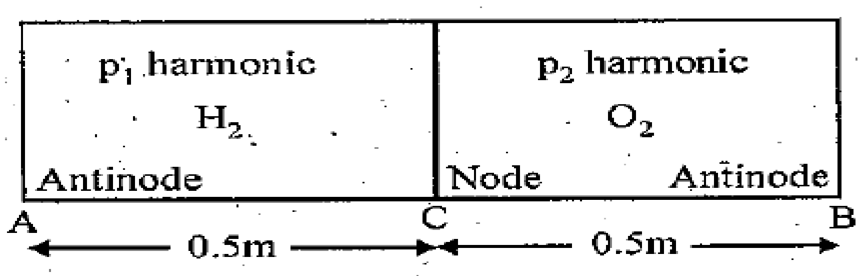
Waves – Solutions

SUBJECTIVE PROBLEMS:

Sol 1.

It is given that C as a node. This implies that at A and B antinodes are formed. Again it is given that the frequencies are same.



$\Rightarrow v_1/4\ell \times p_1 = v_2/4\ell \times p_2$ or $p_1/p_2 = v_1/v_2$ 3/11

Or, $11p_1 = 3p_2$

This means that the third harmonic in AC is equal to 11^{th} harmonic in CB.

Now, the fundamental frequency in AC

$= v_1\, 4\ell = 1100/4 \times 0.5 = 550\text{Hz}$

And the fundamental frequency in CB

$= v_2/4\ell = 300/4 \times 0.5 = 550$ Hz

$\therefore$ Frequency in AC = 3 x 550 = 1650 Hz and frequency in CB = 11 x 150 = 1650Hz.

Sol 2.

(a) Using the formula of the coefficient of linear expansion of wire, $\Delta\ell = \ell\alpha\Delta\theta$ we get

$F = YA\alpha\Delta\theta$

Speed of transverse wave is given by

$V = \sqrt{F/m}$ [where m = mass per unit length = Alp = Ap]

$= \sqrt{YA\alpha\Delta\theta/Ap} = \sqrt{Y\alpha\Delta\theta/p}$

$= \sqrt{1.3 \times 10^{11} \times 1.7 \times 10^{-5} \times 20/9 \times 10^{3}} = 70$ m/s

Sol 3.

Tube open at both ends:

(a) v = v/2(ℓ + 0.6 D) ∴ 320 = 320/2(0.48 + 0.6 x D)

0.48 + 0.6 D = 0.5 ⇒ 0.6 D = 0.02

⇒ D = 0.02/60 x 100 cm = 3.33 cm

Tube closed atone end:

V = v/4(ℓ + 0.3 D) = 320/4 (0.48 + 0.3 x 0.033)

= 163 Hz

Sol 4.

V = 1/2ℓ √T/m = 1/2 x 0.5 √100/m = 10√m . . . . (i)

The frequency of the tuning fork is either v + 5 or v – 5.

NOTE : On decreasing the tension, the frequency will decrease.

Therefore the frequency of tuning fork should ve v – 5.

Now, v^1 = 1/2ℓ √T/m = 1/2 x 0.5 √81/m = 9/√m

This v^1 should be v – 10.

∴ 10/√m – 10 = 9/√m ⇒ 10 – 9/√m = 10

M = 1/100 . . . . (ii)

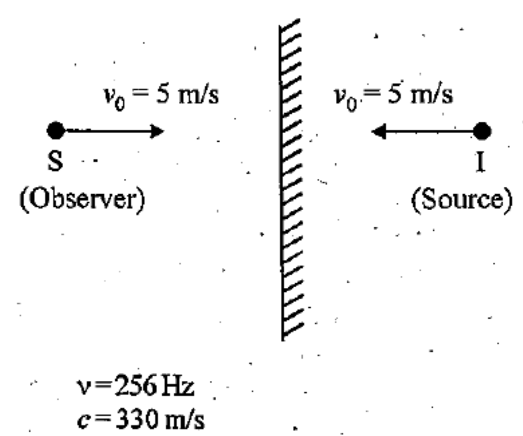
P x π d^2/4 = 1/100 (m =density x volume)

⇒ p = 4/100 x π x $(10^{-3})^2$ = 12738.85 kg/m^3

From (i) and (ii) v = 10/√1/100 = 100 Hz ∴ Frequency of the fork is 95 Hz

Sol 5.

NOTE : If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source in the reflecting surface will become the source of the reflected sound.



$v = v\,[c - v_0/c - v_s]$

v_0, v_s are +ve if they are directed from source to the observer and – ve if they directed from observer to source.

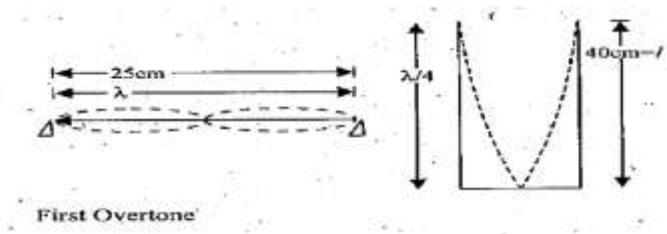
$v = 256\,[330 - (-5)/330 - 5] = 264$ Hz

$\therefore$ Beat frequency = 264 – 256 = 8

Sol 6.

Mass of string unit length = $2.5 \times 10^{-3}/0.25 = 0.01$ kg/m

$\therefore$ Frequency, $v_s = 1/\lambda\, \sqrt{T/m} = 1/0.25\, \sqrt{T/0.01}$ . . . . (i)



Fundamental frequency

$\therefore\ \lambda/4 = 0.4 \Rightarrow \lambda = 1.6$ m

$\therefore v_T = c/\lambda_T = 320/1.6 = 200$ Hz . . . . (ii)

Given that 8 beats/ seconds are heard. The beat frequency decreases with the decreasing tension. This means that beat frequency decreases with decreasing v_s So beat frequency is given by the expression.

$v = v_s - v_T \therefore 8 = 1/0.25\, \sqrt{T/0.01} - 200 \Rightarrow T = 27.04$ N

Sol 7.

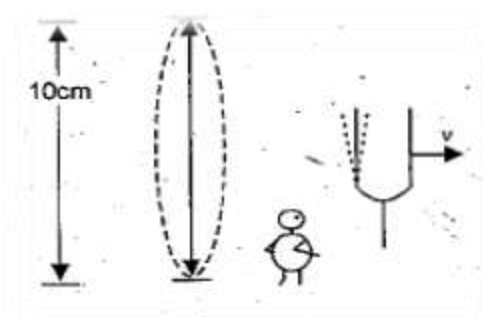
Mass per unit length of somometer wire

$= m/\ell = 0.001/0.1 = 0.01$ kg/m

$v = \sqrt{T/m} = \sqrt{64/(0.01)} = 8 \times 10$

Also, $\lambda/2 = 0.1 \Rightarrow \lambda = 0.2$

$\therefore f = v/\lambda = 8 \times 10/0.2 = 400$ Hz



Since tuning fork is in resonance therefore frequency of tuning fork is 400 Hz. The observer is hearing one beat per second when the tuning fork is moved away with a constant speed v.

The frequency of tuning fork as heard by the observer standing stationary near sonometer wire can be found with the help of Doppler effect,

$v^1 = v[c - v_0/c + v_s] = c/c + v_s$ [$\because v_0 = v$ m/s]

$\therefore v^1 = 400 \times 300/300 + v_s$

Since the beat frequency is I and as the tuning fork is going away from the observer, its apparent frequency is (normal frequency – 1) = 400 – 1 = 399

$\therefore 399 = 400 \times 300/300 + v_s$

Or $v_s = 0.75$ m/s

Sol 8.

The velocity of wave on the string is given by the formula

$v = \sqrt{T/m}$

Where t is the tension and m is the mass per unit length. Since the tension in the string will increase as we move up the string (as the string has mass), therefore the velocity of wave will also increase. (m is the same as the rope is uniform)

$\therefore v_1/v_2 = \sqrt{T_1/T_2} = \sqrt{2 \times 9.8/8 \times 9.8} = 1/2 \therefore v_2 = 2v_1$

Since frequency remains the same

$\therefore \lambda_2 = 2\lambda_1 = 2 \times 0.06 = 0.12$ m

Sol 9.

Using the formula of the coefficient of linear expansion,

$\Delta\ell = \ell\alpha \times \Delta\theta$

Also, $Y = \text{stress/strain} = T/A/\Delta\ell/\ell = T/A/\alpha\ A\ \theta \therefore T = YA\ \alpha\ \Delta\theta$

The frequency of the fundamental mode of vibration.

$v = 1/2\ell \sqrt{T/m} = 1/2\ell \sqrt{YA\ \alpha\ \Delta\ \theta/m}$

$= 2/2 \times 1 \sqrt{2 \times 10^{11} \times 10^{-6} \times 1.21 \times 10^{-5} \times 20/0.1} = 11$ Hz..

Sol 10.

(i) Here amplitude, A = sin (π x/15)

At x = 5m

A = 4 sin (π x 5/15) = 4 x 0.866 = 3.46 cm

(ii) Nodes are the position where A = 0

∴ sin (π x/15) = 0 = sin n π ∴ c = 15 n

Where n = 0, 1, 2   x = 15 cm, 30 cm, 60 cm, ...

(iii) y = 4 sin (π x/15) cos (96 π t)

v = dy/dt = 4 sin (π x/15) [- 96 π sin (96 π t)]

At x = 7.5 cm, t = 0.25 cm

v = 4 sin (π x 7.5/15) [- 96 π sin (96 π x 0.25)]

= 4 sin (π/2) [-96 π sin (24 π)] = 0

(iv) y = 4 sin (π x/15) cos [96 π t]

= 2[2 sin (π x/15) cos (96 π t)]

= 2[sin (96 π t + π r/15) – sin (96 π t – π x/15)]

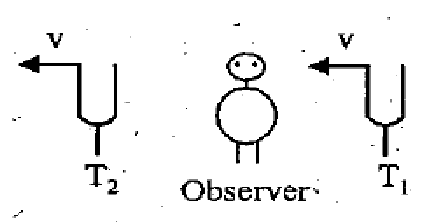
= 2 sin (96 π t + π x/15) – 2 sin (96 π t – π x/15)

= $y_1 + y_2$

Where y_1 = 2 sin (96 π t + π x/15)

And y_2 = -2 sin (96 π t – π x/15)

Sol 11.



The apparent frequency from tuning fork T_1 as heard by the observer will be

v_1 = c/c – v x v . . . . (i)

where c = velocity of sound

v = velocity of turning fork

The apparent frequency from tuning fork T_2 as heard by the observer will be

$v_2 = c/c + v \quad x\ v$ . . . . (ii)

Given $v_1 – v_2 = 3$

$\therefore$ c x v [1/c – v – 1/ c + v] = 3 or, 3 = c x v x $2v/c^2 – v^2$

Since, v < < c $\therefore$ 3 = c x v x $2v/c^2$

$\therefore$ v = 3 x 340 x 340/340 x 340 x 2 = 1.5 m/s

<u>Sol 12.</u>

(i) When two progressive waves having same amplitude and period, but travelling in opposite direction with same velocity superimpose, we get standing waves.

The following two equations quality the above criteria and hence produce standing wave

$Z_1 = A \cos (k x – \omega t)$

$Z_2 = A \cos (k x + \omega t)$

The resultant wave is given by $z = z_1 + z_2$

$\Rightarrow z = A \cos (kx – \omega t) + A \cos (kx + \omega t)$

$= 2A \cos kx \cos \omega t$

The resultant intensity will be zero when $2 A \cos kx = 0$

$\Rightarrow \cos k x = \cos(2n + 1)/2\ \pi$

$\Rightarrow k x = 2n + 1/2\ \pi \Rightarrow x =(2n + 1)\ \pi/2k$

Where n = 0, 1, 2, ...

(ii) The transverse waves

$z_1 = A \cos (k x - \omega t)$

$z_3 = A \cos (k y - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive y axes.

The resultant wave is given by $z = z_1 + z_3$

$z = A \cos (k x - \omega t) + A \cos (k y - \omega t)$

$\Rightarrow z = 2A \cos (x – y)/2 \cos [k(x + y) - 2 \omega t/2]$

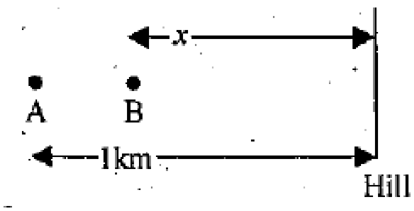
The resultant intensity will be zero when

$2A \cos k(x - y)/2 = 0 \Rightarrow \cos k(x - y)/2 = 0$

$\Rightarrow k(x - y)/2 = 2n + 1/2\ \pi \Rightarrow (x - y) = (2n + 1)/k\ \pi$

Sol 13.

(i) The frequency of the whistle as heard by observer on the hill



$n^1 = n[v + v_m/v + v_m - v_s]$

$= 580\ [\ 1200 + 40/1200 + 40 - 40] = 599$ Hz

(ii) Let echo from the hill is heard by the driver at B which is at a

distance x from the hill.

The time taken by the driver to reach from A to B

$t_1 = 1 - x/40$ . . . . (i)

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$t_2 = 1/(1200 + 40) + x/(1200 - 40)$ . . . . (ii)

where t_{AH} = time taken by sound from A to H with velocity (1200 + 40)

t_{HB} = time taken by sound from H to B with velocity 1200 – 40 From (i) and (ii)

$t_1 = t_2 \Rightarrow 1 - x/40 = 1/1200 + 10 + x/1200 - 40$

$\Rightarrow x = 0.935$ km

The frequency of echo as heard by the driver can be calculated by considering that the source is the

acoustic image.

$n'' = n[(v - v_m) + v_s/(v - v_m) - v_o]$

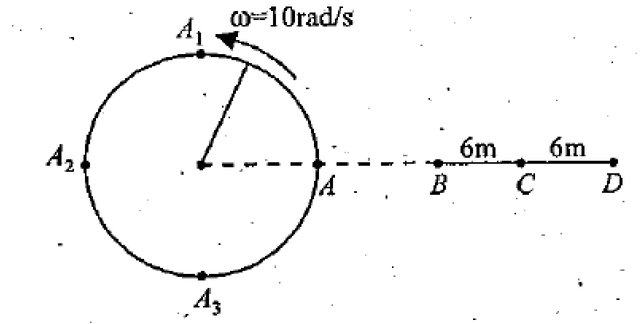
$= 580\ [(1200 - 40) + 40/(1200 - 40) - 40] = 621$ Hz

Sol 14.

The angular frequency of the detector = $2\pi v$

$= 2\pi \times 5/\pi = 10$ rad/s

The angular frequency of the detector matches with that of the source.



⇒ When the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

$v' = v [v - v_0/v + v_s] = 340 \times (340 - 60)/(340 + 30) = 257.3$ Hz

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$v'' = v[v + v_0/v - v_s] = 340 \times (340 + 60)/(340 - 30)= 438.7$ Hz

Sol 15.

(a) Use the equation of a plane progressive wave which is as follows.

$y = A \cos (2\pi/\lambda\, x + 2\pi v t)$

The given equation is $y_1 = A \cos (ax + bt)$

On comparing, we get $2\pi/\lambda = a \Rightarrow \lambda = 2\pi/a$

Also, $2\pi v = b$

$\Rightarrow v = b/2\pi$

(b) Since the wave is reflected by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave I’ = 0.64 I

$\Rightarrow I' \propto A'^2 \Rightarrow 0.64\, I \propto A'^2$

$\Rightarrow 0.64\, A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of resultant wave becomes

$y_2 = 0.8A \cos(ax - bt + \pi) = -0.8A \cos(ax - bt)$

(c) The resultant wave equation can be found by superposition principle

$y = y_1 + y_2 = A \cos(ax + bt) + [-0.8\, A \cos(ax - bt)]$

The particle velocity can be found by differentiating the above equation

$v = dy/dt = -Ab \sin(ax + bt) - 0.8\, Ab \sin(ax - bt)$

$= -Ab [\sin(ax + bt) + 0.8 \sin(ax - bt)]$

$= -Ab [\sin ax \cos bt + \cos ax \sin bt + 0.8 \sin ax \cos bt - 0.8 \cos ax \sin bt]$

$v = -Ab [1.8 \sin ax \cos bt + 0.2 \cos ax \sin bt]$

The maximum velocity will occur when sin ax = 1 and cos bt = 1 under these condition cos ax = 0

And sin bt = 0

$\therefore |v_{max}| = 1.8\, Ab$

Also, $|v_{min}| = 0$

(d) $y = [A \cos(ax + bt)] - [0.8\, A \cos(ax - bt)]$

$= [0.8\, A \cos(ax + bt) + 0.2\, A \cos(ax + bt)] - [0.8A \cos(ax - bt)]$

$= 0.8\, A [-2 \sin\{(ax + bt) + (ax - bt)/2\} \text{Sin} \{(ax + bt) - (ax - bt)/2\}]\ 0.2\, A \cos(ax + bt)]$

$\Rightarrow y = -1.6\, A \sin ax \sin bt + 0.2\, A \cos(ax + bt)$

Where ($-1.6\, A \sin ax \sin bt$) is the equation of travelling wave.

The wave is travelling in –x direction.

NOTE : Antinodes of the standing waves are the positions where the amplitude is maximum,

i.e. $\sin ax = 1 = \sin [n\pi + (-1)^n \pi/2]$

$\Rightarrow x = [n + (-1)^n/2]\, \pi/a$

Sol 16.

Let the two radio waves be represented by the equation

$y_1 = A \sin 2\pi v_1 t$

$y_2 = A \sin 2\pi v_2 t$

The equation of resultant wave according to superposition principle

$y = y_1 + y_2 = a \sin 2\pi v_1 t + A \sin 2\pi v_2 t$

$= A [\sin 2 \pi v_1 t + \sin 2 \pi v_2 t]$

$= A \times 2 \sin (2\pi v_1 + 2 \pi v_2) t/2 \cos (2 \pi v_1 + 2 \pi v_2)t/2$

$= 2A \sin \pi(v_1 + v_2) t \cos \pi(v_1 - v_2) t$

Where the amplitude $A' = 2A \cos \pi (v_1 - v_2) t$

Now, intensity $\propto$ (Amplitude)^2

$\Rightarrow I \propto A'^2$

$\Rightarrow I \propto 4A^2 \cos^2 \pi (v_1 + v_2) t$

The intensity will be maximum when

$\text{Cos}^2 \pi (v_1 - v_2) t = 1$

Or, $\cos \pi (v_1 + v_2) t = 1$

Or, $\pi v_1 - v_2 t n\pi$

$\Rightarrow (\omega_1 - \omega_2)/2 \; t = n\pi$ or, $t = 2n\pi/ \omega_1 - \omega_2$

$\therefore$ Time interval between two maxima

Or, $2n\pi/ \omega_1 - \omega_2 - 2(n - 1)\pi/ \omega_1 - \omega_2$ or, $2\pi \omega_1 - \omega_2 = 2\pi/10^3$ sec

Time interval between two successive maximas is

$2\pi \times 10^{-3}$ sec

(ii) For the detector to sense the radio waves, the resultant intensity $\geq 2 A^2$

$\therefore$ Resultant amplitude $\geq \sqrt{2} A$

Or, $2 A \cos \pi (v_1 - v_2) t \geq \sqrt{2}A$

Or, $\cos \pi (v_1 - v_2) t \geq 1/\sqrt{2}$ or, $\cos [(\omega_1 - \omega_2) t/2] \geq 1/\sqrt{2}$

The detector lies idle when the values of $\cos[(\omega_1 - \omega_2) t/2]$ is between 0 and $1/\sqrt{2}$

$\therefore (\omega_1 - \omega_2) t/2$ is between $\pi/2$ and $\pi/4$

$\therefore t_1 = \pi/ \omega_1 - \omega_2$ and $t_2 = \pi/2 (\omega_1 - \omega_2)$

$\therefore$ The time gap $= t_1 - t_2$

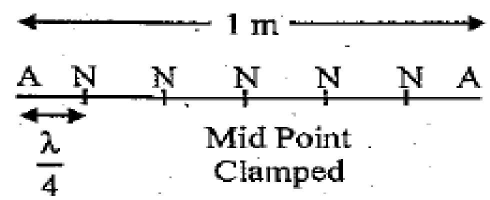
$= \pi/ \omega_1 - \omega_2 - \pi/2 (\omega_1 - \omega_2) = \pi/2 (\omega_1 - \omega_2)$

$= \pi/2 \times 10^{-3}$ sec.

Sol 17.

The placements of the nodes and antinodes on the rod are shown in

the figure



$\therefore \lambda + \lambda/4 = 0.5 \Rightarrow \lambda = 0.4$m

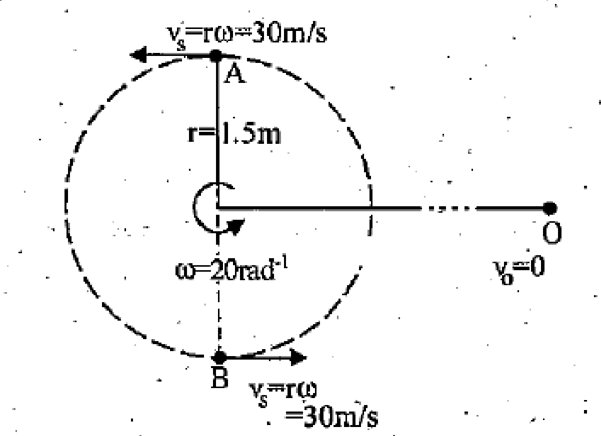
Also, the velocity of waves produced in the rod,

$v = \sqrt{Y/p} = \sqrt{2 \times 10^{11}/8 \times 10^3} = 5000$ m/s

Since, amplitude of antinodes $= 2 \times 10^{-6}$ m

$\therefore 2a = 2 \times 10^{-6}$ m $\Rightarrow a = 10^{-6}$ m

The equation of wave moving in the positive X – direction will be



$r = 1.5$ m (given); $\omega = 20$ rads^{-1} (given)

$y_1 = a \sin 2\pi/\lambda (vt - x)$

$\Rightarrow y_1 = 10^{-6} \sin 2\pi/0.4 (5000t - x)$

The equation of wave after reflection and moving in X – axis is

$y_2 = 10^{-6} \sin[2\pi/0.4 (5000t + x]$

The equation of the stationary wave is

$y = 2a \cos 2\pi/\lambda\, x \sin 2\pi/\lambda\, vt$

$\therefore y = 2 \times 10^{-6} \cos (2\pi/0.4\, x) \sin (2\pi/0.4 \times 5000t)$

Equation of wave at x = 2 cm

$y = 2 \times 10^{-6} \cos (2\pi/0.4 \times 0.02) \sin (2\pi/0.4 \times 5000t)$ $y = 2 \times 10^{-6} \cos (0.1 \pi) \sin (25000 \pi t)$

Sol 18.

The whistle which is emitting sound is being rotated in a circle. We know that $v = r\omega = 1.5 \times 20 = 30\ ms^{-1}$

When the source is instantaneously at the position A, then the frequency heard by the observer will be

$v' = v\,[v/v - v_s] = 440\,[330/330 - 30] = 484$ Hz

when the source is instantaneously at the position B, then the frequency heard by the observer will be

$v'' = v[v / v + v_s] = 440\,[330/330 + 30] = 403.3$ Hz

Hence the range of frequencies heard by the observer is

403.3 Hz to 484 Hz.

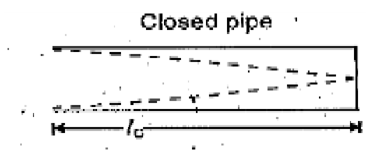
Sol 19.

First overtone frequency

$\ell_0 = \lambda$

$\Rightarrow (v_1)_0 = v/\lambda' = v/\ell_0 = 330/\ell_0$

Fundamental frequency



$\ell_c = \lambda_1/4 \Rightarrow \lambda_1 = 4\,\ell_c$

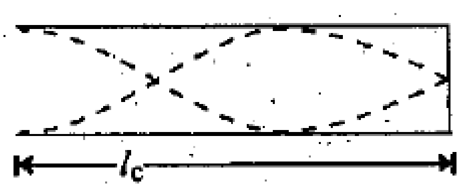
$\Rightarrow (v_1)c = v\ 4\,\ell_c = 10$ Hz (given)

But beat frequency is 2.2

Case 1 : $(v_1)_0 > (v_1)_c$

$(v_1)_0 - (v_1)_c = 2.2$

$\Rightarrow 330/\ell_0 - 330 = 2.2 \Rightarrow \ell_0 = 0.9933$ m



Case 2 : $(v_1)_0 < (v_1)_c$

$(v_1)_0 - (v_1)_c = 2.2$

$330 - 330/\ell_0 = 2.2 \Rightarrow \ell_0 = 1.006$ m

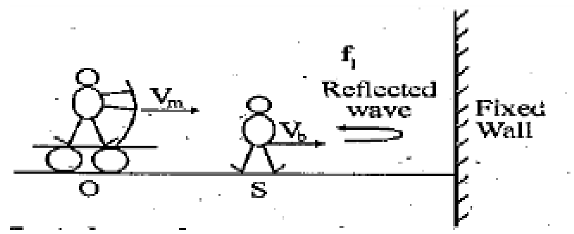
Sol 20.

Motorist will listen two sound waves. One directly the sound source and other reflected from the fixed wall. Let the apparent frequencies of these two waves as received by motorist are f' and f" respectively.

For Direct Sound : V_m will be positive as it moves towards the source and tries to increase the apparent frequency. V_b will be taken positives as it move away from the observer and hence tries to decrease the apparent frequency value.

$f = v + v_m/v + v_b\ f$ . . . . (1)

For reflected sound:



For sound waves moving towards stationary observer (i.e.

wall), frequency of sound as heard by wall

$f_1 = v/v - v_b\ f$

After reflection of sound waves having frequency f_1 fixed wall acts as a stationary source of frequency f_1

for the moving observer i.e. motorist. As direction motion of motorist is of opposite to direction of

sound waves, hence frequency f" of reflected sound waves as received by the motorist is

$f'' = v + v_m/v\ f_1 = v + v_m/v - v_b\ f$ . . . . (2)

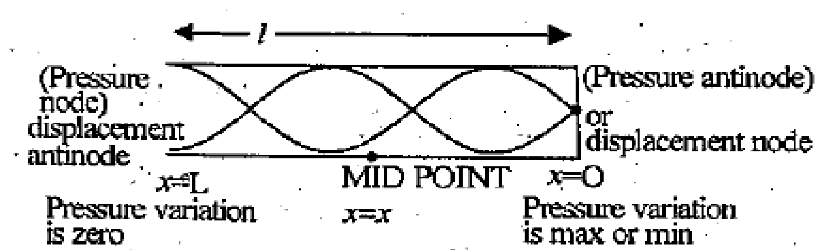
Hence, beat frequency as heard by the motorist

$\Delta f = f'' - f' = (v + v_m/v - v_b - v + v_m/v + v_b)\ f$

Or, $\Delta f = 2v_b (v + v_m) f / v_2 - v_b^2$

Sol 21.

(a) For second overtone as shown,



$5\lambda/4 = \ell \ \therefore\ \lambda = 4\ell/5$

Also, $v = v\lambda$

$\Rightarrow 330 = 440 \times 4\ell/5 \Rightarrow \ell = 15/16$ m.

(b) At any position x, the pressure is given by

$\Delta P = \Delta P_0 \cos kx \cos \omega t$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos 2\pi/\lambda\ x$

For x = 15/2 x 16 = 15/32 m (mid point)

Amplitude = $\Delta P_0 \cos [2\pi/(330 / 440) \times 15/32] = \Delta P_0/\sqrt{2}$

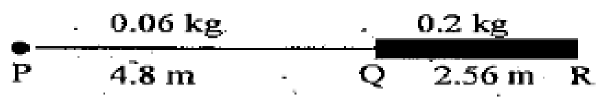
(c) At open end of pipe, pressure is always same i.e. equal to mean pressure

$\because \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end : Maximum Pressure = $P_0 + \Delta P_0$ Minimum Pressure = $P_0 - \Delta P_0$

Sol 22.

(a) (Mass per unit length of PQ



$m_1 = 0.06/4.8$ kg/m

Mass per unit length of QR, $m_2 = 0.2/2.56$ kg/m

Velocity of wave in PQ is

$v_1 = \sqrt{T/m_1} = \sqrt{80/0.06/4.8} = 80\ ms^{-1}$ [$\because T = 80$ N given]

Velocity of wave in QR is

$v_2 = \sqrt{T/m_2} = \sqrt{80/0.2 / 2.56} = 32$ m/s

$\therefore$ Time taken for the wave to reach from P to R

$= t_{PQ} + t_{QR}$

$= 4.8/80 + 2.56/32 = 0.14$ s

(b) When the wave which initiates from P reaches Q (a denser medium) then it is partly reflected and partly transmitted.

In this case the amplitude of reflected wave

$A_r = (v_2 - v_1/v_2 + v_1) A_1$ . . . . (i)

Where A_i = amplitude of incident wave also amplitude of transmitted wave is

$A_t = (2 v_2/v_1 + v_2) A_i$ . . . .(ii)

From (i), (ii)

Therefore, $A_t = 2$ cm and $A_r = -1.5$ cm

Sol 23.

Speed of sound, v = 340 m/s

Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$v/4\ \ell_0 = 212.5$

or $\ell_0 = v/4\ (212.5) = 340/4\ (212.5) = 0.4$ m.

NOTE : In closed pipe only odd harmonic s are obtained. Now, let ℓ_1, ℓ_2, ℓ_3, ℓ_4, etc. be the lengths

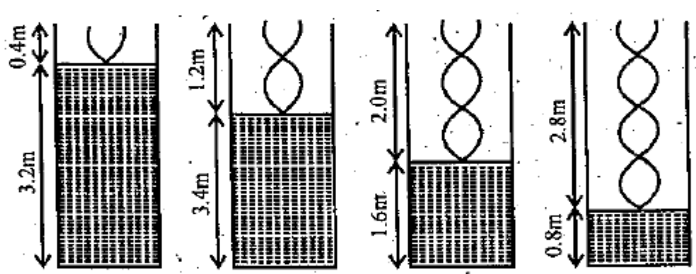
corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$3(v/4\ \ell_1) = 212.5 \Rightarrow \ell_1 = 1.2$ m;

$5\ (4\ \ell_2 = 212.5 \Rightarrow \ell_2 = 2.0$ m;

$7\ (v/\ 4\ \ell_3) = 212.5 \Rightarrow \ell_3 = 2.8$ m;

$9\ (v/4\ \ell_4) = 212.5 \Rightarrow \ell_4 = 3.6$ m;



Or heights of water of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.

Therefore heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.

Let A and a be the area of cross – sections of the pipe and hole respectively. Then

$A = \pi\ (2 \times 10^{-2})^2 = 1.26 \times 10^{-3}\ m^{-2}$

And $a = \pi\ (10^{-3})^2 = 3.14 \times 10^{-6}\ m^2$

Velocity of efflux, $v = \sqrt{2}\ g\ H$

Continuity equation at 1 and 2 gives,

$a \sqrt{2}\ g\ H = A\ (-d\ H/dt)$

Therefore, rate of fall of water level in the pipe,

$(-d\ H/dt) = a/A \sqrt{2}\ g\ H$

Substituting the values, we got

$-dH/dt = 3.14 \times 10^{-6}/1.26 \times 10^{-3} \sqrt{2 \times 10 \times H}$

$\Rightarrow -dH/dt = (1.11 \times 10^{-2}) \sqrt{H}$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$\therefore\ dH/\sqrt{H} = -1.11 \times 10^{-2}\ dt$

$\Rightarrow \int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2}) \int_0^t dt \Rightarrow 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.1 \times 10^{-2})\ t \Rightarrow t = 43$ second

Sol 24.

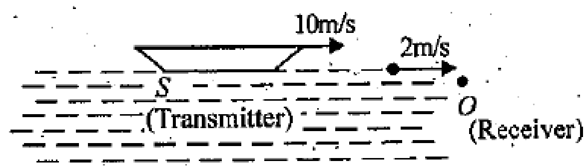
The question is based on Doppler's effect where the medium through which the sound is travelling is also in motion.

By Doppler's formula

$v' = v[c + v_m \pm v_0/c + v_m \pm v_s]$ . . . . (i)

NOTE : Sign convention for V_m is as follows : If medium is moving from s to O then +ve and vice versa. Similarly v_0 and v_s are positive if these are directed from S to O and vice versa.

(a) Situation 1.



Velocity of sound in water $c = \sqrt{B/p} = \sqrt{2.088 \times 10^9/10^3}$

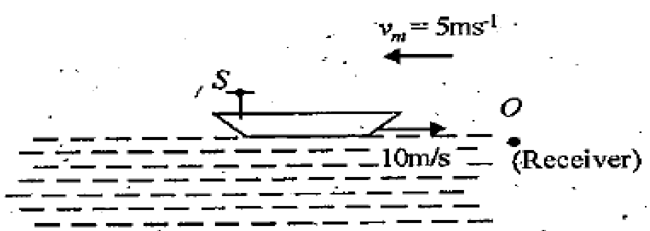
$c = 1445$ m/s; $v_m = +2$m/s' $v_0 = 0$; $v_s = 10$ m/s

$\therefore\ v' = v[1445 + 2 - 0/1445 + 2 - 10] = v[1.007]$

Now $v = c\lambda = 1445/14.45 \times 10^{-3} = 10^5$ Hz

$\therefore\ v' = 1.007 \times 10^5$ Hz

(b) Situation 2.



In air $c = \sqrt{\gamma RT/M} = 344$ m/s

Applying formula (1)

$v' = v[344 - 5 - 0/344 - 5 - 10] = 1.03 \times 10^5$ Hz

Sol 25.

(a) Second harmonic in pipe A is

$2 (v_0)_A = 2 [v/2\ell] = 1/\ell \sqrt{y_A RT/M_A}$

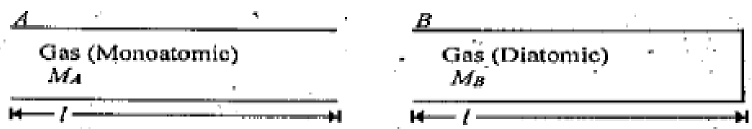
The harmonic in pipe B is

$3(v_0)_B = 3[v/4\ell] = 3/4\ell \sqrt{y_B RT/M_B}$

Given $v_A = v_B$

$1/\ell \sqrt{y_A RT/M_A} = 3/4\ell \sqrt{y_B RT/M_B}$

Or, $M_A/M_B = y_A/y_B \times (4/3)^2 = 5/3 / 7/5 \times 16/9 = 400/189$



Now, $(v_0)_A / (v_0)_B = \sqrt{y_A / y_A \times M_B / M_B} = ¾$

Sol 26.

In the fundamental mode

$(\ell + 0.6r) = \lambda/4 = v/4f \Rightarrow v = 4f(\ell + 0.6r) = 336$ m/s.

Sol 27.

Here $\ell = \lambda/2$ or $\lambda = 2\ell$ Since, $k = 2\pi/\lambda = 2\pi/\ell = \pi/\ell$

The amplitude of vibration at a distance x from x = 0 is given by $A = a \sin kx$

Mechanical energy at x of length dx is

$dE = 1/2 (dm) A^2 \omega^2 = 1/2 (\mu dx)(a \sin kx)^2 (2\pi v)^2$

$= 2\pi^2 \mu v^2 a^2 \sin^2 kx\, dx$

But $v = v\lambda$

$\therefore v = v/\lambda \Rightarrow v^2 = v^2/\lambda^2 = T/\mu / 4\ell^2$ $[\therefore v = \sqrt{T/\mu}]$

$\therefore dE = 2\pi^2 \mu\ T/\mu / 4\ell^2\ a^2 \sin^2 \{(\pi/\ell)x\}\, dx$

$\therefore$ Total energy of the string

$E = \int dE = \int_0^{\ell} 2\pi^2 \mu\ T/\mu / 4\ell^2\ a^2 \sin^2 (\pi x/\ell)\, dx = \pi^2 Ta^2/4\ell$

Sol 28.

Let the speed of the train be v_T

While the train is approaching

Let v be the actual frequency of the whistle. Then

$V = v \; v_S / v_S - v_T$

Where v_S = Speed of sound = 300 m/s (given)

v' = 2.2 k Hz = 220 Hz (given)

∴ $2200 = v\,300/300 - v_T$ . . . . (i)

While the train is receding

$v'' = v \; v_S/v_S + v_T$

Here, v' = 1.8 K Hz = 1800 Hz (given)

∴ $1800 = v\;300/300 + v_T$ . . . . (ii)

Dividing (i) and (ii)

$2200/1800 = 300/300 - v_T \times 300 + v_T/300$

⇒ $v_T = 30$ m/s

Sol 29.

The wave form of a transverse harmonic disturbance is

$y = a \sin(\omega t \pm k x \pm \varphi)$

Given $v_{max} = a\,\omega = 3$ m/s . . . . (i)

$A_{max} = a\,\omega^2 = 90\ m/s^2$ . . . . (ii)

Velocity of wave v = 20 m/s . . . . (iii)

Dividing (ii) by (i)

$A\omega^2/a\,\omega = 90/3 \Rightarrow \omega = 30$ rad/s . . . .(iv)

Substituting the value of ω in (i), we get

$a = 3/30 = 0.1$ m . . . . (v)

Now, $k = 2\pi / \lambda = 2\pi / v/\nu = \omega/v = 30/20 = 3/2$ . . . . (vi)

From (iv), (v) and (vi) the wave form is

$y = 0.1 \sin [30\, t \pm 3/2\, x \pm \varphi]$