Model Paper for IIT JEE

Paper I

Objective Questions I [Only one correct option]

Q 1.

If $\sqrt{2} \cos A = \cos B + \cos^3 B$, $\sqrt{2} \sin A = \sin B - \sin^3 B$. Then, $|\sin(A - B)|$ is equal to

(a) 1/2

(b) 1/3

(c) 2/3

(d) 1/5

Q 2.

If α is a root $x^4 = 1$, with negative principal argument, then

Principal argument of ω where $\omega = \begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix}$ (where $n \in Z_+$) is

(a) $\frac{\pi}{4}$

(b) $-\frac{3\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $-\frac{\pi}{4}$

Q 3.

If a and b be two perpendicular unit vector such that

$x = b - (a \times x)$, then $|x|$ is equal to

(a) 1

(b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\sqrt{3}$

Q 4.

Number of points on hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ from where mutually perpendicular tangents can be drawn to circle $x^2 + y^2 = a^2$ is

(a) 2

(b) 3

(c) infinite

(d) 4

Q 5.

If x + y + z + w = 5, then the least value of $x^2 \cot 9° + y^2 \cot 27° + z^2 \cot 63° + w^2 \cot 81°$ is

(a) $\frac{5}{4}$

(b) $\frac{5\sqrt{5}}{4}$

(c) $\frac{25}{4}$

(d) $\frac{25\sqrt{5}}{4}$

Q 6.

If $\sin^{-1} \sin\left(\sqrt{1-\alpha}\right) = \sqrt{1-\alpha}$, when

(a) $-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

(b) $\alpha > \frac{4-\pi}{4}$

(c) $\alpha < 4 - \frac{\pi}{4}$

(d) $\frac{4-\pi^2}{4} \le \alpha \le 1$

Q 7.

From any point A on the circle $|z - z_1| = R$, tangents are drawn to the circle $|z - z_1| = 10$, which meet the former circle at B, C. If chord BC of the former circle touches the later circle, then R is equal to

(a) 5

(b) 20

(c) 10

(d) 15

Objective Questions II [One or more than one correct options]

Q 8.

If $f(x) = \int_1^x \frac{1n\, t}{1+t}\, dt$, then

(a) $f\left(\frac{1}{x}\right) = -\int_1^x \frac{1n\, t}{t\,(1+t)} dt$

(b) $f\left(\frac{1}{x}\right) = \int_1^x \frac{1n\, t}{t\,(1+t)} dt$

(c) $f(x) + f\left(\frac{1}{x}\right) = 0$

(d) $f(x) + \left(\frac{1}{x}\right) = \frac{1}{2}(1n - x)^2$

Q 9.

If both the roots of the equation $ax^2 + x + - a = 0$ are imaginary and $c > -1$, then

(a) $3a > 2 + 4c$

(b) $3a < 2 + 4c$

(c) $c < a$

(d) $a > 0$

Q 10.

If α, $ß_i$ ∈ R and

$\sin^2 \theta_1$ =

$$\frac{(\cos^2 \alpha_1+\cos^2 \alpha_2+\cos^2\alpha_3)(\sin^2 ß_1+\sin^2 ß_2+\sin^2 ß_3)}{(\cos \alpha_1 \sin ß_1+\cos \alpha_2 \sin ß_2+\cos \alpha_3 \sin ß_3)^2}$$

$\cos^2 \theta_2 = \frac{(\sin^2 \alpha_1+\sin^2 \alpha_2+\sin^2 \alpha_3)(\cos^2 ß_1+\cos^2 ß_2+\cos^2 ß_3)}{[\sin \alpha_1 \cos ß_1+\sin \alpha_2 \cos ß_2+\sin \alpha_3 \cos ß_3]^2}$

Then

(a) $1 = \sin^4\theta_1$

(b) $\cos^4\theta_2 = 1$

(c) $\sin^8\theta_1 + \cos^8\theta_2 = 2$

(d) Cannot be determined

Q 11.

Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are Drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$, such that all the chords of contact pass through a fixed point $(x^2, y+2)$, then

(a) x_1, a, x^2 are in GP

(b) $\frac{y_1}{2}$, a, y_2 are in GP

(c) -4, $\frac{y_1}{y_2}$, $\frac{x_1}{x_2}$, are in GP

(d) $x_1 x_2 + y_1 y_2 = a^2$

Passage Based Problem

Direction (Q. N. 12 to 14) If the normal at any point P on the ellipse meets the major axis at G and S, S' Are the foci of the ellipse. Then, $\frac{SG}{SP} = \frac{S'G}{S'P}$= e P is any variable point and A, B are any two fixed points. The Point F moves such that PA + PB = k (k > AB), then locus of P is the ellipse.

Q 12.

If the lines x + y = 4 and 3x + 4y + 5 = 0 represents equations of the focal chords intersecting at P on the Ellipse whose centre is origin, then equation of tangent at 'P' of the ellipse is

(a) $x (5 - 3\sqrt{2}) + y (5 - 4\sqrt{2}) - 20 - 5\sqrt{2} = 0$

(b) $x (5 - 4\sqrt{2} - y (5 - 3\sqrt{2}) + 4 + 5\sqrt{2} = 0$

(c) $2x - 3y - 5 = 0$

(d) $2x + 3y + 5 = 0$

Q 13.

S (5, 12), S' (-12, 5) are the foci of an ellipse passing through the origin. Then, the eccentricity of ellipse

through the origin. Then, the eccentricity of ellipse

(a) $\frac{1}{\sqrt{13}}$

(b) $\frac{1}{\sqrt{5}}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{\sqrt{7}}$

Q 14.

P is any point on the ellipse whose foci are S, S'. Then, w. r. t. Δ SPS'

(a) the excentre opposites side SS' lies on tangent at P

(b) the excentre opposite to side S' P lies on tangent at P

(c) the excentre opposite to side P' S lies on tangent at P

(d) None of the above

Directions (Q. No. 15 to 17) In the Argand plane Z_1, Z_2 and Z_3 are respectively the vertices of an isosceles triangle ABC with AC = BC and ∠ CAB = θ. If I (Z_4) is the in-centre of triangle, then

Q 15.

The value of $\left(\frac{AB}{IA}\right)^2 \left(\frac{AC}{AB}\right)$ is equal to

(a) $\frac{(Z_2-Z_1)(Z_1-Z_3)}{(Z_4-Z_1)^2}$

(b) $\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)}$

(c) $\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)^2}$

(d) $\frac{(Z_2+Z_1)(Z_3+Z_1)}{(Z_4+Z_1)}$

Q 16.

The value of $(Z_4 – Z_1)^2 (1 + \cos\theta) \sec\theta$ is

(a) $(Z_2 – Z_1) (Z_3 – Z_1)$

(b) $(Z_2 – Z_1) (Z_3 – Z_1) / Z_4 – Z_1$

(c) $\frac{(Z_2-Z_1)(Z_3-Z_1)}{(Z_4-Z_1)^2}$

(d) $(Z_2 – Z_1) (Z_3 – Z_1)^2$

Q 17.

The value of $(Z_2 – Z_1)^2 \tan\theta \tan(\theta/2)$ is

(a) $(Z_1 + Z_2 – 2 Z_3) (Z_1 + Z_2 – 2 Z_4)$

(b) $(Z_1 + Z_2 – Z_3) (Z_1 + Z_2 – Z_4)$

(c) $(2 Z_3 – Z_1 – Z_2) (Z_1 + Z_2 – 2Z_4)$

(d) $(Z_1 + Z_2 + Z_3) (Z_2 + Z_3 – Z_1)$

Integer Answer Type Questions

Q 18.

Let a, b be arbitrary real numbers, Find the smallest natural number b for which the equation

$x^2 + 2 (a + b) x + (a – b + 8) = 0$ has unequal real roots for all $a \in R$.

Q 19.

If x and y are non – zero real numbers satisfying

$xy\ (x^2 - y^2) = x^2 + y^2$, find the minimum value of $x^2 + y^2$.

Q 20.

From a point P (α, ß), three real and distinct normal are drawn to the parabola $y^2 = 8x$. Let A, B and C be a feet of normals. The normals at A, B and C out the x – axis at Q, R and S respectively. The value of OQ + OR + OS (O is vertex) must be greater than a positive integer k, then find the greatest value of k.

Q 21.

If the value of $\int_0^8 \left(\sqrt{\cot^{-1}(\cot \pi x)} + \cot^{-1}(\cot \pi\sqrt{x}\,)\right) dx$

$= a\sqrt{\pi} + (2a\sqrt{2} - b)\,\pi$, then find 3a – b.

Q 22.

If α, ß are two distinct real roots of the equation $ax^3 + x - 1 - a = 0$, $(a \neq 1, 0)$, none of which is equal to unity, then the value of $\lim_{x \to \frac{1}{\alpha}} \frac{(1+a)x^3 - x^2 - a}{(e^{1-\alpha x} - 1)(x-1)}$ is $\frac{al\,(k\alpha - ß)}{\alpha}$. Find the value of kl.

Q 23.

If α, ß be the roots of the equation $x^2 + ax - \frac{1}{2a^2} = 0$, a being a real parameter, then find the least value of $[\alpha^4 + ß^4]$ (where [.] represents greatest integer function).

Q 24.

If the area of the region bounded by the curve C: y = tan x, the tangent drawn to C at x = π/4 and The x – axis is $\frac{k}{10}\left(1n\ 2 - \frac{1}{2}\right)$, then find the value of k.

Paper II

Objective Questions I [Only one correct option]

Q 1.

It the system of equations $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non – zero

Solution, then the possible values of k are

(a) -1, 2

(b) 0, 1

(c) 1, 2

(d) -1, 1

Q 2.

If A is a square matrix of order n x n such that $A_2 = A$ and I is a unit matrix of order n x n,

then $(I + A)^n$ is equal to

(a) $I + 2^n A$

(b) $I + (2^n - 1) A$

(c) $I - (2^n - 1) A$

(d) None of these

Q 3.

If f (x) is a non – negative continuous function for all $x \geq 1$, such that $f'(x) \leq pf(x)$ where

$p > 0$ and $f(1) = 0$, then $[f(\sqrt{e}) + f(\sqrt{\pi})$ is equal to

(a) 0

(b) negative

(c) positive

(d) None of these

Q 4.

In a Δ ABC is BC = a, CA = b and AB = c and Δ is the area of triangle. Then, least value of

$\frac{s^2}{P(A/B)} + \frac{(s-a)^2}{P(\bar{A}/B)} + \frac{(s-b)^2}{P(B/A)} + \frac{(s-c)^2}{P(\bar{B}/A)}$ is, (where A, B are any two events in a sample space such that

P (A), P (B) $\neq 0$

(a) $4\sqrt{3}\,\Delta$

(b) $3\sqrt{3}\,\Delta$

(c) $6\sqrt{3}\,\Delta$

(d) $12\sqrt{3}\,\Delta$

Q 5.

If $t_n = \sum_{r=0}^{n} \frac{1}{({}^nC_r)^k}$ and $S_n = \sum_{r=0}^{n} \frac{r}{({}^nC_r)^k}$, where $k \in z_+$, then $\cos^{-1}\left(\frac{S_n}{nt_n}\right)$ is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

Q 6.

Let $S = \sin\sqrt{2} - \sin\sqrt{3}$ and $C = \cos\sqrt{2} - \cos\sqrt{3}$ then which of the following is correct?

(a) $S > 0$ and $C > 0$

(b) $S > 0$ and $C < 0$

(c) $S < 0$ and $C > 0$

(d) $S < 0$ and $C < 0$

Q 7.

The sum of all the roots of the equation $\sin\left(\pi \log_3\left(\frac{1}{x}\right)\right) = 0$ in $(0, 2\pi)$ is

(a) 3/2

(b) 4

(c) 9/2

(d) 13/3

Q 8.

If $e^{(\sin^2 x+\sin^4 x+\sin^6 x+............\infty)\ln 3}, x \in \left(0, \frac{\pi}{2}\right)$ satisfies the equation $t^2 - 28t + 27 = 0$, then the value of

$(\cos x + \sin x)^{-1}$ equals to

(a) $\sqrt{3} - 1$

(b) $2(\sqrt{3} - 1)$

(c) $\sqrt{3} + 1$

(d) 1

Objective Question II [One or more than one correct options]

Q 9.

Let $(1 + x^2)^2 (1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$. If $_1$, a_2 a_3 are in AP, then n is (given that ${}^nC_r = 0$, if $n < r$)

(a) 6

(b) 4

(c) 3

(d) 2

Q 10.

If $f(x) = \int_0^1 e^{|t-x|} dt$ $(0 \leq x \leq 1)$, the maximum value of f (x) is equal to

(a) $\sqrt{e} - 1$

(b) $2(e - 1)$

(c) $(e - 1)$

(d) $2(\sqrt{e} - 1)$

Q 11.

f: R → R, $f(x) = \frac{\sin(\pi\{x\})}{x^4+3x^2+7}$, where { } is fractional function, then

(a) f is injective

(b) f is not one – one and non – condition

(c) f is a surjective

(d) f is a zero function

Q 12.

Let f: R → R and g: R → R be two one – one and onto function, such that they are the mirror images of each other about the line y = a. If h (x) = f (x) + g (x), then h (x) is

(a) one – one and onto

(b) only one – one and not onto

(c) only onto but not one – one

(d) None of the above

Integer Answer Type Questions

Q 13.

Suppose that the side lengths of a triangle are three consecutive integers and one of the Angles is twice another. The number of such triangles is . . . .

Q 14.

Let A $(2\hat{i} + 3\hat{j} + 5\hat{k})$, B $(-\hat{i}, 3\hat{j} + 2\hat{k})$ and C $(\lambda\hat{i} + 5\hat{j} + \mu\hat{k})$ are vertices if a triangle and its median through A is equally inclined to the positive directions of the axes. The value of $2\lambda - \mu$ is equal to . . . .

Q 15.

The number of integral solutions of $\sum_{i=1}^{3} x_j = 24$, $\sum_{i=1}^{3} x_i^2 = 21$ and $\prod_{i=1}^{3} x_i = 440$ is ....

Q 16.

If f (x + y + z) = f (x) + f (y) + f (z) with f (1) = 1 and f (2) = 2 and x, y, z ϵ R, then evaluate

$\lim_{n\to\infty} \frac{\sum_{r=1}^{n}(4r)f(3r)}{n^3}$ is equal to. . . . .

Q 17.

In Δ ABC, if BC is unity, $\sin\frac{A}{2} = x_1$, $\sin\frac{B}{2} = x_2$, $\cos\frac{A}{2} = x_3$ and $\cos\frac{B}{2} = x_4$ with $\left(\frac{x_1}{x_2}\right)^{2007} - \left(\frac{x_3}{x_4}\right)^{2006} = 0$, then the length of AC is . . . .

Q 18.

x, y ∈ R, $x^2 + y^2 + xy = 1$, then the minimum value of $x^3 y + xy^3 + 4$ is . . . .

Match the Columns

Q 19.

Match the statements of Column I with values of Column II.

	Column I	Column II
(A)	Out of machines, two are faulty, they are tested one by one in a random order till both faulty machines are identified, then the probability that only two test are needed.	(p) 1/2
(B)	A dice with 6 faces marked 1, 1, 4, 3, 3, 3, is tossed twice. Find the probability of getting sum 4.	(q) 1/4
(C)	(a_1, b_1, c_1), $(a_2\ b_2, c_2)$ and (a_3, b_3, c_3) are direction ratios of three perpendicular lines are direction ratio of line equally inclined to them is given by k $(a_1 + a_2 + a_3)$, k$(b_1+b_2 + b_3)$, k $(c_1 + c_2 + c_3)$. Then k is given by	(r) 1/3
(d)	If three points are lying in a plane what is the probability that a triangle will be formed by joining them.	(s) 1/6

Q 20.

	Column I	Column II
(A)	Let a, b, c be three mutually perpendicular vectors with same magnitude. If x satisfies the relation a x {(x – b) x a} + b x {(x – c) x b} + c x {(x – a) x c} = 0, then x is equal to	(p) $\frac{a+b+c}{3}$
(B)	The value of $\lim_{x\to\infty}(\sqrt[3]{(x+a)(x+b)(x+c)} - x$	(q) 1
(C)	$\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$ equals to	(r) $\frac{a+b+c}{2}$
(D)	Let a, b, c are distinct reals satisfying $a^3 + b^3 + c^3 = 3abc$. If the quadratic equation $(a + b + c)x^2 + (a + b + c)x + (c + a - b) = 0$, then a root of the quadratic equation is	(s) $\sqrt[3]{abc}$