as shown in the figure. The centripetal force required for the mass to rotate = $(dm) \times \omega^2$

\therefore The total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm) \times \omega^2 \text{ where } dm = \rho \times \pi d^2 / 4 \times dx$$

\therefore Total centripetal force

$$= \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx \right) \times (x \omega^2)$$

$$= \rho \times \pi d^2 / 4 \times \omega^2 \times L^2 / 2 \quad \dots(ii)$$

This centripetal force is provided by the weight of liquid of height H.

From (i) and (ii)

$$\pi d^2 / 4 \times H \times \rho \times g = \rho \times \pi d^2 / 4 \times \omega^2 \times L^2 / 2$$

$$H = \omega^2 L^2 / 2g$$

