

## Practice Paper 3

### Section-I

#### Straight Objective Type

##### Q1

The number of positive integers satisfying  $\left[\frac{x}{99}\right] = \left[\frac{x}{101}\right]$  must be

- a. 2491
- b. 2495
- c. 2498
- d. 2499

##### Q2

The foci of the ellipse  $x^2 + 4x + 4\lambda^2 y^2 = 0$  will lie on the line  $x = -2$ , if

- a.  $\lambda \in (-2, 2)$
- b.  $\lambda \in \left(-\frac{1}{2}, \frac{1}{2}\right)$
- c.  $\lambda = 1$
- d. None of these

##### Q3

The point in the closed disc  $D = \{(x, y) : x^2 + y^2 \leq 1\}$  at which the function  $x + y$  attains its maximum is

- a.  $\left(\frac{1}{2}, \frac{3}{4}\right)$
- b.  $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
- c.  $(0, 1)$
- d.  $(1, 0)$

##### Q4

Let  $P(x) = x^4 + ax^3 + bx^3 + cx + d$ , where a, b, c, d are integers. Then sum of pairs of roots of  $P(x)$  are given by 1, 2, 5, 6, 9, 10 then  $P''\left(\frac{1}{2}\right)$  must be

- a. 33
- b. 28
- c. -28
- d. None of these

##### Q5

Let two arithmetic means and two geometric means between  $\alpha$  and  $\beta$  ( $\alpha > \beta > 0$ ) be  $A_1, A_2$  and  $G_1, G_1$  respectively then

- a.  $A_1 A_2 < G_1 G_1$
- b.  $A_1 A_2 > G_1 G_1$
- c.  $A_1 A_2 = G_1 G_1$
- d. None of these

**Q6**

The number of integral values of  $a$  for which the lines  $x - 4y = 1$  and  $ax + 3y = 1$  intersect at an integer point must be

- a. 0
- b. 1
- c. 3
- d. infinite

**Q7**

The range of the function  $\sin^{-1}\left(\frac{x^2+x+1}{x^4+1}\right)$  is

- a.  $\left[0, \frac{\pi}{4}\right]$
- b.  $\left[0, \frac{\pi}{2}\right]$
- c.  $\left[0, \frac{\pi}{2}\right]$
- d. None of these

**Q8**

If  $m$  and  $n$  are arbitrary positive integers then  $(1 + \omega + \omega^2 + \dots + \omega^n)^m$  ( $\omega$  being a non-real complex cube root of unity) will take

- a. 3 values
- b. 5 values
- c. 7 values
- d.  $mn$  values

**Q9**

If  $f(x)$  monotonically increasing and is differentiable on  $[\alpha, \beta, ]$  then  $\int_{\alpha}^{\beta} f(x)dx + \int_{f(\alpha)}^{f(\beta)} f^{-1}(x)dx$  is equal to:

- a.  $f(\alpha) - f(\beta)$
- b.  $\beta f(\alpha) - \alpha f(\beta)$
- c.  $\beta f(\beta) - \alpha f(\alpha)$
- d.  $f(\alpha) + f(\beta)$

## Section-II

### Multiple Objective Type

#### Q10

If  $P$  is a point inside a convex quadrilateral  $ABCD$  such that  $PA^2 + PB^2 + PC^2 + PD^2$  is twice the area of the quadrilateral, then the correct statement is/are

- $PA, PB, PC, PD$  all are equal
- $ABCD$  must be a square and  $P$  must be its Centre
- $ABCD$  must be a square but  $P$  may not be its Centre
- $ABCD$  may not be a square

#### Q11

If  $f(x)$  and  $g(x)$  are two monotonically increasing functions such that

$g(x) = f(x)\sqrt{1 - 2(f(x))^2}$  are then for values of  $x$  in the domain of  $f(x)$

- $|f(x)| \leq 1$
- $|f(x)| < \frac{2}{3}$
- $|f(x)| < \frac{1}{2}$
- $|f(x)| < \frac{1}{\sqrt{2}}$

#### Q12

Eight players of equal strength participate in a round robin tournament (Each player plays against any other player exactly once). Suppose no game ends in a draw then

- Total results in the tournament can be 256
- Total results in the tournament can be  $2^{28}$
- The number of cases in which each team has won different number of games must be 56
- The number of cases in which each team has won different number of games must be 8!.

#### Q13

The equation of an ellipse is  $(x + 2y - 3)^2 + 4(2x - y - 4)^2 = 10$  then

- Centre of the ellipse is  $\left(\frac{11}{5}, \frac{2}{5}\right)$
- eccentricity is  $\frac{\sqrt{3}}{2}$
- foci is  $\left(\frac{11}{5} + \sqrt{\frac{3}{5}}, \frac{2}{5} - \frac{3}{2\sqrt{5}}\right)$
- foci is  $\left(\frac{11}{5} - \sqrt{\frac{3}{5}}, \frac{2}{5} + \frac{3}{2\sqrt{5}}\right)$

**Q14**

Which of the following is (are) true in a triangle)

- $R^2 \geq \frac{abc}{a+b+c}$
- if the triangle is right angled at  $C$  then  $r + 2R = s$
- $\sin 2A + \sin 2B + \sin 2C \leq \frac{3\sqrt{3}}{2}$
- $\frac{2}{R} \leq \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

**Q15**

In a cube of side  $a$  let  $A$  and  $F$  be the opposite corners such that  $AF$  is a solid diagonal, then

- length of any solid diagonal is  $a\sqrt{3}$
- length of any plane diagonal is  $a\sqrt{2}$
- shortest distance between  $A$  and  $F$  through plane faces of the cubic  $a(\sqrt{2} + 1)$
- shortest distance between  $A$  and  $F$  through plane faces of the cube is  $a\sqrt{5}$

**Q16**

If  $x \in \left(0, \frac{\pi}{2}\right)$  then the minimum value of  $\tan x + \cot x + \sec x + \operatorname{cosec} x$  must be

- more than 4
- $2(\sqrt{2} + 1)$
- less than 5
- $2\sqrt{2}$ .

**Q17**

The equation  $x^4 + 4x + a = 0$ , where  $a$  is real has

- no real roots if  $a > 2$
- no real roots if  $a > 3$
- two equal roots if  $a = 3$
- two distinct real roots if  $a < 3$

**Section-III****Assertion-Reason Type****Q18****Statement-1:**

If  $a > b > 0$  then eccentricity of the ellipse  $ax^2 + by^2 + cx + dy + e = 0$  is  $\sqrt{\frac{a-b}{b}}$  because

**Statement-2:**

Eccentricity  $e$  of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b$ ) is given by  $b^2 = a^2(1 - e^2)$

- Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- Statement-1 is True, Statement-2 is False
- Statement-1 is True, Statement-2 is True

**Q19****Statement-1:**

If a tangent to the curve  $x^3 + y^3 = a^3$  at  $(x_1, y_1)$  passes through  $(a, a)$  then  $(x_1, y_1)$  lies on a circle of radius  $a$  because

**Statement-2:**

The curve  $x^3 + y^3 = a^3$  and  $x^2 + y^2 = a^2$  cut orthogonally.

- a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- c. Statement-1 is True, Statement-2 is False
- d. Statement-1 is True, Statement-2 is True

**Q20****Statement-1:**

The curve  $x^3 + y^3 = a^3$  and  $x^3 + y^3 = a^2$  cut orthogonally.

- a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- c. Statement-1 is True, Statement-2 is False
- d. Statement-1 is True, Statement-2 is True

**Statement-2:**

The solution to the differential equation  $yy'' + xy'^2 = 3yy'$  is  $y = C_1x^3 + C_2$

- a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- c. Statement-1 is True, Statement-2 is False
- d. Statement-1 is True, Statement-2 is True

**Q21****Statement-1:**

The inequality  $3x^3 - 4x^2 - 3x - 2 < 0$  is equivalent to  $x - 2 < 0$ .

**Statement-2:**

$ax^3 + bx + c > 0$  for all  $x$  if  $a > 0, b^2 - 4ac > 0$ .

- a. Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- b. Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
- c. Statement-1 is True, Statement-2 is False
- d. Statement-1 is True, Statement-2 is True

## Section-IV

### Linked comprehension type

#### M<sub>22-24</sub>: Paragraph for Question Nos. 22 to 24

Consider the differential equation  $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 0$  and answer the following questions:

#### Q22

By putting  $z = \log x$ , the differential takes the form.

a.  $\frac{d^2y}{dz^2} - 2 \frac{dy}{dz} + y = 0$

b.  $\frac{d^2y}{dz^2} - y = 0$

c.  $\frac{d^2y}{dx^2} + y = 0$

d. None of these

#### Q23

The solution of the differential equation  $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 0$  is

a.  $y = (c_1 + c_2x)ex$

b.  $y = C_1 \sin(x + C_2)$

c.  $y = C_1x + \frac{C_2}{x}$

d. None of these

#### Q24

The family of curves satisfying the given differential equations cannot be

a. circles

b. lines

c. rectangular hyperbolas

d.  $y = C_1x + \frac{C_2}{x}$

#### M<sub>25-27</sub>: Paragraph for Question Nos. 25 to 27

Let  $a_1, a_2, a_3 \dots$  be a sequence of complex numbers defined by  $a_1 = 0, a_{n+1} = a_n^2 - i$  for  $n > 1$ .

#### Q25

$a_{2n}$  Must be

a.  $1 + i$

b.  $1 - i$

c.  $-1 - i$

d.  $i$

#### Q26

$a_{2n+1}$  must be

a.  $1 + i$

b.  $1 - i$

c.  $-1 - i$

d.  $i$

**Q27**

The distance between the points represented by complex numbers  $a_{2008}$  and  $a_{2009}$  must be

- a.  $\sqrt{2}$
- b.  $\sqrt{3}$
- c. 2
- d.  $\sqrt{5}$

**Section-V****Subjective Type**

This section contains 3 Question. Write the answer of the questions (28-31) from the following combinations

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

**Q28**

The value of definite integral  $\frac{4}{\pi} \int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)}$  must be

**Q29**

The angle of intersection of curves  $y = (x - 2)^2$  and  $y = 4x - x^2 + 4$  must be  $\tan^{-1} \frac{8}{\lambda}$ , the numerical quantity  $\lambda$  should be

**Q30**

A normal to the curve  $y = x \log x$  parallel to the line  $2x - 2y + 3 = 0$  is  $x - y = \frac{\lambda}{e^2}$ , the numerical quantity  $\lambda$  should be

**Q31**

If the shortest distance between the circle  $x^2 + y^2 - 3x - \sqrt{8}y + 4 = 0$  and the parabola  $y = 1 - \frac{x^2}{2}$  is  $\frac{\sqrt{\lambda}-1}{2}$  then  $\lambda$  should be

## Section-VI

### Matrix-Match Type

#### Q32

If  $a + b = 1$ ,  $a^2 + b^2 = 2$ , then match the following:

#### Column I

- a.  $a^3 + b^3$
- b.  $a^4 + b^4$
- c.  $a^5 + b^5$
- d.  $a^7 + b^7$

#### Column II

- p.  $19/4$
- q.  $7/2$
- r.  $5/2$
- s.  $71/8$

#### Q33

Let  $ABCDEF$  be a convex Hexagon in 2 dimensional plane where A is origin

$AB \parallel DE$ ,  $BC \parallel EF$  and  $CD \parallel FA$ . They coordinates of vertices A, B, C, D, E, F are the five distinct elements of the set  $\{2, 4, 6, 8, 10\}$ .

Match the x-co-ordinates of these vertices.

#### Column I

- a. A
- b. B
- c. C
- d. D
- e. E

#### Column II

- p.  $6\sqrt{3}$
- q. 2
- r.  $\frac{10}{\sqrt{3}}$
- s. 0
- t.  $-\frac{8}{\sqrt{3}}$

#### Q34

Match the domain of following function  $[0, 2\pi]$ :

#### Column I

- a.  $\sqrt{\tan x}$
- b.  $\tan x + \cot x$
- c.  $\log(1 + \tan x)$
- d.  $\tan(\sin x)$

#### Column II

- p.  $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
- q.  $\left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$
- r.  $[0, 2\pi]$
- s.  $\left(\frac{3\pi}{4}, \frac{3\pi}{2}\right) \cup \left[\frac{7\pi}{4}, 2\pi\right]$