Solution Paper - I
PHYSICS

1. (B)

Δxm = (X – L)

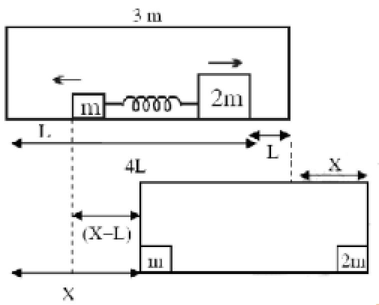
Δx2m = (X + L)

Δx3m = X

⇒m (X – L) + 2m (X + L) + 3mx = 0

⇒6mx – mL = 0

⇒$x = \frac{1}{6}$



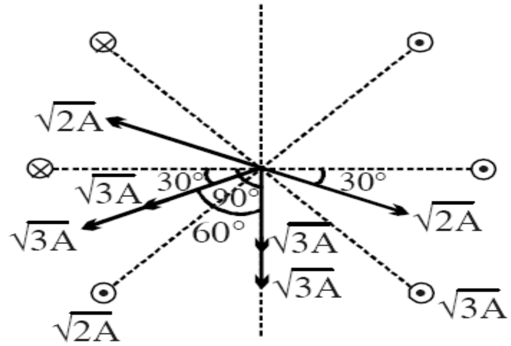
2. (A)

q2 = q0 C2 / (C1 + C2) (1 – e–t / |), where τ is

$$\frac{RC_1C_2}{C_1 + C_2}$$

3. (C) Just after closing switch no current flows through R2 so I1 = 3mA Long time after closing switch no current flows through C so I2 = 2mA Directly after re-opening the switch no current flows through R1 and the capacitor will discharge through R2 so I3 = 2mA]

$B_1 = \frac{\mu_0}{4\pi} \times \frac{\sqrt{3}\times 2}{3\times 10^{-2}} = 10^{-5} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 10^{-5}$



$= B = \sqrt{(2B_1)^2 + (2B_2)^2 + 2(2B_1)\times(2B_2)\cos 60^\circ}$

$= B\sqrt{4+4+4}$

$= 4 \times 10^{-5} = 40 \times 10^{-6}$ along 9

5. (C)

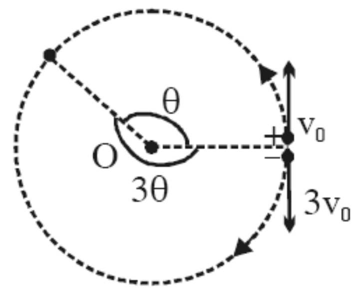
$r = \frac{p}{qB}$ = same, $T_+ = \frac{2\pi m_+}{qB} = \frac{6\pi m}{qB}$, $T_- = \frac{2\pi m}{qB}$

as $T_+ = 3T_-$, They will meet at $\theta = \pi/2$

$q = 1\ \mu C$, $B = 2\pi\mu T$, $m = 10^{-15}$ kg

The time is $= \frac{T_+}{4} = \frac{6\pi m}{4qB} = \frac{6\times\pi\times 10^{-15}}{4\times 1\times 10^{-6}\times 2\pi\times 10^{-6}}$

$= 0.75 \times 10^{-3}\ S = 750\ \mu S$



6. (C)

for 1 loop $\oint_0^{\ell_1} B\cdot\vec{ds} = \mu_0 I$

$\Rightarrow$ for N loop $\oint_0^{\ell} B\cdot\vec{ds} = N \oint_0^{\ell_1} \vec{B}\cdot\vec{ds} = \mu_0 NI$

7. (A)

$R = \frac{20^2 \times \sin 120^\circ}{g} = 20\sqrt{3} = \frac{\Delta R}{R} = \frac{2\Delta U}{U} \Rightarrow \Delta R = \frac{2\times 5}{100} \times 20\sqrt{3} = 2\sqrt{3}$

$20\sqrt{3} - 2\sqrt{3} < R < 20\sqrt{3} + 2\sqrt{3} \Rightarrow 31.1\text{m} < R < 38.1\ \text{m}$

8. (C)
$TV^{r-1} = \text{const.}$

9. (C)
$\Delta\phi = 2n\pi$

$$\Rightarrow \frac{\pi}{2} + \frac{2\pi}{\lambda} d \sin\theta = 2n\pi$$

$$\frac{2\pi}{\lambda} d \sin\theta = \left(2n - \frac{1}{2}\right)\pi$$

$$\sin\theta = \left(2n - \frac{1}{2}\right)\frac{\lambda}{2d} = \frac{1}{2} \times \frac{\lambda}{2\times 3\lambda} = \frac{1}{12}$$

$$\Rightarrow = \frac{y}{\sqrt{(100\lambda)^2}} = \frac{1}{12}$$

$$144y^2 = (100\lambda)^2$$

$$y \approx \frac{100\lambda}{12} = \frac{25\lambda}{3}$$

10. (A)

$$\text{Radius } R_1 = \frac{mv}{qB_1}$$

In a given fields radius can be same for every entry, if magnitude of B1 and B2 are equal.

11. (A), (C), (D)

12. (B), (D) The flux through the differential cube is

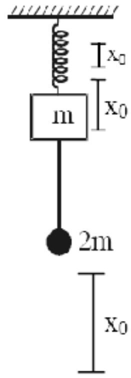
$$d\phi = \frac{\partial E}{\partial x}(dx\,dy\,dz) + \frac{\partial E}{\partial y}(dx\,dy\,dz) + \frac{\partial E}{\partial z}(dx\,dy\,dz)$$

$$= (3 + 4 + 5)\,dx\,dy\,dz$$

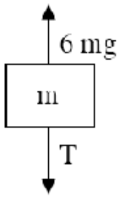
$$d\phi = 12\,dx\,dy\,dz = 12\,dV$$

$$dq = 12\,\varepsilon_0\,dV$$

13. (A), (D)



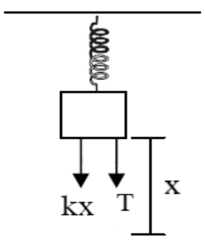
(A)



$2(6\,mg - T - mg = ma)$
$-(T - 2\,mg = 2\,ma)$
$12\,mg - 3\,T = 0$
$T = 4\,mg$

(B) $Kx + T + mg = ma$
$2mg - T = 2ma$
for $T = 0$, $a = g$
$\Rightarrow \; x = 0$

as particle is released x0 below equilibrium so it will go x0 above equilibrium.
i.e. at x = 0 ⇒Tmin = 0



(C) & (D) for x0 > $\frac{3mg}{k}$ it will be no longer SHM as string will block.

14. (A), (D)

(A) $\frac{\mu_0 K}{2}$. Hence, $K = \sigma V$

(D) $F = q\vec{u} \times \vec{B}$ hence upwards.

15. (A), (B)

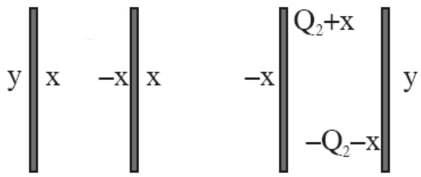
$xa + xb + (Q_2 + x)c = 0$

$$x = \frac{-Q_2 c}{a+b+c}$$

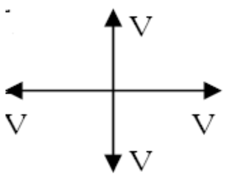
$Q_1 = y + x + y - Q_2 \quad -x$

$$y = \frac{Q_1 + Q_2}{2}$$

$$V = \frac{Q_2 ca}{(a+b+c)S\varepsilon_0}]$$



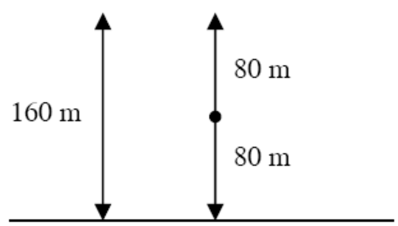
16. [2] Initially VCM = 0 and particles are spreading symmetrically in all possible direction. Finally if topmost particle come back to initial point, its speed must be v and directed downward at this moment distance travelled by CM is 80 m



(as acc. of CM is $t = \sqrt{\frac{2h}{g}} = 4$ sec.)

$\Rightarrow 4 = \frac{2V}{g}$

$\Rightarrow 4 = \frac{2V}{g} \Rightarrow v - 2g \qquad \therefore v = 20$ m/s



17. [2] For minimum condition it should just touch topmost This imply three facts

$D = \frac{u^2 \sin\theta \cos\theta}{g}$ .... (I)

$h_{max} = \frac{u^2 \sin^2\theta}{2g}$ .... (II)

r = radius of curvature at topmost point

$= \frac{u^2 \cos^2\theta}{g}$ .... (III)

from (II) & (III)

$\frac{u^2 \cos^2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow \tan\theta = \sqrt{2}$

from (I) & (II)

$\frac{D}{r} = \tan\theta$

$D = r\tan\theta \qquad \therefore D = 2$ m

18. [1] as object is moving in x – y plane with centre at origin

$\Rightarrow z = 0$

$$\vec{F} = \left[(2x - y + 3z)\hat{i} + (x + y - z)\hat{j} + (5x - 2y - z)\hat{k}\right]$$

$$\vec{F} = \left[(2x - y)\hat{i} + (x + y)\hat{j} + (5x - 2y)\hat{k}\right]$$

$$y = R\sin\theta \Rightarrow dy = R\cos\theta\, d\theta$$

$$x = R\cos\theta \Rightarrow dx = -R\sin\theta\, d\theta$$

$$\vec{dS} = dx\,\hat{i} + dy\,\hat{j}$$

$$= -R\sin\theta\, d\theta\,\hat{i} + R\cos\theta\, d\theta\,\hat{j}$$

$$\Rightarrow d\omega = \vec{F}\cdot\vec{dS}$$

$$= \int(2x - y)(-y\, d\theta) + \int(x + y)x\, d\theta$$

$$= \int\left(y^2 - 2xy\right)d\theta + \int\left(x^2 + xy\right)d\theta$$

$$= R^2\left[\int_0^{2\pi}\left(\sin^2\theta - 2\sin\theta\cos\theta\right)d\theta + \int_0^{2\pi}\left(\cos^2\theta + \sin\theta - \cos\theta\right)d\theta\right.$$

$$\left[\int_0^{2\pi}\left(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta\right)d\theta\right.$$

$$= R^2\left[\int_0^{2\pi}\left(1 - \frac{\sin 2\theta}{2}\right)d\theta\right.$$

$$= R^2(2\pi) = 32\pi \simeq 100 \text{ joule}$$

19. [2]

$$13(2x) + R = 200$$

$$13(2(10 - x)) + R = 100$$

$$260 + 2R = 300$$

$$R = 20\,\Omega \quad = 2 \text{ deca ohm}$$

20. [5] By Energy Conservation

$$= mg\frac{R}{\sqrt{2}} = \frac{1}{2}\frac{m(\sqrt{2}R)^2\omega^2}{3} \Rightarrow \omega^2 = \frac{3g}{\sqrt{2}R}$$

$$\text{Now, } 2N\cos 45^\circ - mg = m\times\frac{3g}{\sqrt{2}R}\times\frac{R}{\sqrt{2}} \Rightarrow N = \frac{5mg}{2\sqrt{2}} = 50$$

CHEMISTRY

21. (A) 3 mole atoms of oxygen are present in 1 mole of BaCO3 So, 1.5 mole atoms of oxygen will be present in

$= \frac{1}{3}\times 1.5 = 0.5$ mole of BaCO3

22. (D) H2O is polar, hence it has higher critical temperature.

23. (B) Let the wavelength of particle = x

$$\text{So, velocity} = \frac{x}{100}$$

$$\lambda = \frac{h}{mv}; \quad x = \frac{h\times 100}{m\times x}$$

$$x^2 = \frac{100h}{m}; \quad x = 10\sqrt{\frac{h}{m}}$$

24. (B)

25. (C)

$$K_{sp} = S^2$$

$$S = \sqrt{10^{-8}} = 10^{-4}\text{ mol L}^{-1}$$

$$= 10^{-4}\times 283\text{ g L}^{-1}$$

$$= 2.83\times 10^{-2}\text{ g L}^{-1}$$

1000 mL of the solution contains AgIO3 = $2.83 \cdot 10^{-2}$ g

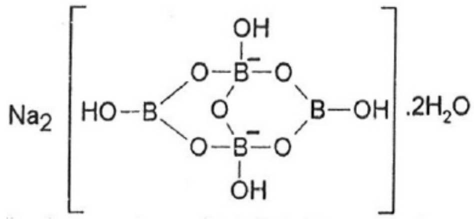
100 mL of the solution contains AgIO3 = $2.83 \cdot 10^{-3}$ g

26. (A)

$$\Delta S_f = \frac{\Delta H_f}{T_f} = \frac{2930}{300} = 9.77 \text{ J K}^{-1} \text{ mol}^{-1}$$

27. (A)

28. (A)



The boron atoms bearing negative charges are sp3 hybridized, while other two boron atoms are sp2 hybridized.

29. (B)

30. (A) In presence of light (a free radical producing agent), toluene undergoes free radical substitution in the side chain to form benzyl chloride.

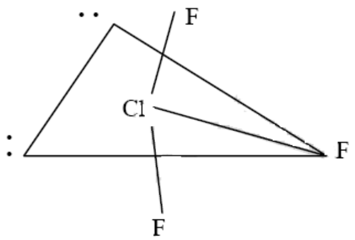
31. (B), (C)

32. (A), (B), (C), (D) Fact

33. (A), (B), (D)

34. (A), (C), (D) $Mg(NO_3)_2$ is more covalent and more readily decomposes.
$Mg(NO_3)_2(s) \rightarrow MgO(s) + 2NO_2(g) + 1/2O_2(g)$

35. (A), (B), (C)
$BrF_5 + 3H_2O \rightarrow 5HF + HBrO_3$ In ClF_3, Cl is sp3d hybridized. It is arrow–shaped (also mentioned as T–shaped)



Equatorial Cl–F bond distance is shorter than the other two.\

36. [5]

$CH_3CH_2CH_2CH_2CH_2OH$ $\quad$ $CH_3CH(CH_3)CH_2CH_2OH$

$C_2H_5CH(CH_3)CH_2OH$ (dl) $\quad$ $(CH_3)_3CCH_2OH$

37. [4] **38.** [3] **39.** [6]

40. [2]

$\overset{sp^3}{CH_3}-\overset{sp^3}{CH_2}-\overset{sp}{C}\equiv\overset{sp}{CH}$

41. (A)

Assuming arg $z_1 = \theta_1$ and arg $z_2 = \theta_1 + \theta_2$

$$\frac{\alpha\beta z_1}{\gamma\delta z_2}+\frac{\gamma\delta z_2}{\alpha\beta z_1}=\frac{\alpha\beta|z_1|e^{i\theta_1}}{\gamma\delta|z_2|e^{i(\theta_1+\theta_2)}}+\frac{\gamma\delta|z_2|e^{i(\theta_1+\theta_2)}}{\alpha\beta|z_1|e^{i\theta_1}}$$

$$= e^{-i\theta_2}+e^{i\theta_2}=2\cos\theta_2$$

which lies in [–2, 2]

42. (D)

We have $\frac{\pi}{2}<\theta<\frac{2\pi}{3}$,

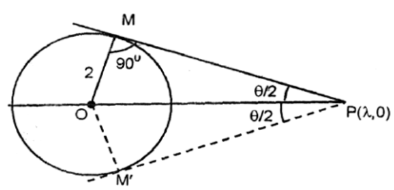
i.e. $\frac{\pi}{4}<\frac{\theta}{2}<\frac{\pi}{3}$

$\Rightarrow \frac{1}{\sqrt{2}}<\sin\frac{\theta}{2}<\frac{\sqrt{3}}{2}$

But $\sin\frac{\theta}{2}=\frac{2}{|\lambda|}\Rightarrow\frac{1}{\sqrt{2}}<\frac{2}{|\lambda|}<\frac{\sqrt{3}}{2}$

$\Rightarrow \frac{4}{\sqrt{3}}<|\lambda|<2\sqrt{2}$

$\Rightarrow \frac{4}{\sqrt{3}}<\lambda<2\sqrt{2}$ or $-2\sqrt{2}<\lambda<-\frac{4}{\sqrt{3}}$



43. (B) f (x) is piecewise continuous and decreasing function and the graph of function will cut x–axis at 2008 points. Hence answer is (B).

44. (D)

For real roots $b^2 \geq 4ac$ or $\left(\frac{b}{2}\right)^2 \geq ac$

$\therefore$ Required probability = $\frac{4}{27}$

45. (D) **Case – I :** We choose first square from corner square. In this total number of ways of choosing 2 squares = 4 · 2.

Case – II : We choose first square from first or last row or column except corner one. Total number of ways = 24 · 3

Case – III : Any square except from boundary one

Total ways = 36 · 4

Total ways = 8 + 72 + 144 = 224

Now any square can be chosen as first or second

$\therefore$ Required probability = $\frac{112}{^{64}C_2} = \frac{1}{18}$

46. (C)

$\cos^{-1}(\log_3 (x^2 + 17x + 75))$ is defined if $-1 \le \log_3 (x^2 + 17x + 75) \le 1$.

$\Rightarrow \frac{1}{3} \le x^2 + 17x + 75 \le 3$

Now, $x^2 + 17x + 75 \le 3$

$\Rightarrow x^2 + 17x + 72 \le 0$

$\Rightarrow (x + 8)(x + 9) \le 0$

$\Rightarrow x \in [-9, -8]$

and $x^2 + 17x + 75 \ge \frac{1}{3}$

$\Rightarrow 3x^2 + 51x + 225 - 1 \ge 0$

$\Rightarrow 3x^2 + 51x + 224 \ge 0$

Here $D < 0$

$\Rightarrow 3x^2 + 51x + 224 > 0 \ \forall\ x \in R$

Hence $x \in [-9, -8]$.

47. (B) Let d be the common difference of the A.P. Then a2r = a2r–1 + d

$\Rightarrow \sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} (a_{2r-1} + d) \Rightarrow \sum_{r=1}^{10^{99}} a_{2r} = \sum_{r=1}^{10^{99}} a_{2r-1} + 10^{99} d$

$\Rightarrow 10^{100} = 10^{99} + 10^{99} d$

$\Rightarrow d = \frac{10^{100} - 10^{99}}{10^{99}} = \frac{10^{99}(10-1)}{10^{99}} = 9$

48. (C)

$\log_{0.09} (x^2 + 2x)$ is defined when $x^2 + 2x > 0$

$\Rightarrow (x)(x + 2) > 0 \Rightarrow x \in (-\infty, -2) \cup (0, \infty)$ .... (1)

and $\log_{0.3} \sqrt{x+2}$ is defined when $x + 2 > 0$

$\Rightarrow x \in (-2, \infty)$ .... (2)

Also $\log_{0.09} (x^2 + 2x) \ge \log_{0.3} \sqrt{x+2}$

$\Rightarrow \frac{1}{2} \log_{0.3} (x^2 + 2x) \ge \log_{0.3} \sqrt{x+2}$

$\Rightarrow \log_{0.3}(x^2 + 2x) \ge \log_{0.3}(x + 2)$

$\Rightarrow x^2 + 2x \le x + 2 \Rightarrow x^2 + x - 2 \le 0$

$\Rightarrow x \in [-2, 1]$ .... (3)

From (1), (2) and (3) the solution is $x \in (0, 1]$

49. (C)

Given $\alpha + i\beta = \left(\frac{-1+i\sqrt{3}}{2}\right)^{3n_1/4} (1-i)^{-2n_2}$

Here $\frac{-1+i\sqrt{3}}{2} = e^{\frac{i2\pi}{3}}$ and $1-i = \sqrt{2}e^{\frac{-i\pi}{4}}$

Now, $\alpha + i\beta = e^{\frac{i\pi n_1}{2}} \cdot \left(\sqrt{2}\right)^{-2n_2} e^{\frac{i\pi}{2}n_2}$

$\Rightarrow \alpha + i\beta = 2^{-n_2} e^{i(n_1+n_2)\pi/2}$

$\Rightarrow \alpha = 2^{-2n_2} \cos(n_1+n_2)\frac{\pi}{2} = 0$, when $n_1 + n_2$ is odd

$\beta = 2^{-n_2} \sin(n_1+n_2)\frac{\pi}{2} = 0$, when $n_1 + n_2$ is even

Thus (C) is false.

50. (B) The form being 0/0, the required limit by L' Hospital's rule is

$$\lim_{x\to 0} \frac{2x}{2xf'(x^2) - 360xf'(9x^2) + 396xf'(99x^2)} = \lim_{x\to 0} \frac{1}{f'(x^2) - 180f'(9x^2) + 198f'(99x^2)}$$

for f (x) = 0; the slope of tangent is $f'(x)$ and slope of normal at x = 0 is $-\frac{1}{f'(0)} = 1$ (given)

$\Rightarrow f'(0) = -1$

Limit $= \frac{1}{(1-180+198).f'(0)} = -\frac{1}{19}$

51. (A), (B)

$f(x+y) + f(x-y) = 2f(x) \cdot f(y)$

Put x = 0, $\Rightarrow f(y) + f(-y) = 2f(0) \cdot f(y)$

Put x = y = 0

$\Rightarrow f(0) = 1$ ($\because f(0) \neq 0$))

$\Rightarrow f(-y) = f(y)$

f is even $\Rightarrow f(-2) = f(2) = a$

52. (A), (C), (D)

We have

$^{100}C_{50} = \frac{(100)!}{(50)!(50)!} = 2^{\alpha}, 3^{\beta}, 5^{\gamma}, 7^{\delta}, \ldots\ldots$ where $\alpha, \beta, \gamma, \delta \ldots\ldots$ are non–negative integers.

Exponent of 2 in (100)! is $= \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$

Exponent of 2 in (50)! is $= \left[\frac{50}{2}\right] + \left[\frac{50}{2^2}\right] + \left[\frac{50}{2^3}\right] + \left[\frac{50}{2^4}\right] + \left[\frac{50}{2^5}\right]$

$= 25 + 12 + 6 + 3 + 1 = 47$

Exponent of 2 in $^{100}C_{50}$ is = 3 $\left\{\because\ ^{100}C_{50} = \frac{2^{97}}{2^{47} \cdot 2^{47}} I = 2^3 \cdot I\right\}$

In the similar way exponent of 3, 5 and 7 in $^{100}C_{50}$ are 4, 0 and 0 respectively.

$\therefore\ ^{100}C_{50} = 2^3 \cdot 3^4 \cdot 5^0 \cdot 7^0 \ldots\ldots\ldots\ldots\ldots$

$\Rightarrow \alpha = 3, \beta = 4, \gamma = 0, \delta = 0$

53. (B), (C), (D)

Let $\frac{3x}{\pi} = t$, $f(t) = [2t] \cos [t]$

$x \in \left[\frac{\pi}{6}, \pi\right] \Rightarrow t \in \left[\frac{1}{2}, 3\right]$

$\therefore t = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$

55. (B), (C) Let O be the circumcentre. Then OP + OR ε PR ε AD = 1, so the radius is at least 1/2. P, Q, R always lie inside or on the circle through A, B, C, D which has radius 1/√2, so the radius is at most 1/√2.

56. [9]

$\log_{1/3}(\log_{1/2}y) = \log_3(\log_{1/2}y) \times \log_{1/3}3$ also $\log_{1/2}y = \log_2 y \times \log_{1/2}2$

$$\Rightarrow \log_3(\log_2 x) - \log_3(-\log_2 y) = 1 \Rightarrow \log_3\left(-\frac{\log_2 x}{\log_2 y}\right) = 1$$

$$-\frac{\log_2 x}{\log_2 y} = 3 \Rightarrow xy^3 = 1 \text{ also, } xy^2 = 9 \Rightarrow y = \frac{1}{9} \therefore x = 729.$$

57. [5] Suppose the axes are rotated in the anti–clockwise direction through an angle ∝. The equation of the line L with respect to the old axes is given by $\frac{x}{1} + \frac{y}{1/2} = 1$.To find the equation of L with respect to the new axes, replace x by (x cos∝ – y sin ∝) and y by
(x sin ∝ + y cos∝).

$\therefore$ equation is $(x\cos\alpha - y\sin\alpha) + \frac{1}{1/2}(x\sin\alpha + y\cos\alpha) = 1$

Since p, q are the intercepts made by this line on co–ordinate axes, we have on putting (p, 0) and then (0, q) $\Rightarrow \frac{1}{p} = 1\cos\alpha + \frac{1}{1/2}\sin\alpha$ and $\frac{1}{q} = -(1)\sin\alpha + \frac{1}{1/2}\cos\alpha$.

Eliminate α.

Squaring and adding, we get $\frac{1}{p^2} + \frac{1}{q^2} = 1 + \frac{1}{(1/2)^2} = 5$

58. [0]

$(1 + \cot^2 A)\cot^2 A - (1 + \tan^2 A)\tan^2 A - (\cot^2 A - \tan^2 A)[(1 + \tan^2 A)(1 + \cot^2 A) - 1]$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A - (\cot^2 A - \tan^2 A)(\cot^2 A + \tan^2 A + 1)$
$= \cot^2 A + \cot^4 A - \tan^2 A - \tan^4 A - (\cot^2 A - \tan^2 A) - (\cot^4 A - \tan^4 A) = 0$

59. [2]

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} 0 \Rightarrow \begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ 0 & b-q & r-c \end{vmatrix} = 0$$

$R_3 \to R_3 - R_2$
$R_2 \to R_2 - R_1$

$$\begin{vmatrix} \frac{p}{p-a} & \frac{b}{q-b} & \frac{c}{r-c} \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0 \Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

$$\Rightarrow \frac{p}{p-a} + \left(\frac{q}{q-b} - 1\right) + \left(\frac{r}{r-c} - 1\right) = 0 \Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

60. [4] Using Lagrange's mean value theorem,

for f in [1, 2], $\forall \subset \in (1, 2)$

$$\frac{f(2) - f(1)}{2-1} = f'(c) \le 2$$

$f(2) - 2 \le 2 \Rightarrow f(2) \le 4$ .... (1)

Again using Lagrange's mean value theorem,

for f in [2, 4], $\forall\ d \in (2, 4)$

$$\frac{f(4) - f(2)}{4-2} = f'(d) \le 2$$

$f(4) - f(2) \le 4$

$4 \le f(2)$ .... (2)

$\therefore$ from (1) and (2)

$f(2) = 4$