Solution Paper II
PHYSICS

1. (A) Let m1 and m2 be the masses of particles. By principle of conservation of momentum

m1 u1 + 0 = m2v – m1v

By Newton's law

v + v = –e (0 – u1)

e = 1

2v = u1

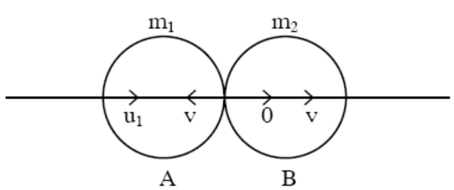
Substituting this in (1)

2m1v = m2v – m1v

2m1 = m2 – m1

3m1 = m2

$$\frac{m_1}{m_2} = \frac{1}{3}$$



2. (B)

Frequency = 200 Hz

Velocity = 50 m/sec.

$\therefore$ Wavelength = $\frac{50}{200}$ = 0.25 m

The equation for stationary wave.

$y = 2\,A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$

$\therefore$ y = 10 cos 8πx sin 400 πt

and y = 5 sin 2π (200 t – 4x)

3. (A) Applying Newton's second law to the circular orbit.

we have $\frac{mv^2}{r} = \frac{GMm}{r^2}$ But $v = \frac{2\pi r}{T}$

$$m\frac{4\pi^2 r^2}{rT^2} = \frac{GMm}{r^2} : T^2 = \frac{4\pi^2 r^3}{GM} : T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

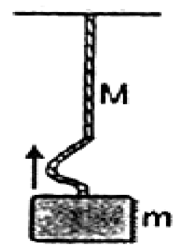
But $M = \frac{4}{3}\pi r^3 \rho$ and $r = r_p$

$$\therefore T = \frac{2\pi r_p^{3/2}}{\sqrt{G \cdot \frac{4}{3}\pi r_p^3 \rho}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

$$= \sqrt{\frac{3\pi}{G\rho}}$$

4. (D)

$$f = \left(\frac{v}{\lambda}\right) \text{ at lowest point}$$



$$f = \frac{1}{\lambda}\sqrt{\frac{mg}{\mu}}$$

$$f^1 = \frac{v^1}{\lambda^1} \text{ at highest point}$$

$$f^1 = \frac{1}{\lambda^1}\sqrt{\frac{(M+m)g}{\mu}}$$

$f^1 = f$: no any change in source freq.

$$\frac{1}{\lambda}\sqrt{\frac{mg}{\mu}} = \frac{1}{\lambda^1}\sqrt{\frac{(M+m)g}{\mu}} \Rightarrow \left[\lambda^1 = \lambda\sqrt{\frac{M+m}{m}}\right]$$

5. (D) At t = 2 second, the position of both pulses are separately given by figure (a) and figure (b); the superposition of both pulses is given by figure (c).

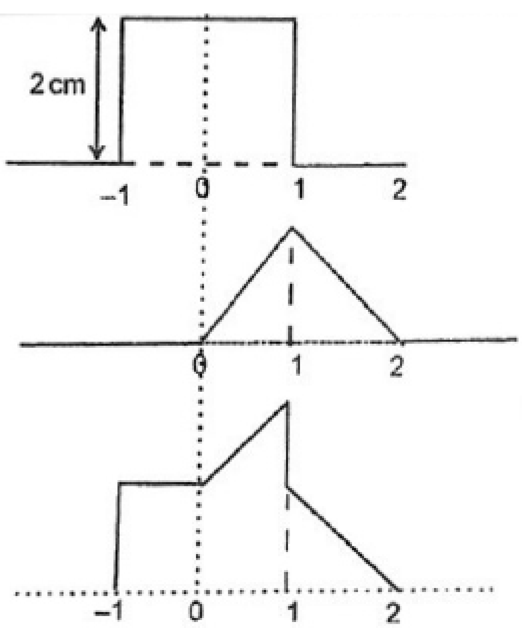


Fig. (a)

Fig. (b)

Fig. (c)

6. (D) Before heating let the pressure of gas be P1, from the equilibrium piston, PA = kx1

$$\therefore x_1 = \frac{PA}{K} = \left(\frac{nRT}{V}\right)\frac{A}{K} = \frac{1\times 8.3\times 100\times 10^{-2}}{0.83\times 100}$$

$= 10^{-1} = 0.1$ m

Since during heating process, The spring is compressed further by 0.1 m

$\therefore x_2 = 0.2$ m

work done by gas $= \frac{1}{2}.100\left(0.2^2 - 0.1^2\right)$

$= \frac{1}{2}$. 100 (0.1) (0.3) = 1.50 = 1.5 J

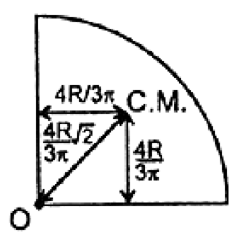
7. (B)

M. I. about 'O' is $\frac{MR^2}{2}$

By parallel–axis theorem :

$$\frac{MR^2}{2} = I_{cm} + M\left(\frac{4R}{3\pi}\cdot\sqrt{2}\right)^2$$

$$\Rightarrow I_{cm} = \frac{MR^2}{2} - M\left(\sqrt{2}\frac{4R}{3\pi}\right)^2$$



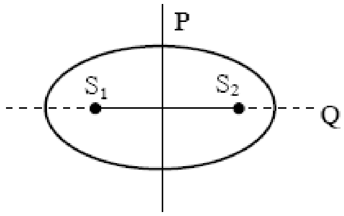
8. (A)

$S_1P - S_2P = 0$

$S_1Q - S_2Q = 4\lambda$

∴ 4 3 maximum lie on are between P & Q. Counting all quadrants; the total number is 12. P & Q symmetrical points are also maxima.

∴ 4 Total number is 16



9. (A)

$$\frac{Q}{A} = 0.06 \times \frac{30-25}{1.5} = 0.2 \text{ [From first slab]}$$

$t_1 = 30°C$, $t_2 = 25°C$, t_3, t_4, $t_5 = -10°C$ be temperatures at interfaces.

$$\therefore \frac{Q}{A} = 0.2 = 0.1 \times \frac{t_4 + 10}{3.5} \Rightarrow t_4 = -3°C$$

$$\therefore \frac{Q}{A} = 0.2 = 0.04 \times \frac{t_3 + 3}{2.8} \Rightarrow t_3 = 11°C$$

10. (A)

From problem (9) :

$$\frac{Q}{A} = 0.2 = K_2 \frac{14}{1.4} \Rightarrow k_2 = 0.02$$

11. (C) For a wire of radius R carrying a current I, the magnetic field at distance r is given by :

$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi R} \cdot \left(\dfrac{r}{R}\right) & 0 \le r \le R \\ \dfrac{\mu_0 I}{2\pi r} & r \ge R \end{cases}$$

Clearly a & c carry same currents, b & d carry same currents } (i)

a & b have same current densities, c & d have same current densities } (ii)

From condition (ii): $R_a > R_b$ & $R_c > R_d$

From condition (i): $R_c > R_a$

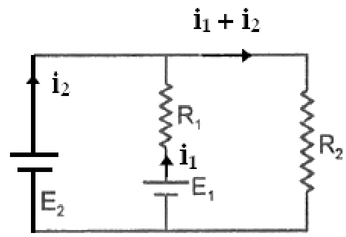
∴ (C) has largest radius.

12. (A) a & c carry same current in graph but radius of 'a' is smaller than radius of c.

∴ Current density in 'a' is larger.

13. (B) **14.** (D)

Soln. for Q. No. 13 & 14



$i_1 + i_2 = \frac{E_2}{R_2}$ .... (1)

$E_2 = E_1 - i_1 R_1$

$\Rightarrow i_1 = \left(\frac{E_1 - E_2}{R_1}\right)$ .... (2) [Negative slope]

$i_2 = \frac{E_2}{R_2} - i_1 = E_2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - \frac{E_1}{R_1}$ [Positive slope]

$i_1 = 0 \Rightarrow E_1 = E_2$ from Eq. (1) according to graph: $i_1 = 0$ at $E_2 = 6V$ ∴ $E_1 = 6V$

Slope of i_1 vs E_2 graph $= -\frac{1}{R_1} = -\frac{0.1}{2} \Rightarrow R_1 = 20\ \Omega$

Slope of i_2 *vs* E_2 graph $= \frac{1}{R_1} + \frac{1}{R_2} = \frac{0.3}{4} = \frac{3}{40} \Rightarrow \frac{1}{R_3} = \frac{3}{40} - \frac{1}{R_1} = \frac{3}{40} - \frac{1}{20}$

$\Rightarrow R_2 = 40\ \Omega$

15. (A), (C) We know the acceleration of particle executing S.H.M. $= -\omega^2 x$.

Velocity $v^2 = \omega^2 (a^2 - x^2)$

$\therefore u^2 = \omega^2 \left(a^2 - x_1^2\right)$

$v^2 = \omega^2 \left(a^2 - x_2^2\right)$

$\alpha = -\omega^2 x_1$

$\beta = -\omega^2 x_2$

(1) − (2) $\quad u^2 - v^2 = \omega^2 \left(x_2^2 - x_1^2\right)$

(3) + (4) $\quad \alpha + \beta = -\omega^2 (x_1 + x_2)$

Dividing (5) by (6)

$$\frac{u^2 - v^2}{\alpha + \beta} = -(x_2 - x_1) \qquad \text{so } d = \left|\frac{u^2 - v^2}{\alpha + \beta}\right|$$

Hence when the accelerations are ∝ and β the distance between the particles

$= x_2 - x_1 = \dfrac{u^2 - v^2}{\alpha + \beta}$

From (3) and (4) we get

$\beta - \alpha = \omega^2 (x_2 - x_1)$

$\omega^2 = \dfrac{\beta - \alpha}{(x_2 - x_1)}$

$$T = \frac{2\pi}{\omega} = 2\pi \Big/ \sqrt{\frac{\beta - \alpha}{(x_2 - x_1)}} = 2\pi\sqrt{\frac{(x_2 - x_1)}{\beta - \alpha}}$$

But $x_2 - x_1 = \dfrac{u^2 - v^2}{\alpha + \beta}$

$$\therefore T = 2\pi \sqrt{\frac{u^2 - v^2}{(\alpha + \beta)(\beta - \alpha)}} = 2\pi\sqrt{\frac{u^2 - v^2}{\beta^2 - \alpha^2}}$$

16. (A), (C), (D) For any curve slope of the tangent at the point $= \frac{dy}{dx}$

Here slope $\frac{ds}{dt}$ but $\frac{ds}{dt}$ is the velocity at the point.

∴ The velocity of particle at any point is given by the slope of the tangent at that point.

i) Between O and A the slope is positive. Hence velocity is positive. As we move from O to A the slope decreases. So the velocity decreases and hence acceleration is negative.

ii) Between A and B slope of tangent is zero at every point. Velocity and acceleration are both
zero.

iii) Between C and D the slope of tangent is negative but constant. The velocity is negative and
the acceleration is zero.

iv) Between D and E the slope of the tangent is positive and increasing. Hence velocity is
positive and increasing. The acceleration is positive.

17. (A), (C)

$$f \propto (T)^{1/2} \Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\therefore \frac{15\text{Hz}}{f} = \frac{1}{2}\times\frac{21}{100} \Rightarrow f = \frac{30\times100}{21} \Rightarrow f = \frac{1000}{7}\text{Hz}$$

$$\therefore f = 143\text{ Hz}$$

$$\text{and } v \propto \sqrt{T} \text{ So } \frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T} = \frac{21}{2} = 10.5\%$$

18. (A), (C) For O1; observer stationary, source is moving;

$$f_1 = \frac{V}{V - \frac{V}{5}}\cdot f = \frac{5f}{4} \qquad \therefore \lambda_1 = \frac{V}{f_1} = \frac{4V}{5f}$$

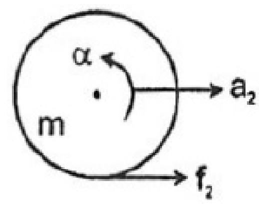
Frequency passed in water is $\frac{5f}{4}$, now for observer O_2

$$\therefore f_2 = \frac{4V + \frac{V}{5}}{4V}\cdot\frac{5f}{4} = \frac{21f}{16}$$

$$\lambda_2 = \frac{4V}{f_2} = \frac{4V}{21f}\times16$$

19. (A), (B), (C)

20. (B), (C) Friction on plate due to ground



$f_1 = 7.5 \times 0.2 \times 10 = 15$

$25 - 15 - f_2 = 1.5\, a_1$

$f_2 = 6a_2$

$10 = 1.5\, a_1 + 6a_2$ .... (i)

$f_2 \cdot r = mr \cdot \alpha$

$\Rightarrow f_2 = ma_2$ .... (ii)

$f_2 = ma_1 - ma_2$

$a_2 + r\alpha = a_1 \quad \Rightarrow \quad a_1 - a_2 = a_2$

$\Rightarrow a_2 = a_1 - a_2 \quad \Rightarrow \quad a_1 = 2a_2$

$10 - 3a_1 = 1.5\, a_1$

$\Rightarrow a_1 = \frac{100}{45} = \frac{20}{9}$

$a_2 = \frac{a_1}{2} = \frac{20}{18}$

$v_1 = a_1 t = \frac{20}{9} \times \frac{3}{4} = \frac{5}{3}$ (Plate)

$v_2 = a_2 t = \frac{20}{18} \times \frac{3}{4} = \frac{5}{6}$ (ring)

21. (B) dsp^2hybridisation gives square planar structure with s, p_x, p_y and $dx^2 - y^2$ orbitals involved
forming angles of 90°.

22. (B)

$P \times V = \frac{4}{M} \times R \times T$ Case I

$P \times V = \frac{4-0.8}{M} \times R \times (T+50)$ Case II

$\therefore \frac{4}{3.2} \times \frac{T}{(T+50)} = 1$ or $4T = 3.2\,T + 160$

$\therefore 0.8\,T = 160$

or $T = \frac{160}{0.8} = 200\,K$

23. (B) As conversion of X to AB is fast, it means the process has a very low activation energy.

24. (B)

25. (C)

Number of atoms in 1 g ${}^{235}_{92}U = \frac{6.023 \times 10^{23}}{235}$

Energy obtained by fission of one atom $= 3.20 \times 10^{-11}$ J

$\therefore$ Energy obtained by fission of $\frac{6.023 \times 10^{23}}{235}$ atoms

$= \frac{3.20 \times 10^{-11} \times 6.023 \times 10^{23}}{235} = 8.20 \times 10^{10}\,J = 8.20 \times 10^{7}\,kJ$

26. (C)

Energy available for muscular work = $\frac{2880 \times 25}{100}$ = 720 kJ mol^{-1}

$\therefore$ Energy available for muscular work for 120 g of glucose

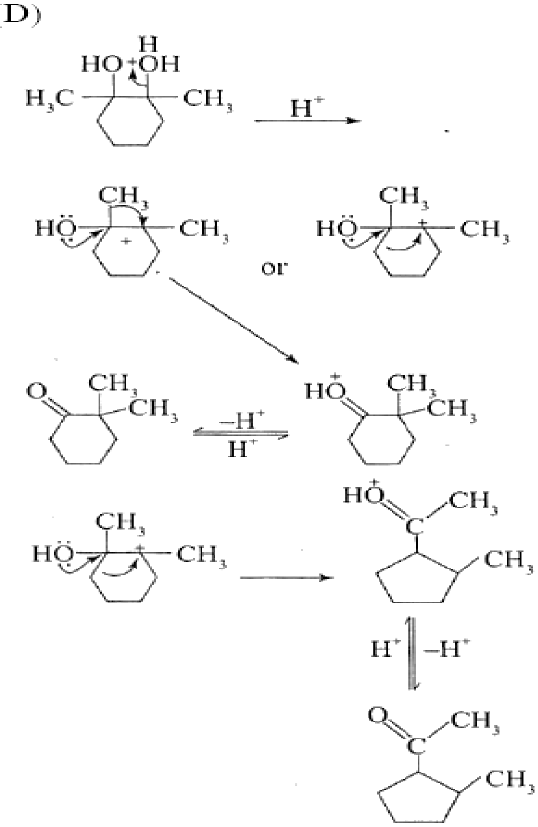
= $720 \times \frac{120}{180}$ = 480 kJ (molecular mass of glucose = 180)

$\because$ 100 kJ of energy is used for walking = 1 km

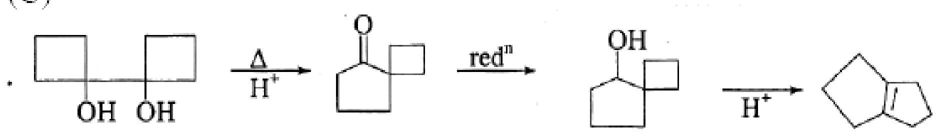
$\therefore$ 480 kJ of energy is used for walking = $1 \times \frac{480}{100}$ = 4.8 km

27. (A) **28.** (A) **29.** (C) **30.** (D)

31. (D)



32. (C)



33. (B) **34.** (D) **35.** (B), (C)

36. (A), (B), (C) Particle size in crystalloids are smaller than that in colloids, crystalloid particles passed through a membrane whereas colloidal particles do not. Crystalloids do not exhibit Tyndall effect whereas colloids exhibit Tyndall effect.

37. (A), (B), (D)

38. (A), (C)

39. (A), (C), (D) (A) For exothermic reaction ΔH is –ve.

As $\Delta H = H_p - H_r$

For ΔH to be –ve

$H > H_p$

(B) Enthalpy of combustion is always negative

(C) As $\Delta G = \Delta H - T\Delta S$

for a reaction to be feasible at all temperatures

ΔG = –ve which is possible if

ΔH = –ve and ΔS = +ve

(D) for the reaction

$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)}$

$\Delta n = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore\ \Delta H = \Delta E + \frac{1}{2}RT$

$\therefore\ \Delta H > \Delta E$

40. (A), (B) When the dispersion medium is gas, the colloidal system is called aerosol, smoke, dust are examples of aerosols of solids whereas fog, clouds are examples of aerosol of liquids.

MATHEMATICS

41. (C)

$\alpha - p > 0,\ \beta - p > 0$ and $D \ge 0$

$\Rightarrow\ \alpha + \beta - 2p > 0,\ \alpha\beta - p(\alpha + \beta) + p^2 > 0$ and $p^2 + 4p^2 \ge 0$

$\therefore\ p - 2p > 0,\ -p^2 - p \cdot p + p^2 > 0 \quad \Rightarrow\ p < 0,\ p^2 < 0$ (absurd).

42. (D)

$a \sin\theta = b\cos\theta = \frac{2c\tan\theta}{1-\tan^2\theta}$ where, $\theta = x^2$

$\tan\theta = \frac{b}{a}$ and $\frac{b}{\sec\theta} = \frac{2c\tan\theta}{1-\tan^2\theta}$

Squaring both sides we get,

$b^2(1+\tan^4\theta - 2\tan^2\theta) = (4c^2\tan^2\theta)(1+\tan^2\theta)$

$b^2\left(1+\frac{b^4}{a^4} - 2\frac{b^2}{a^2}\right) = 4\frac{c^2b^2}{a^2}.\left(1+\frac{b^2}{a^2}\right)$

$b^2\left(\frac{a^4+b^4-2a^2b^2}{a^4}\right) = \frac{4c^2b^2(a^2+b^2)}{a^4}$ or $(a^2-b^2)^2 = 4c^2(a^2+b^2)$

43. (A)

$$a\left(\tan A-\tan\frac{A+B}{2}\right)+b\left(\tan B-\tan\frac{A+B}{2}\right) = 0$$

or, $$a\left(\frac{\sin A}{\cos A}-\frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}}\right)+b\left(\frac{\sin B}{\cos B}-\frac{\sin\frac{A+B}{2}}{\cos\frac{A+B}{2}}\right) = 0$$

or, $$a\,.\,\frac{\sin\left(\frac{A-B}{2}\right)}{\cos A\cos\frac{A+B}{2}}-b\,.\,\frac{\sin\left(\frac{A-B}{2}\right)}{\cos B\ \cos\frac{A+B}{2}} = 0$$

or, $$a\,.\frac{\sin\left(\frac{A-B}{2}\right)}{\cos A} = b\,.\,\frac{\sin\left(\frac{A-B}{2}\right)}{\cos B}$$

Either, $\sin\frac{A-B}{2} = 0 \Rightarrow A = B$

or, $\frac{a}{\cos A} = \frac{b}{\cos B}$ or, $\frac{a\,.2bc}{b^2+c^2-a^2} = \frac{2ac\,.b}{a^2+c^2-b^2}$

or, $b^2 = a^2$

$\therefore$ $a = b$

$\therefore$ The Δ is isoceles.

44. (A)

$\sin(\alpha+\beta) = 1 \quad \Rightarrow \quad \alpha+\beta = \frac{\pi}{2}$

$\sin(\alpha-\beta) = \frac{1}{2} \quad \Rightarrow \quad \alpha-\beta = \frac{\pi}{6}$

$\alpha = \frac{\pi}{3}, \quad \beta = \frac{\pi}{6}$

Now, $\tan(\alpha+2\beta)\tan(2\alpha+\beta) = \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{5\pi}{6}\right) = \left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right) = 1$

45. (C)

$$\cos\left[\frac{1}{2}\cos^{-1}\left(\cos\left(-\frac{14\pi}{5}\right)\right)\right] = \cos\left[\frac{1}{2}\cos^{-1}\left(-\cos\frac{\pi}{5}\right)\right]$$

$$= \cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right) = \sin\frac{\pi}{10} = \cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right) = \cos\frac{2\pi}{5}$$

46. (C)

See the figure since, $OP \perp PY$

$\therefore \ \angle APY = 90° - \theta$, where $\angle OPA = \theta$

$\therefore \ \angle PAY = \theta$

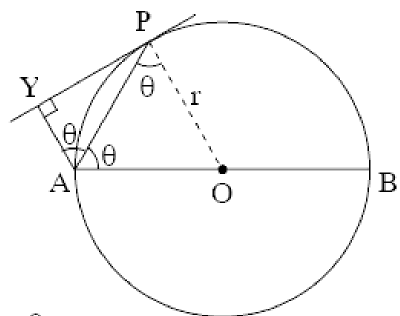
Now, in ΔOPA,

$AP^2 = r^2 + r^2 - 2 \cdot r \cdot r\cos(\pi - 2\theta) = 4r^2\cos^2\theta$

$\therefore \quad AP = 2r\cos\theta$

$\Rightarrow \quad PY = AP\sin\theta = r\sin 2\theta$

and $\quad AY = AP\cos\theta = 2r\cos^2\theta$



$\therefore$ Area of ΔAPY, $\Delta = \frac{1}{2}.PY.AY = r^2\sin 2\theta\cos^2\theta$

$$\frac{d\Delta}{d\theta} = r^2[2\cos 2\theta\cos^2\theta - \sin^2 2\theta] = 0$$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{\pi}{6} \qquad \because \ \theta \neq \frac{\pi}{2}$. Also, Δ is maximum at $\theta = \frac{\pi}{6}$ (Check)

$$\therefore \ \Delta_{max} = r^2\frac{\sqrt{3}}{2}\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3\sqrt{3}r^2}{8}$$

47. (B)

We have, $e^{-\pi/2} < \theta < \pi/2$

$\Rightarrow \ -\frac{\pi}{2}\log e < \log\theta < \log\frac{\pi}{2}$

Now, $\log_e e = 1$ and $\dfrac{\pi}{2} < e$, $\therefore \log\dfrac{\pi}{2} < \log e = 1$

$\therefore -\dfrac{\pi}{2} < \log\theta < 1 < \dfrac{\pi}{2}$

$\Rightarrow$ $\log\theta$ lies in 1^{st} and 4^{th} quadrant

$\therefore \cos(\log\theta)$ is positive …… (1)

Now, $0 < \cos\theta < 1 \Rightarrow \log\cos\theta < \log 1 = 0$

$\Rightarrow$ $\log\cos\theta$ is negative …… (2)

From (1) and (2), we conclude $\cos\log\theta > \log\cos\theta$

48. (B) Let the lines be y = c1 and y = c2 [∵the lines are parallel to x↕axis] From the equation of circle, x2+y2–6x–4y–12 = 0

centre (3, 2) and radius = 5

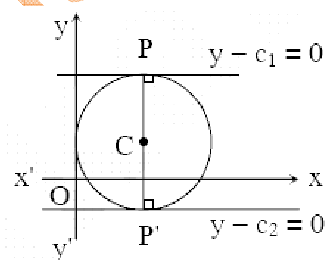
The perpendicular drawn from centre to the lines are CP and CP'

$CP = \dfrac{2-c_1}{\sqrt{1}} = \pm 5 \quad \Rightarrow 2 - c_1 = \pm 5$

$\therefore$ $c_1 = 7$ and $c_1 = -3$

Hence the lines are $y - 7 = 0$ and $y + 3 = 0$

$\therefore$ Pair of lines is $(y - 7)(y + 3) = 0$, i.e., $y^2 - 4y - 21 = 0$.



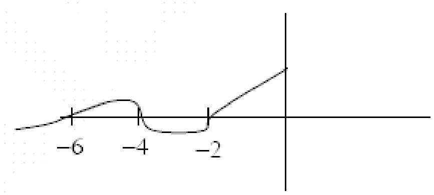
49. (D)

$\dfrac{(]x[+2)(]x[+4)}{(]x[+6)} > 0$

$\Rightarrow -6 <]x[< -4$ or $]x[> -2$

$\therefore$ $]x[= -5, -1, 0$

$\therefore$ $x = -5, -1, 5, 1, 0$



50. (B)

$-\left|x + \dfrac{1}{x}\right| = \left|x + \dfrac{1}{x}\right| \quad \Rightarrow \quad 2\left|x + \dfrac{1}{x}\right| = 0$

which is not possible, as $\left|x + \dfrac{1}{x}\right| \geq 2$

51. (B) $g(x) \equiv f(x) - x^3 + 1 = 0$ has at least 3 roots in [1, 4]

$\Rightarrow$ $f'(x) = 3x^2$ has at least 1 root each in (1, 2) and (2, 4).

52. (C) f(x) – x retains it's sign, so consider f(x) – x > 0.

$$\Rightarrow f^2(x) - f(x) > 0$$
$$f^3(x) - f^2(x) > 0$$
$$\vdots$$
$$f^n(x) - f^{n-1}(x) > 0 \quad \Rightarrow f^n(x) - x > 0$$

53. (D)

$$P_k = \frac{1-x^{k+1}}{1-x}$$

$$P_1 P_2 P_3 .....P_n = \frac{(1-x^2)(1-x^3)(1-x^4).....(1-x^{n+1})}{(1-x)^n}$$

No. of terms $= 1 + \text{max. power of } x$

$$= 1 + \frac{n(n+1)}{2} = \frac{n^2+n+2}{2}$$

54. (D)

$$(1+x)(1+x+x^2)....(1+x+.....+x^n) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$$

Put, x = 1,

$$2\times3\times4\times.....(n+1) = a_0 + a_1 + a_2 + a_3 + ...$$

Put x = –1,

$$0 = a_0 - a_1 + a_2 - a_3 + ...$$

$$(n+1)! = 2[a_0 + a_2 + ...] \quad \Rightarrow a_0 + a_2 + ... = \frac{(n+1)!}{2}$$

55. (B),(C),(D)

(A) $f(0) \rightarrow$ undefined

$$f(0^-) = \lim_{h\to 0} \frac{1}{1+2^{-\cot h}} = 1$$

$$f(0^+) = \lim_{h\to 0} \frac{1}{1+2^{\cot h}} = 0$$

$\Rightarrow$ non-removable discontinuity at x = 0.

(B) $f(0) \rightarrow$ not defined

$$f(0^-) = \lim_{h\to 0} \cos\left(\frac{\sin h}{-h}\right) = \cos 1$$

$$f(0^+) = \lim_{h\to 0} \cos\left(\frac{\sin h}{h}\right) = \cos 1$$

$\Rightarrow$ removable discontinuity at x = 0.

(C) $f(0) \rightarrow$ not defined

$$f(0^-) = \lim_{h\to 0} h.\sin\frac{\pi}{h} = 0$$

$$f(0^+) = \lim_{h\to 0} h.\sin\frac{\pi}{h} = 0$$

$\Rightarrow$ removable discontinuity at x = 0.

(D) $$f(0^-) = \lim_{h\to 0} \frac{1}{\ln h} = 0$$

$$f(0^+) = \lim_{h\to 0} \frac{1}{\ln h} = 0$$

$\Rightarrow$ removable discontinuity at x = 0.

56. (A), (B), (C)

Let the roots be α, β, γ

$$\sum \alpha = -a$$

$$|a| = \left|\sum \alpha\right| \le \sum |\alpha| = 3$$

$$b = \sum \alpha\beta$$

$$|b| = \left|\sum \alpha\beta\right| \le \sum |\alpha||\beta| = 3$$

$$c = \alpha\beta\gamma \quad \Rightarrow \quad |c| = \Pi|\alpha| = 1$$

57. (A),(B),(C),(D)

We have, f (x) = | [x] x | in $-1 < x \le 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \le x < 1 \\ x, & 1 \le x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at x = 0. Also, it is discontinuous at x = 1 and non–differentiable at x = 2.

58. (A), (B), (C), (D)

We have, $\frac{P-Q}{p-q} = \frac{Q-R}{q-r} = \frac{R-P}{r-p} = d$ = common difference.

$\therefore$ (q – r) (P – Q) = (p – q) (Q – R)

qP – qQ – rP + rQ = pQ – pR – qQ + qR

$\therefore$ pQ + qR + rP = pR + rQ + qP ……(A)

Simplifying, $\begin{vmatrix} P & Q & R \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$

= P(q – r) – Q(p – r) + R(p – q) ……(B)

= 0 from (A)

(B) can be written as ΣP(q – r)

$\therefore$ Σ P(q – r) = 0 ……(C)

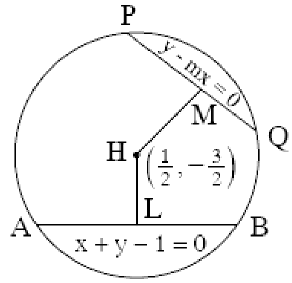
(B) readjusting,

ΣP(q – r) = Σp(Q – R) = 0. ……(D)

59. (A), (B)

Let H be the centre of circle.

$H \equiv \left(\frac{1}{2}, -\frac{3}{2}\right)$ and radius = $\sqrt{\frac{1}{4}+\frac{9}{4}} = \sqrt{\frac{5}{2}}$



Given AB = PQ

$\Rightarrow$ HL = HM

$\Rightarrow \left|\dfrac{\frac{1}{2}-\frac{3}{2}-1}{\sqrt{2}}\right| = \left|\dfrac{-\frac{3}{2}-\frac{m}{2}}{\sqrt{1+m^2}}\right|$

$\Rightarrow \sqrt{2} = \dfrac{3+m}{2\sqrt{1+m^2}} \quad \Rightarrow 8(1 + m^2) = 9 + m^2 + 6m$

$\Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow m = \dfrac{6 \pm \sqrt{36+28}}{14} = 1, -\dfrac{1}{7}$

$\therefore$ Equation of L_1 can be

$y - x = 0$ or $y + \frac{1}{7}x = 0$

i.e., $x + 7y = 0$

$\therefore$ correct option is (A) or (B)

60. (A),(C)

Since f '(x) > 0 $\Rightarrow$ f (x) is increasing function

and g '(x) < 0 $\Rightarrow$ g (x) is decreasing function.

Since x + 1 > x

$\therefore$ g (x + 1) < g (x) ($\because$ g is decreasing)

$\therefore$ f {g (x + 1)} < f {g (x)} {$\because$ f is increasing}

or f {g (x)} > f {g (x + 1)}

Alternate (A) is correct.

Now : x + 1 > x

f (x + 1) > f (x) {$\because$ f is increasing}

$\Rightarrow$ g {f (x + 1)}< g {f (x)} {$\because$ g is decreasing}

Or g {f (x)} > g {f (x + 1)}

Alternate (C) is correct.

Now replacing x by x – 1 in alternate (A) and (C) then

$f\{g(x-1)\} > f\{g(x)\}$

and $g\{f(x-1)\} > g\{f(x)\}$

Hence alternates (B) and (D) are wrong.