

## Quadratic Equation and Inequalities (Inequalities)

### SUBJECTIVE PROBLEMS:

#### Q. 1.

Solve for x:  $4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$

(IIT JEE – 1978 – 5 Marks)

#### Q. 2.

If  $(m, n) = (1 - x^m)(1 - x^{m-1}) \dots (1 - x^{m-n+1}) / (1 - x)(1 - x^2) \dots (1 - x^n)$

Where m and n are positive integers ( $n \leq m$ ), show that  $(m, n + 1) = (m - 1, n + 1) + x^{m-n-1} (m - 1, n)$

(IIT JEE – 1978 – 6 Marks)

#### Q. 3.

Solve for x:  $\sqrt{x} + 1 - \sqrt{x} - 1 = 1$ .

(IIT JEE – 1978 – 4 Marks)

#### Q. 4.

Solve the following equation for x:

$2 \log_x a + \log_{ax} a + 3 \log_{a2x} a = 0, a > 0$

(IIT JEE – 1978 – 3 Marks)

#### Q. 5.

Show the square of  $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$  is a rational number.

(IIT JEE – 1978 3 Marks)

#### Q. 6.

Sketch the solution set of the following system of inequalities:

$x^2 + y^2 - 2x \geq 0; 3x - y - 12 \geq 0.$

(IIT JEE – 1978 – 4 Marks)

#### Q. 7.

Find all integers x for which

(IIT JEE – 1978 – 3 Marks)

$(5x - 1) < (x + 1)^2 < (7x - 3).$

#### Q. 8.

If  $\alpha, \beta$  are the roots of  $x^2 + px + q = 0$  and  $\gamma, \delta$  are the roots of  $x^2 + rx + s = 0$ , evaluate  $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$  in terms of p, q, r and s.

Deduce the condition that the equations have a common root.

(IIT JEE – 1979 - Marks)

**Q. 9.**

Show that for any triangle with sides a, b and c

$$3(ab + bc + ca) < (a + b + c)^2 < 4(bc + ca + ab)$$

When are the first two expressions equal?

(IIT JEE – 1979 – 4 Marks)

**Q. 10.**

Given  $n^4 < 10^n$  for a fixed positive integer  $n \geq 2$ , prove that  $(n + 1)^4 < 10^{n+1}$ .

(IIT JEE – 1980 – 5 Marks)

**Q. 11.**

$$\text{Let } y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$$

(IIT JEE – 1980 – 5 Marks)

Find all the real values of x for which y takes real values.

**Q. 12.**

For what values of m, does the system of equations?

$$3x + my = m$$

$$2x - 5y = 20$$

Has solution satisfying the conditions  $x > 0, y > 0$

(IIT JEE – 1980 – 5 Marks)

**Q. 13.**

Find the solution set of the system

$$x + 2y + z = 1;$$

$$2x - 3y - w = 2;$$

$$x \geq 0; y \geq 0; z \geq 0; w \geq 0.$$

(IIT JEE – 1980 – 4 Marks)

**Q. 14.**

Show that the equation  $e^{-\sin x} - e^{\sin x} - 4 = 0$  has no real solution.

(IIT JEE – 1982 – 2 Marks)

**Q. 15.**

$mn$  squares of equal size are arranged to form a rectangle of dimension  $m$  by  $n$ , where  $m$  and  $n$  are natural numbers. Two squares will be called 'neighbours' if they have exactly one common side. A

natural number is written in each square such that the number written in any square is the arithmetic mean of the numbers written in its neighbouring squares. Show that this is possible only if all the numbers used are equal. **(IIT JEE – 1982 – 5 Marks)**

**Q. 16.**

If one root of the quadratic equation  $ax^2 + bx + c = 0$  is equal to the  $n$  – th power of the other, then show that **(IIT JEE – 1983 – 3 Marks)**

$$(ac^n)^{1/n+1} + (a^n c)^{1/n+1} + b = 0$$

**Q 17.**

Find all real values of  $x$  which  $x^2 - 3x + 2 > 0$  and  $x^2 - 2x - 4 \leq 0$  **(IIT JEE – 1983 – 2 Marks)**

**Q 18.**

Solve for  $x$ ;  $(5 + 2\sqrt{6})^{x^2-3} + (5 - 2\sqrt{6})^{x^2-3} = 10$  **(IIT JEE – 1985 – 5 Marks)**

**Q. 19.**

For  $a \leq 0$ , determine all real roots of the equation  $\Rightarrow x^2 - 2a |x - a| - 3a^2 = 0$  **(IIT JEE – 1986 – 5 Marks)**

**Q 20.**

Find the set of all  $x$  for which  $2x/(2x^2 + 5x + 2) > 1/(x + 1)$  **(IIT JEE – 1987 – 3 Marks)**

**Q. 21.**

Let  $\alpha_1, \alpha_2$  and  $\beta_1, \beta_2$  be the roots of  $ax^2 + bx + c = 0$  and  $px^2 + qx + r = 0$  respectively. If the system of equations  $\alpha_1 y + \alpha_2 z = 0$  and  $\beta_1 y + \beta_2 z = 0$  has nontrivial solution, then prove that  $b^2/q^2 = ac/pr$ . **(IIT JEE – 1987 – 3 Marks)**

**Q. 22.**

Solve  $|x^2 + 4x + 3| + 2x + 5 = 0$  **(IIT JEE – 1988 - 5 Marks)**

**Q 23.**

Let  $a, b, c$  be real. If  $ax^2 + bx + c = 0$  has two real roots  $\alpha$  and  $\beta$ , where  $\alpha < -1$  and  $\beta > 1$ , then show that

$$1 + \frac{c}{a} + \left| \frac{b}{a} \right| < 0. \quad \text{(IIT JEE – 1995 – 5 Marks)}$$

**Q. 24.**

Find the set of all solutions of the equation

**(IIT JEE – 1997C – 3 Marks)**

$$2^{|y|} - |2^{y-1} - 1| 2^{y-1} + 1$$

**Q. 25.**

Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the lengths of sides of the quadrilateral; prove that **(IIT JEE – 1997 – 5 Marks)**

$$2 \leq a^2 + b^2 + c^2 + d^2 \leq 4.$$

**Q. 26.**

If  $\alpha, \beta$  are the roots of  $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) and  $a + \delta, \beta + \delta$  are the roots of  $Ax^2 + Bx + c = 0$ , ( $A \neq 0$ ) for some constant  $\delta$ , then prove that  $b^2 - 4ac/a^2 = B^2 - 4AC/A^2$ . **(IIT JEE – 2000 – 4 Marks)**

**Q. 27.**

Let a, b, c be real numbers with  $a \neq 0$  and let  $\alpha, \beta$  be the roots of the equation

$ax^2 + bx + c = 0$ . Express the roots of  $a^3x^2 + abcx + c^3 = 0$  in terms of  $\alpha, \beta$ .

**(IIT JEE – 2001 – 4 Marks)**

**Q. 28.**

If  $x^2 + (a - b)x + (1 - a - b) = 0$  where  $a, b \in \mathbb{R}$  then find the values of a for which equation has unequal real roots for all values of b. **(IIT JEE – 2003 – 4 Marks)**

**Q. 29.**

If a, b, c is positive real numbers. Then prove that  $(a + 1)^7 (b + 1)^7 (c + 1)^7 > 7^7 a^4 b^4 c^4$

**(IIT JEE – 2004 – 4 Marks)**

**Q. 30.**

Let a and b the roots of the equation  $x^2 - 10cx - 11d = 0$  and those of  $x^2 - 10ax - 11b = 0$  are c, d then the value of  $a + b + c + d$ , when  $a \neq b \neq c \neq d$ , is. **(IIT JEE – 2006 – 6 Marks)**