PRACTICE PAPER 1- SOLUTIONS

ANSWER KEY

SECTION I

1.(b) 2.(d) 3.(d) 4.(b) 5.(a) 6.(d) 7.(a) 8.(b) 9.(b)

SECTION II

10.(d) 11.(a) 12.(c) 13.(b)

SECTION III

14.(d) 15.(a) 16.(b) 17.(b) 18.(d) 19.(b)

SECTION IV

20.(A)→(P) , (B) →(P),(C) →(P),(D) →(P)

21.(A)→(S) , (B) →(S),(C) →(S),(D) →(S)

22.(A)→(S) , (B) →(Q),(C) →(P),(D) →(P)

SOLUTIONS WITH CLEAR REASONING

Sol 1 (b)

Explanation:

Define the sets

A : set of odd positive integers which are less than 10,000 are divisible by 3.

B : set of odd positive integers which are less than 10,000 are divisible by 5.

Now $A = \{3,9,15 \dots\dots 9999\}$

$B = \{5,15,25 \dots\dots 9995\}$

$A \cap B = 15,45,75 \dots\dots 9975$

$9999 = 3 + (n-1)6 \Rightarrow n = 1667$

$9995 = 5 + (n-1)10 \Rightarrow n = 1000$

$9975 = 15 + (n-1)30 \Rightarrow n = 333$

$n(A \cup B) = 1667 + 1000 + 333 = 2334$

But there are 5000 odd numbers from 1 to 10000

⇒ Required number = 5000−2334 = 2666

⇒ (b) is correct.

Sol 2 (d)

Explanations:

Observe the table

Possible values of d	Possible AP's	No. of AP's
1.	(1, 2, 3), (2 ,3 , 4) ……. (22, 23, 24)	22
2.	(1, 3, 5), (2 ,4 ,6)	
……	……. (20, 22, 24)	20
…….		

11(max) (1, 11, 23), (2, 13, 14) 2

⇒ Number of AP's

= 2+4+6+….. +22

= 2(1+2+3+…..11)

= 11×12=132

(d) is correct.

Sol 3 (d)

Explanation:

Let $f(x) = 8x3 - 6x + 1$

$\Rightarrow f(-1) < 0, f(0), f\left(\frac{1}{2}\right) < 0, f(1) > 0$

⇒ There is a root between (−1) and 0, between 0 and ½ and between ½ and 1

⇒ (d) is correct.

Sol 4 (b)

Explanation:

$x^2 - 3x + 2 = (x-1)(x-2)$

when x^{100} is divided by $x-1$, the remainder is 1^{100} and when it is divided by $x-2$, the remainder is 2^{100}.

$\Rightarrow \frac{x^{100}}{x-1} = q_1(x) + \frac{1}{x-2}, \frac{x^{100}}{x-2} = q_2(x) + \frac{2^{100}}{x-2}$

On subtracting , we get

$-\frac{x^{100}}{(x-1)(x-2)} = q_1(x) - q_2(x) + \frac{1}{x-1} - \frac{2^{100}}{x-2}$

$$\frac{x^{100}}{(x-1)(x-2)} = q_2(x) - q_1(x) + \frac{2^{100}(x-1)(x-2)}{(x-1)(x-2)}$$

⇒ Remainder is $(2^{100}-1)x - 2(2^{99}-1)$

⇒ (b) is correct.

Sol 5 (a)

Explanation:

$2B = A + C, C = 3A, A + B + C = 180^0$

$\Rightarrow A = 30^0$

Sol 6 (d)

Explanation:

Applying AM ≥ HM, we get

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq \frac{3}{x+y+z} = 3$ etc.

Sol 7 (a)

Explanation:

From equation of line $4y = 12 - 3x$

On putting in equation of ellipse, we get

$9x^2 + (12 - 3x)^2 = 144$

$\Rightarrow 72x + 9x^2 = 0$

$\Rightarrow x = 0, x = 4$ whence $y = 3, y = 0$

⇒ Points of intersection of line and ellipse are $(0,3)$ and 4,0 whence r= 5

Sol 8 (b)

Explanation:

Let $OA = p, OB = q$

If P be x, y then

$$x = \frac{1 \times 0 + \times P}{3}, y = \frac{1 \times q + 2 \times 0}{3}$$

$\Rightarrow = \frac{2p}{3}, y = \frac{q}{3}$, But $p^2 + q^2 = a^2$

$\Rightarrow \left(\frac{3x}{2}\right)^2 + (3y)^2 = a^2 \Rightarrow \frac{9x^2}{4} = a^2 \Rightarrow$ (b) is correct.

<u>Sol 9</u> (b)

Explanation:

2 sec 2∝ = $\frac{sin\beta}{cos\beta} + \frac{cos\beta}{sin\beta} = \frac{1}{sin\beta cos\beta}$

⇒ sec 2∝ = $\frac{1}{sin2\beta} = cosec2\beta$

⇒ sec 2∝ = sec$\left(\frac{\pi}{2} - 2\beta\right)$

⇒ 2∝ = $\frac{\pi}{2} - 2\beta \Rightarrow \propto\ + \beta = \frac{\pi}{4}$

⇒ (b) is correct.

<u>Sol 10</u> (d)

Explanation:

Assertion is false since last period is 2**π**. Reason is true since $f(x+1) = f(x)$ for all x.

⇒ (d) is correct.

<u>Sol 11</u> (a)

Explanation:

$x\ sin\ |x| = \{x\ sin\ (-x), x < 0$

$\{x\ sin\ (+x), x > 0$

The derivatives (right and left at zero) are given below :

$-(x\cos x + \sin x)$ and $+(x\cos x + \sin x)$ which are equal at $x = 0 \Rightarrow$ Assertion is true.

The true reason is a well known result and can be proved as follows:

Let $g(x) = xf(x)$, then

$$g'(0) = \lim_{h\to\infty} \frac{(0+h)f(0+h) - g(0)}{h}$$

= $\lim_{n\to\infty} f(x) = f(0)$($\because f(x)$ is continuous)

⇒ (a) is correct.

Sol 12 (c)

Explanation:

Assertion is true since A, B, C are collinear

$\Rightarrow \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = 0,\ \alpha + \beta + \gamma = 0$

(α, β, γ are not all zero)

And $\vec{a}, \vec{b}, \vec{c}$ are position vector of A, B, C.

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar.

The reason is obviously false since there are infinite planes containing a given line.

Sol 13 (b)

Explanation:

Assertion is true and reason is correct. Since circle and parabola intersect at (a, a) in first quadrant and focus of the parabola $y^2 = ax$ is $\left(\frac{a}{4}, 0\right)$.

Sol 14) (d), **Sol 15** (a)**, Sol 16** (b)

Explanation:

If 0 < a < 1, then there is only one number which is equal to its logarithm.

If $1 < a < e^{1/e}$ then, there are two numbers which are equal to their logarithms. (Fig.(ii))

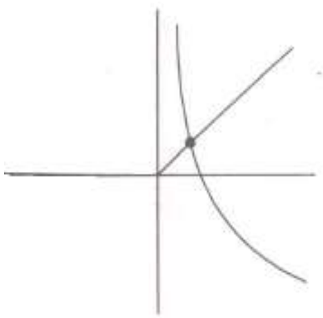

Fig. (*i*)

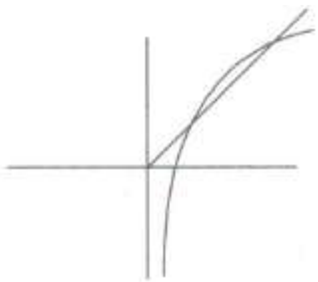

Fig. (*ii*)

If $a = e^{1/e}$ then there is only number which is equal to its logarithm (y = x will taken)

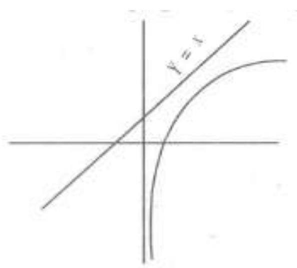

Fig. (*iii*)

The number being equal to e.

Note that $\log_{e^{1/e}} e = e$.

Finally if a > $e^{1/e}$ then there will be no number since the graph $y = x$ will not cut the graph of $y = \log_a x$.

Thus answer to 14, 15 and 16 follows.

<u>Sol 17</u> (b)

Explanation:

Put $x = 0$ in the first equation.

We get t = 0, t = 1, t = −1

For t = 0, $y = 1-0^4 = 1$

For t = 0, $y = 1-1^4 = 0$

For t =−1, $y = 1-(-1)^4 = 0$

⇒ Curve cuts y – axis at two points (0,0) and (0,1).

⇒ (b) is correct.

<u>Sol 18</u> (d)

Explanation:

(a), (b), (c) are easily ruled out suppose it is symmetrical about y – axis then if (∝, β) lies on the curve then (− ∝, β) must also lie on the curve choose t = + 2 to set (− 6, −15) on the curve. It can be shown that (−6, 15)does not lie on the curve. Since for $y = 15$, t is not real. We similarly rule out (a) and (c).

<u>Sol 19</u> (b)

Explanation:

Area of the loop

$$= \left|2\int_0^1 x\,dy\right| = \left|2\int_0^1 x - \frac{dy}{dt} - dt\right|$$

$$= \left|2\int_0^1 (t - t^3)(-4t^3)\,dt\right| = \frac{16}{35}$$

<u>Sol 20</u> (A)→(P) , (B) →(P),(C) →(P),(D) →(P)

Explanation:

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\frac{a\cos x+b}{a+b\cos x}\right)^2}}$$

$$\frac{[-a\sin x\,(a+b\cos x)+b\sin x\,(a\cos x+b)]}{(a+b\cos x)^2}$$

$$-\frac{2}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\cdot\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}\cdot\frac{1}{2}$$

$$=\frac{(a^2-b^2)\sin x}{\sqrt{(a+b\cos x)^2\,(a\cos x+b)^2}}\cdot\frac{1}{(a+b\cos x)}$$

$$-\frac{\sqrt{\frac{a-b}{a+b}}\sec^2\frac{x}{2}(a+b)}{(a+b)+(a-b)\tan^2\frac{x}{2}}$$

$$=\frac{(a^2-b^2)\sin x}{\sqrt{(a^2-b^2)-(a^2-b^2)\cos^2 x}}\cdot\frac{1}{a+b\cos x}$$

$$-\frac{\sqrt{a^2-b^2}\sec^2\frac{x}{2}}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}$$

$$=\frac{\sqrt{a^2-b^2}}{a+b\cos x}-\frac{\sqrt{a^2-b^2}}{a+b\cos x}\left(\because \cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$$

= 0

⇒ y is a constant

⇒ $y(x) = y(0)$ for all x

But $y(0) = 0$ by actually putting x=0

Thus $y(x), y(0), \frac{dy}{dx}, \frac{d'y}{dx^2}$ all are equal to zero.

Sol 21 (A)→(S) , (B) →(S),(C) →(S),(D) →(S)

Explanation:

7^{1000} will definitely be odd

⇒ Remainder will be 1

Now when 7^{1000} is divided by 3, observe the following :

$7^{1000} = (6+1)^{1000}$ = A multiple of 6+1

⇒ Remainder when 7^{1000} is divided by 3 is 1

Now $7 = 5K + 2$, $7^2 = 5K + 4$, $7^3 = 5K + 3$, $7^4 = 5K + 1$

⇒The cycle is 2, 4, 3, 1

⇒Remainder when 7^{1000} id divided by 5 is again 1.

Now $7^2 = 11K + 5$, $7^3 = 11K + 2$, $7^4 = 11K + 3$,

$7^5 = 11K + 10$, $7^6 = 11K + 4$, $7^7 = 11K + 6$,

$7^8 = 11K + 9$, $7^9 = 11K + 8$, $7^4 = 11K + 1$

⇒The cycle is of length 10.

⇒The remainder when 7^{1000} is divided by 11 is again 1.

Note: We do not have to calculate higher powers of 7.

Indeed from any equality of the type $7^m = 11K + r$

We can easily switch over to $7^{m+1} = 11K' + r'$

For example, $7^2 = 11K + 5$ $\quad (\because 49 = 44 + 5)$

$\Rightarrow 7^3 = 77K + 35 = 77K + 33 + 2 = 11K' + 2$

Again multiplying by 7, we get

$7^4 = 77K' + 14 = 77K' + 11 + 3 = 11K' + 3$

And so on.

<u>Sol 22</u> (A)→(S) , (B) →(Q),(C) →(P),(D) →(P)

Explanation:

Let S = 2 sin 2^0 + 4 sin 4^0 + … + 178 sin 178^0 on multiplying by sin 1^0

S = 2 sin 1^0 sin 2^0 + 4 sin 1^0 sin 4^0

+….. + 178.2 sin1^0 sin 178^0

= (cos 1^0 –cos 3^0) + 2(cos 3^0 –cos 5^0)+3

(sin 5^0– sin 7^0)+ … +178(cos 177^0 – cos 179^0)

= cos 1^0 +(cos 3^0 + cos 177^0)+…+(cos 89^0 + cos 91^0) + 89 cos 1^0

= cos 1^0 + 89 cos 1^0 = 90 cos1^0 (Other terms Vanish) ⇒ S =90 cot 1^0 and matching follow.