

## Properties of Triangle – Questions

### Q. 1.

A triangle ABC has sides  $AB = AC = 5\text{ cm}$  and  $BC = 6\text{ cm}$ . Triangle  $A' B' C'$  is the reflection of the triangle ABC in a line parallel to AB placed at a distance 2 cm from AB, outside the triangle ABC. Triangle  $A'' B'' C''$  is the reflection of the triangle  $A' B' C'$  in a line parallel to  $B' C'$  placed at a distance of 2 cm from  $B' C'$  outside the triangle  $A' B' C'$ . Find the distance between A and  $A''$  **(IIT JEE – 1978 – 2 Marks)**

### Q. 2.

(a) If a circle is inscribed in a right angled triangle ABC with the right angle at B, show the diameter of the circle is equal to  $AB + BC - AC$ .

(b) If a triangle is inscribed in a circle, then the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the third side from the opposite vertex. Prove the above statement. **(IIT JEE – 1979 – 5 Marks)**

### Q. 3.

(a) A balloon is observed simultaneously from three points A, B and C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at C is thrice that at A. If the distance between A and B is  $a$  and the distance between B and C is  $b$ , find the height of the balloon in terms of  $a$  and  $b$ .

(b) Find the area of the smaller part of a disc of radius 10 cm, cut off by a chord AB which subtends an angle of  $22\frac{1}{2}^\circ$  at the circumference. **(IIT JEE – 1979 – 5 Marks)**

### Q. 4.

ABC is a triangle. D is the middle point of BC. If AD is perpendicular to AC, then prove that

$$\cos A \cos C = 2(c^2 - a^2) / 3ac \quad \text{(IIT JEE – 1980 - 3 Marks)}$$

### Q. 5.

ABC is a triangle with  $AB = AC$ . D is any point on the side BC. E and F are point on the side AB and AC, respectively; such that DE is parallel to AC, and DF is parallel to AB prove that

$$DF + FA + AE + ED = AB + AC \quad \text{(IIT JEE – 1980 – 5 Marks)}$$

### Q. 6.

(i) PQ is a vertical tower. P is the foot and Q is the top of the tower. A, B, C are three points in the horizontal plane through P. The angles of elevation of Q from A, B, C are equal, and each is equal to  $\theta$ . The sides of the triangle ABC are  $a, b, c$  and the area of the triangle ABC is  $\Delta$ . Show that the height of the tower is  $abc \tan \theta / \Delta$  **(IIT JEE – 1980 – 5 Marks)**

(ii) AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion CB subtends an angle  $\beta$  at P. If  $AP = n AB$ , then show that  $\tan \beta = n/2n^2 + 1$

**Q. 7.**

Let the angles A, B, C of a triangle ABC be in A. P. and let  $b : c = \sqrt{3} : \sqrt{2}$ . Find the angle A.

*(IIT JEE – 1987 – 2 Marks)*

**Q. 8.**

A vertical pole stands at a point Q on a horizontal ground. A and B are points on the ground, d meters apart. The pole subtends angles  $\alpha$  and  $\beta$  at A and B respectively. AB subtends an angle  $\gamma$  at Q. Find the height of the pole.

*(IIT JEE – 1982 – 3 Marks)*

**Q. 9.**

Four ships A, B, C and D is at sea in the following relative positions: B is on the straight line segment AC, B is due north of D and D is due west of C. The distance between B and D is 2 km.  $\angle BDA = 40^\circ$ ,  $\angle BCD = 25^\circ$ . What is the distance between A and D? [Take  $\sin 25^\circ = 0.423$ ]

*(IIT JEE – 1983 – 3 Marks)*

**Q. 10.**

The ex – radii  $r_1, r_2, r_3$  of  $\Delta ABC$  are in H. P. Show that its sides a, b, c are in A.P. *(IIT JEE – 1983 – 3 Marks)*

**Q. 11.**

For a triangle ABC it is given that  $\cos A + \cos B + \cos C = 3/2$  Prove that the triangle is equilateral.

*(IIT JEE – 1984 – 4 Marks)*

**Q. 12.**

With usual notation, if in a triangle ABC;

$b + c/11 = c + a/12 = a + b/13$  then prove that  $\cos A/7 = \cos B/19 = \cos C/25$ . *(IIT JEE – 1984 – 4 Marks)*

**Q. 13.**

A ladder rests against a wall at angle  $\alpha$  to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle  $\beta$  with the horizontal. Show that  $a = b \tan 1/2 (\alpha + \beta)$

*(IIT JEE - 1985 – 5 Marks)*

**Q. 14.**

In a triangle ABC, the median to the side BC is of length

$$\frac{1}{\sqrt{11-6\sqrt{3}}}$$

*(IIT JEE – 1985 – 5 Marks)*

And it divides the angle A into angles  $30^\circ$  and  $45^\circ$ . Find the length of the side BC.

**Q. 15.**

If in a triangle ABC,  $\cos A \cos B + \sin A \sin B \sin C = 1$ , Show that  $a : b : c = 1 : 1 : \sqrt{2}$

*(IIT JEE -1986 – 5 Marks)*

**Q. 16.**

A man observes a tower AB of height  $h$  from a point P on the ground. He moves a distance  $d$  towards the foot of the tower and finds that the angle of elevation has doubled. He further moves a distance  $3/4 d$  in the same direction and finds that the angle of elevation is three times that at P. Prove that  $36h^2 = 35d^2$ .

*(IIT JEE 1986 – 5 Marks)*

**Q. 17.**

A 2 meter long object is fired vertically upwards from the mid-point of two locations A and B, 8 meters apart. The speed of the object after  $t$  seconds is given by  $ds/dt = (2t + 1)$  meter per second. Let  $\alpha$  and  $\beta$  be the angles subtended by the object A and B, respectively after one and two seconds. Find the value of  $\cos(\alpha - \beta)$ .

*(IIT JEE – 1987 – 3 Marks)*

**Q. 18.**

A sign – post in the form of an isosceles triangle ABC is mounted on a pole of height  $h$  fixed to the ground. The base BC of triangle is parallel to the ground. A man standing on the ground at a distance  $d$  from the sign – post find that the top vertex A of the triangle subtends an angle  $\beta$  and either of the other two vertices subtends the same angle  $\alpha$  at his feet. Find the area of the triangle

*(IIT JEE – 1988 – 5 Marks)*

**Q. 19.**

ABC is a triangular park with  $AB = AC = 100$  m. A television tower stands at the midpoint of BC. The angles of elevation of the top of the tower at A, B, C are  $45^\circ$ ,  $60^\circ$ , and  $60^\circ$ , respectively. Find the height of the tower.

*(IIT JEE – 1989 – 5 Marks)*

**Q. 20.**

A vertical tower PQ stands at a point P. Points A and B are located to the South and East of P respectively. M is the mid-point of AB PAM is an equilateral triangle; and N is the foot of the perpendicular from P on AB. Let  $AN = 20$  meters and the angle of elevation of the top of the tower at N is  $\tan^{-1}(2)$ . Determine the height of the tower and the angles of elevation of the top of the tower at A and B.

*(IIT JEE – 1990 – 4 Marks)*

**Q. 21.**

The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangles.

*(IIT JEE – 1991 – 4 Marks)*

**Q. 22.**

In a triangle of base  $a$  the ratio of the other two sides is  $r$  ( $< 1$ ). Show that the altitude of the triangle of the triangle is less than or equal to  $ar / 1 - r^2$  *(IIT JEE - 1991 - 4 Marks)*

**Q. 23.**

A man notices two objects in a straight line due west. After walking a distance  $c$  due north he observes that the objects subtend an angle  $\alpha$  at his eye; and, after walking a further distance  $2c$  due north, an angle  $\beta$ . Show that the distance between the objects is  $8c/3 \cot \beta - \cot \alpha$ ; the height of the man is being ignored. *(IIT JEE - 1991 - 4 Marks)*

**Q. 24.**

Three circles touch the one another externally. The tangents at their point of contact meet at a point whose distance from a point of contact is 4. Find the product of the radii to the sum of the radii of the circles *(IIT JEE - 1992 - 4 Marks)*

**Q. 25.**

An observer at  $O$  notices that the angle of elevation of the top of a tower is  $30^\circ$ . The line joining  $O$  to the base of the tower makes an angle of  $\tan^{-1}(1/\sqrt{2})$  with the North and is inclined eastwards. The observer travels a distance of 300 meters towards the North to a point  $A$  and finds the tower to his East. The angle of elevation of the top of the tower at  $A$  is  $\phi$ , Find  $\phi$  and the height of the tower

*(IIT JEE 1993 - 5 Marks)*

**Q. 26.**

A tower  $AB$  leans towards west making an angle  $\alpha$  with the vertical. The angular elevation of  $B$ , the topmost point of the tower is  $\beta$  as observed from a point  $C$  due west of  $A$  at a distance  $d$  from  $A$ . If the angular elevation of  $B$  from a point  $D$  due east of  $C$  at a distance  $2d$  from  $C$  is  $\gamma$ , then prove that

$$2 \tan \alpha = -\cot \beta + \cot \gamma. \quad \text{(IIT JEE - 1994 - 4 Marks)}$$

**Q. 27.**

Let  $A_1, A_2, \dots, A_n$  be the vertices of an  $n$ -sided regular polygon such that  $1/A_1 A_2 = 1/A_1 A_3 + 1/A_1 A_4$ , Find the value of  $n$ . *(IIT JEE - 1994 - 4 Marks)*

**Q. 28.**

Consider the following statements concerning a triangle  $ABC$  *(IIT JEE - 1994 - 5 Marks)*

(i) The sides  $a, b, c$  and area  $\Delta$  are rational.

(ii)  $a, \tan B/2, \tan C/2$  are rational

(iii)  $a, \sin A, \sin B, \sin C$  are rational.

Prove that (i)  $\Rightarrow$  (ii)  $\Rightarrow$  (iii)  $\Rightarrow$  (i)

**Q. 29.**

A semicircular arch AB of length  $2L$  and a vertical tower PQ are situated in the same vertical plane. The feet A and B of the arch and the base Q of the tower are at the same horizontal level, with B between A and Q. A man at A finds the tower hidden from his view due to the arch. He starts crawling up the arch and just sees the topmost point P of the tower after covering a distance  $L/2$  along the arch. He crawls further to the topmost point of the arch and notes the angle of elevation of P to be  $\theta$ . Compute the height of the tower in terms  $L$  and  $\theta$ . *(IIT JEE - 1997 C - 5 Marks)*

**Q. 30.**

Let A, B, C be three angles such that  $A = \pi/4$  and  $\tan B \tan C = p$ . Find all possible values of  $p$  such that A, B, C are the angles of triangle. *(IIT JEE - 1997 C - 5 Marks)*

**Q. 31.**

A bird flies in a circle on a horizontal plane. An observer stands at a point on the ground. Suppose  $60^\circ$  and  $30^\circ$  are the maximum and the minimum angles of elevation of the bird and that they occur when the bird is at the points P and Q respectively on its path. Let  $\theta$  be the angle of elevation of the bird when it is a point on the arc of the circle exactly midway between P and Q. Find the numerical value of  $\tan^2 \theta$ . (Assume that the observer is not inside the vertical projection of the path of the bird.) *(IIT JEE - 1998 - 8 Marks)*

**Q. 32.**

Prove that a triangle ABC is equilateral if and only if

$$\tan A + \tan B + \tan C = 3\sqrt{3}. \quad \text{(IIT JEE 1998 - 8 Marks)}$$

**Q. 33.**

Let ABC be a triangle having O and I as its circumcenter and in centre respectively. If, R and r are the circumradius and the inradius, respectively, then prove that  $(10)^2 = R^2 - 2Rr$ . Further show that the triangle BIO is a right - angled triangle if and only if b is arithmetic mean of a and c.

*(IIT JEE - 1999 - 10 Marks)*

**Q. 34.**

Let ABC be a triangle with incenter I and inradius r. Let D, E, F be the feet of the perpendiculars from I to the sides BC, CA and AB respectively. If  $r_1, r_2$  and  $r_3$  are the radii of circles inscribed in the quadrilaterals AFIE, BDIF and CEID respectively, prove that

*(IIT JEE - 2000 - 7 Marks)*

$$r_1/r - r_1 + r_2/r - r_2 + r_3/r - r_3 = r_1 r_2 r_3 / (r - r_1) (r - r_2) (r - r_3).$$

**Q. 35.**

If  $\Delta$  is the area of a triangle with side lengths a, b, c, then show that  $\Delta \leq 1/4 \sqrt{(a+b+c)abc}$ . Also show that the equality occurs in the above inequality if and only if  $a = b = c$ . *(IIT JEE - 2001 - 6 Marks)*

**Q. 36.**

If  $I_n$  is the area of  $n$  sided regular polygon inscribes in a circle of unit radius and  $O_n$  be the area of the polygon circumscribing the given circle, prove that **(IIT JEE – 2003 – 4 Marks)**

$$I_n = O_n/2 \left( 1 + \sqrt{1 - \left(\frac{2I_n}{n}\right)^2} \right)$$