Practice Paper-4

Section-I

Straight Objective Type

This section contains 9 multiple choice question numbered 1 to 9. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Q1

If $f(x) = \int_{\sin x}^{\cos x} e^{t^2+xt}\, dt$, then $f'(0)$ is

a. $e + 1$

b. -1

c. $\frac{e-1}{2}$

d. $\frac{e-3}{2}$

Q2

$\sin x + \sin y = \frac{1}{\sqrt{2}}, \cos x + \cos y = \frac{\sqrt{6}}{2}$. Then the value of $\sin(x + y)$ must be

a. $\frac{1}{2}$

b. $\frac{1}{\sqrt{3}}$

c. $f \frac{\sqrt{3}}{2}$

d. None of the above

Q3

Using the result $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^3} + \ldots \ldots \infty = \frac{\pi^2}{6}, \int_0^1 \log x \log(1 \times x) dx$ is equal to

a. $2 - \pi^2/6$

b. $1 - \pi^2/6$

c. $1 + \pi^2/6$

d. None of the above

Q4

If a, b, c, be the sides of a right angled triangle whose smallest angle is θ. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also the sides of a right angled triangle then the value of $\sin \theta$ must be

a. $\frac{\sqrt{5+1}}{2}$

b. $\frac{\sqrt{5+1}}{4}$

c. $\frac{\sqrt{5-1}}{2}$

d. $\frac{\sqrt{5-1}}{4}$

Q5

Let f be a continuous function whose domain and range be [0, 1] and [0, 1], then the equation $f(x) = x$

a. has atleast one solution

b. has atleast two solutions

c. may not have any solution

d. always has infinite solutions

Q6

$A(3, 4\ )$, $B(4, 3)$ and $C(5, 0)$ be the vertices of a triangle ABC. A circle passes through mid-points of sides of ΔABC. ABC. The radius of this circle must be

a. $\frac{5}{8}$

b. $\frac{5}{4}$

c. $\frac{5}{2}$

d. $\frac{\sqrt{5}}{2}$

Q7

The value of $\cos 65° \cos 55° \ cos 5°$ must be

a. $\frac{\sqrt{4}}{3}$

b. $\frac{1}{4}\sqrt{2-\sqrt{3}}$

c. $\frac{5\sqrt{3}}{9}$

d. $\frac{1}{4}\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$

Q8

If $x \ and\ y$ are positive real numbers satisfying $x^2 + y^2 = 1$, then the minimum value of $x - y + \frac{1}{xy}$

a. 2

b. $2 + \sqrt{2}$

c. 3

d. $1 + \sqrt{2}$

Q9

Suppose the circumcentre of a triangle ABC lies on BC. Then the orthocenter of the triangle must be

a. the point A

b. the incentre of the triangle

c. the centroid of the triangle

d. outside the triangle

Section-II

Multiple Objective Type

Q10

There are exactly n circles whose radii are the positive integers and which do not cut the curve $xy = 9, n$ must be

a. 2
b. 3
c. 4
d. 5

Q11

If $c\int_0^1 xf\ (2x)dx = \int_0^2 tf\ \ (t)\ dt$ where f is a positive continuous function then value of c must be

a. 1/2
b. 4
c. 2
d. 1

Q12

The equation $\tan x = \frac{x}{100}$ has

a. 100 roots
b. 200 roots
c. one root
d. infinite roots

Q13

A and S is said to have a minimum if there is an element a in S such that $a \leq y$ a for all in S. Similarly, is said to have a maximum if there is an element b in S such that $b \geq y$ for all $y\ in\ S. If\ S = \left\{y: y = \frac{2x+3}{x+2}, x \geq 0\right\}$, which one of the following statements is correct?

a. S
has both a maximum and a minimum
b. S has neither a maximum nor a minimum
c. S has a maximum but not minimum
d. S has a minimum but not maximum

Q14

If S is the area of triangle ABC then $\sqrt{a^2b^2 - 4S^2} + \sqrt{b^2c^2 - 4S^2} + \sqrt{c^2a^2 - 4S^2}$ must be equal to

a. $a^2 + b^2 + c^2$
b. $\frac{1}{2}(a^2 + b^2 + c^2)$
c. $ab\cos C + bc\cos A + ca\cos B$
d. $ab\sin C + bc\sin A + ca\sin B$.

Q15

If $A\ and\ B$ are positive acute angles then the equation $sin^6 A + 3sin^2 A\ cos^2\ B + cos^6\ B = 1\ is$

a. never possible

b. possible only when $A + B = 90°$

c. possible only when $A = B$

d. possible when $A = B = 15°$

Q16

If $x^2 + y^2 + z^2 = 2xyz$ where x, y, z are integers then

a. all the three integers x, y, z cannot be odd

b. two of them cannot be odd

c. none of them can be odd

d. none of them can be even

Q17

Two ellipses $\frac{x^2}{cos^2 \propto} + \frac{y^2}{sin^2 \propto} = 1 \left(0 < \propto < \frac{\pi}{4}\right)$and $\frac{x^2}{sin^2 \propto} + \frac{y^2}{cos^2 \propto} = 1$ intersect at four points, then

a. $PQRS$ is a square of side $\sin 2 \propto$

b. P, Q, R, S lie on a circle whose centre is origin and whose radius is $\frac{sin2\propto}{\sqrt{2}}$

c. there are two point on the ellipse $\frac{x^2}{sin^2 \propto} + \frac{y^2}{cos^2 \propto} = 1$ whose reflection in the line $y = x$ lie on the same ellipse

d. The eccentricities of the two ellipses are same

Section-III

Linked Comprehension Type

$M_{18\text{-}20}$: Paragraph for Question Nos. 18 to 20

Q18

$P + P'$ must be equal to

a. S^2

b. $-S^2$

c. $2S^2$

d. None of the above

Q19

S satisfies the polynomial equation

a. $S^6 + 20S^2 - 144 = 0$

b. $S^2 + 2S - 3 = 0$

c. $S^6 - 5S^3 + 12 = 0$

d. None of the above

Q20

The value of S' must be equal to

a. 3

b. 2

c. -2

d. None of the above

M$_{21\text{-}23}$: Paragraph for Question Nos. 21 to 23

Q21

If which of the following $P(n) \nRightarrow P(n+1)$

a. 8” – 1 is divisible by 7

b. 3”+ 8” is not divisible by 5

c. 2”+ 1 is not divisible by 7

d. None of these.

Q22

In which of the following $P(n) \Rightarrow P(n+1)$

a. $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}$ is not an integer for any $n>1$.

b. $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<2$

c. $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}(n>1)$

d. None of these

Q23

Let $P(n)$ be statement $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2^2}\right)\ldots\left(1+\frac{1}{2^n}\right)<\frac{5}{2}$ then while proving $P(n)<P(n+1)$. which of the following will come across ?

a. $\frac{5}{2}\left(1-\frac{1}{2^n}\right)\left(1+\frac{1}{2^{n+1}}\right)\leq\frac{5}{2}\left(1-\frac{1}{2^{n+1}}\right)$

b. $\frac{5}{2}\left(1+\frac{1}{2^n}\right)\left(1+\frac{1}{2^{n+1}}\right)\leq\frac{5}{2}\left(1-\frac{1}{2^{n+1}}\right)$

c. $\frac{5}{2}\leq\frac{5}{2}$

d. None of these

Section-IV

Subjective Type

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Q24

Number of divisors of the number $2^{100}3^{11}5^{12}$ which leave remainder 1 when divided by 4 is

Q25

The length of the longest chord of the ellipse $4x^2 + 9y^2 = 1$ drawn through $\left(0, -\frac{1}{3}\right)$ is $\frac{3}{\sqrt{\lambda}}$ the numerical integer λ should be

Q26

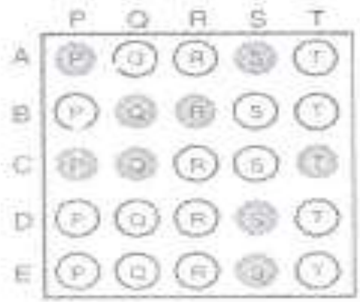
If $cos\ \frac{\pi}{7}$ is a root of the equation $8x^3 + 4x^2 - 4x - \lambda = 0$, then the numerical integer λ should be

Q27

The graph of a quadratic function defined from [- 2, 3] to [0, 3] touches $x - axis$ at $x = 3$. If the function is $\frac{3}{\lambda}(x^2 - 6x + 9)$, then the numerical integer should be

Section-V

Matrix-Match Type



Q28

Match the following functions with their ranges :

Column I	**Column II**
a. $(x^2 + 3)^2 + 4$	p. $(-\infty, \infty)$
b.$\sin x - [\sin x]$	q. [0, 1]
c. $\cot(\cot x)$	r. [13, ∞]
d. $\tan(x^2 + x + 1)$	

Q29

If $log_{10}5 = a, log_{10}3 = b$, then match the following :

Column I	**Column II**
a. $log_{30}\ 8$	p. $\frac{3+3b-3a}{1-a}$
b. $log_2 216$	q. 8/3
c. $log_{30} 216$	r. $\frac{3+3b-3a}{b}$
d. $log_8 256$	s. $\frac{3(1-a)}{b+1}$

Q30

Consider the expression $f(x) = x^2 + mx + m^2 + 6m$, where m is a parameter, match the following

Column I	**Column II**
a. $f(x) > 0$ for all x	p. $(-\infty, \infty)$
b. $f(x) < 0$ for $x \in (1,\ 2)$	q. Null set
c. $f(x) > 0$ for all $x > 0$	r. $(-\infty, -\infty) \cup (0, \infty)$
d. $f(x) < 0$ for all x	s. $\left(-\frac{7+3\sqrt{5}}{2}, -4 + 2\sqrt{3}\right)$