Probability

Q 1.
Balls are drawn one-by-one without replacement from a box containing 2 black 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black. 4 white and 3 red. **(IIT JEE – 1978 – 3 Marks)**

Q 2.
Six boys and six girls sit in a row randomly. Find the probability that
(i) The six girls sit together
(ii) The boys and girls sit alternately. **(IIT JEE – 1979 – 3 Marks)**

Q 3.
An anti -aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.1 respectively. What is the probability that the gun hits the plane? **(IIT JEE – 1981 – 2 Marks)**

Q 4.
A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is almost 0.3. Is it possible that the probability of B getting selected is 0.9? **(IIT JEE - 1982 - 2 Marks)**

Q 5.
Cards are drawn one by one at random from a well – shuffled full pack of 52 playing cards until 2 aces are obtained to be drawn, first time. If N is the number of cards required to be drawn, then show that P_r {N = n} = (n – 1) (52 – n) (51 – n)/50 x 49 x 17 x 13 where $2 \leq n \leq 50$ **(IIT JEE – 1983 – 3 Marks)**

Q 6.
A, B, C are events such that **(IIT JEE – 1983 – 2 Marks)**
P (A) = 0.3, P (B) = 0.4, P (C) = 0.8
P (AB) = 0.008, P (AC) = 0.28, P (ABC) = 0.09
If P (A ∪ B ∪ C) $\geq$ 0.75, then show that P (BC) lies in the interval $0.23 \leq x \leq 0.48$

Q 7.
A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of the occurrence of A.
(IIT JEE – 1984 – 2 Marks)

Q 8.
In a certain city two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population that reads an advertisement? **(IIT JEE – 1984 – 4 Marks)**

Q 9.

In a multiple – choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the questions only if he ticks the correct answers. The candidate decides to tick the answers at random, if he is allowed up to three chances to answer the questions, find the probability that he will get marks in the questions. **(IIT JEE – 1985 – 5 Marks)**

Q 10.

A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelfth testing. **(IIT JEE – 1986 – 5 Marks)**

Q 11.

A man takes a step forward with probability 0.4 and backwards with probability 0.6 Find the probability that at the end of eleven steps he is one step away from the starting point.

(IIT JEE – 1987 – 3 Marks)

Q 12.

An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white it is not replaced into the urn. Otherwise it is replaced along with another ball of the same color. The process is repeated. Find the probability that the third ball drawn is black. **(IIT JEE – 1987 - 4 Marks)**

Q 13.

A box contains 2 fifty paisa coins, 5 twenty five paisa coins and a certain fixed number N (≥ 2) of ten and five paisa coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paisa. **(IIT JEE – 1988 – 3 Marks)**

Q 14.

Suppose the probability for A to win game B is 0.4. If A has an option of playing either a "best of 3 games" of a "best of 5 games" match against B, which option should be choose so that the probability of his winning the match is higher? (No game ends in a draw). **(IIT JEE – 1989 – 5 Marks)**

Q 15.

A is set containing n elements. A subset P of A is chosen at random. The set A reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. **(IIT JEE - 1990 – 5 Marks)**

Q 16.

In a test an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it. **(IIT JEE – 1991 – 4 Marks)**

Q 17.

A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as **(IIT JEE – 1992 – Marks)**

A = (the first bulb is defective)

B = (the second bulb is non – defective)

C = (the two bulbs are both defective or both non defective)
Determine whether
(i) A, B, C are pair wise independent
(ii) A, B, C are independent.

Q 18.
Numbers are selected at random, one at a time, from the two digit numbers 00, 01, 02. . . 99 with replacement. An even E occurs if only if the product of the two digits of a selected numbers is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. **(IIT JEE – 1993 – 5 Marks)**

Q 19.
An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4 . . . . 12 are picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8? **(IIT JEE - 1994 – 5 Marks)**

Q 20.
In how many ways three girls and nine boys can be seated in two vans, each having numbers seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? **(IIT JEE - 1994 – 5 Marks)**

Q 21.
Sixteen players S1, S_2 . . . . . S_{16} play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength **(IIT JEE - 1997 C – 5 Marks)**
(a) Find the probability that the player S_1 is among the eight winners.
(b) Find the probability that exactly one of the two players S_1 and S_2 is among the eight winners.

Q 22.
If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement determine the probability that the roots of equation $x^2 + px + q = 0$ are real. **(IIT JEE – 1997 – 5 Marks)**

Q 23.
Three players, A, B and C, toss a coin cyclically in that order (that A, B, C, A, B, C, A, B . . .)
Till a head shows. Let be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta = (1 - p)\,\alpha$. Determine α, β and γ (in terms of p) **(IIT JEE - 1998 – 8 Marks)**

Q 24.
Eight players P_1, P_2 . . . P_8 play a knock – out tournament It is known that whenever the players P_i and P_j play, the player P_i will win if i < j. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final? **(IIT JEE – 1999 – 10 Marks)**

Q 25.
A coin has probability p of showing head when tossed. It is tossed n times. Let P_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1 = 1$, $p_2 = 1 - p^2$ and $p_n = (1 - p).\ p_{n-1} + p\ (1 - p)\ p_{n-2}$ for all $n \geq 3$. **(IIT JEE 2000 - 5 Marks)**

Q 26.

An urn contains m white and n black balls. A ball is drawn at random and is put back into urn along with k additional balls of the same color as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white? **(IIT JEE - 2001 - 5 Marks)**

Q 27.

An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in this list? **(IIT JEE - 2001 - 5 Marks)**

Q 28.

A box contains N coins, m of which is fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? **(IIT JEE - 2002 - 5 Marks)**

Q 29.

For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1^{st} exam is p. If he fails in one of the exams then the probability of passing in the next exam is p/2 otherwise it remains the same. Find the probability that he will qualify. **(IIT JEE - 2003 - 2 Marks)**

Q 30.

A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are 2/3, 1/2 and 1/3 respectively. If A is hit then find the probability that B hits the target and C does not, **(IIT JEE - 2003 - 2 Marks)**

Q 31.

A and B are two independent events. C is event in which exactly one of A or B occurs. Prove that P (C) $\geq$ P (A $\cup$ B) P ($\bar{A} \cap \bar{B}$) **(IIT JEE - 2004 - 2 Marks)**

Q 32.

A box contains 12 red and 6 white balls. Balls are drawn from the box one at a time without replacement. If in 6 draws there are at least 4 white balls, find the probability that exactly one white is drawn in the next two draws. (Binomial coefficients can be left as such) **(IIT JEE - 2004 - 4 Marks)**

Q 33.

A person goes to office either by car, scooter, bus or train, the probability of which being 1/7, 3/7, 2/7 and 1/7 respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is 2/9, 1/9, 4/9 and 1/9 respectively. Given that he reached office in time, then what is the probability that he travelled by a car. **(IIT JEE - 2005 - 2 Marks)**