

Sequences and Series

SUBJECTIVE PROBLEMS:

Q. 1.

The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation. $2A + G^2 = 27$. Find the two numbers. (IIT JEE -1979 -3Marks)

Q. 2.

The interior angles of a polygon are in arithmetic progression. The smallest angle is 120° , and the common difference is 5° , Find the number of sides of the polygon. (IIT JEE-1980 -4Marks)

Q. 3.

If a_1, a_2, \dots, a_n are in arithmetic progression, where $a_i > 0$ for all i , Show that

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}} \quad \text{(IIT JEE-1982-2Marks)}$$

Q. 4.

Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exists, how many such progressions are possible? (IIT JEE-1982-3Marks)

Q. 5.

Find three number a, b, c between 2 and 18 such that

(i) their sum is 25

(ii) the numbers 2, a, b are consecutive terms of an A. P. and

(iii) the numbers b, c, 18 are consecutive terms of a G. P.

(IIT JEE-1983 -2Marks)

Q. 6.

If $a > 0$, $b > 0$ and $c > 0$, prove that

$$(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9 \quad \text{(IIT JEE-1984-2Marks)}$$

Q. 7.

If n is a natural such that

$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} \dots p_k^{\alpha_k}$ and p_1, p_2, \dots, p_k are distinct primes, then show

that $\ln n \geq k \ln 2$

(IIT JEE- 1984 -2Marks)

Q 8.

Find the sum of the series:

$$\sum_{r=0}^n (-1)^r \binom{n}{r} \left[-\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} \dots \text{up to } m \text{ terms} \right] \quad \text{(IIT JEE-1985 -5Marks)}$$

Q. 9.

The sum of the squares of three distinct real numbers, which are in G. P., is S^2 . If their sum is aS , show that

$$a^2 \in \left(\frac{1}{3}, 1 \right) \cup (1, 3). \quad \text{(IIT JEE-1986-5Marks)}$$

Q. 10.

Solve for x the following equation :

$$\log_{(2x+3)} (6x^2 + 23x + 21) = 4 - \log_{(3x+7)} (4x^2 + 12x + 9) \quad \text{(IIT JEE -1987-3Marks)}$$

Q. 11.

If $\log_3 2$, $\log_3 (2x - 5)$, and $\log_3 (2^x - 7/2)$ are in arithmetic progression, determine the value of x .

(IIT JEE -1990-4Marks)

Q. 12.

Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $(n + 1)/n - 1)^2 p$.

(IIT JEE -1991-4Marks)

Q 13.

If $S_1, S_2, S_3, \dots, S_n$ are the sums of infinite geometric series whose first terms are $1, 2, 3, \dots, n$ and whose common ratios are $1/2, 1/3, 1/4, \dots, 1/n + 1$ respectively, then find the values of $S_1^2 + S_2^2 + S_3^2 + \dots + S_{2n-1}^2$

(IIT JEE-1991-4Marks)

Q. 14.

The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A. P. Find the intervals in which β and γ lie.

(IIT JEE-1996 -3Marks)

Q. 15.

Let a, b, c, d be real numbers in G. P. If u, v, w , satisfy the system of equations

$$u + 2v + 3w = 6 \quad \text{(IIT JEE – 1999 – 10Marks)}$$

$$4u + 5v + 6w = 12$$

$$6u + 9v = 4$$

Then show that the roots of the equations

$$(1/u + 1/v + 1/w)x^2 + [(b-c)^2 + (c - a)^2 + (d - b)^2] x + u + v + w = 0$$

And $20x^2 + 10(a - d)^2 x - 9 = 0$ are reciprocals of each other.

Q. 16.

The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. (IIT JEE-2000 - 4Marks)

Q 17.

Let a_1, a_2, \dots, a_n be positive real numbers in geometric progression. For each n , let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of a_1, a_2, \dots, a_n . Find an expression for the geometric mean of G_1, G_2, \dots, G_n in terms of $A_1, A_2, \dots, A_n, H_1, H_2, \dots, H_n$. (IIT JEE -2001-5 Marks)

Q. 18.

Let a, b , be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression,

$$\text{show that } G_1 G_2 = G_2 / H_1 H_2 = A_1 + A_2 / H_1 H_2 = (2a + b) (a + 2b) / 9ab \quad \text{(IIT JEE-2002 -5Marks)}$$

Q. 19.

If a, b, c are in A. P., a^2, b^2, c^2 are in H. P. , then prove that

$$\text{either } a = b = c \text{ or } a, b, -c/2 \text{ form a G. P.} \quad \text{(IIT JEE -2003 -4Marks)}$$

Q. 20.

$$\text{If } a_n = 3/4 - (3/4)^2 + (3/2)^2 + \dots + (-1)^{n-1} (3/4)^n \text{ and} \quad \text{(IIT JEE- 2000 – 6 Marks)}$$

$b_n = 1 - a_n$, then find the least natural number n_0 such that $b_n > a_n \forall n \geq n_0$.