

ANSWERS KEY

CHEMISTRY

1. a 2. d 3. a 4. b 5. b 6. b 7. c 8. b 9. c 10. a 11. c 12. a 13. c
14. a 15. d 16. c 17. c 18. c 19. b 20. d 21. c 22. b 23. a 24. b 25. a 26. a
27. d 28. a 29. a 30. c

PHYSICS

1. b 2. a 3. d 4. c 5. d 6. a 7. c 8. b 9. a 10. c 11. b 12. c 13. c
14. a 15. c 16. a 17. b 18. b 19. b 20. b 21. b 22. d 23. c 24. c 25. c 26. d
27. b 28. d 29. b 30. d

MATHEMATICS

1. c 2. b 3. b 4. a 5. c 6. d 7. c 8. a 9. d 10. b 11. a 12. b 13. c
14. a 15. d 16. a 17. b 18. c 19. a 20. a 21. d 22. a 23. c 24. c 25. c 26. b
27. c 28. a 29. c 30. a

HINTS & EXPLANATIONS

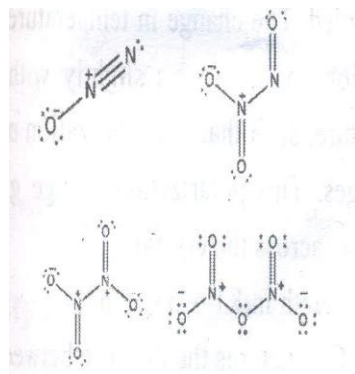
CHEMISTRY

Sol 1.

Ca imparts red colour to the flame.

Sol 2.

N_2O_5 does not have a N – N bond.



Sol 3.

In this process, the impure nickel is reached with excess carbon monoxide at 50- 60 °C to form nickel carbonyl. $\text{Ni (s)} + 4 \text{CO (g)} \rightarrow \text{Ni (CO)}_4 \text{(g)}$

The mixture of excess carbon monoxide and nickel tetracarbonyl decomposes to give nickel:



Sol 4.

Orbitals corresponding to options are :

- (a) 4s
- (b) 3d
- (c) 3d
- (d) 3p;

Increasing order of energy is 3p, 4s, 3d = 3d or

$$d < a < b = c$$

Sol 5.

$$\Delta T_f = i \cdot K_f \cdot m$$

For glycerine, $\Delta T_f = 1 \times 0.030 \times K_f = 0.030 \times K_f$ For KBr, $\Delta T_f = 2 \times 0.02 \times K_f = 0.04 \times K_f$ (KBr undergoes dissociation into K^+ and Br^- ions) for benzoic acid, $\Delta T_f = \frac{1}{2} \times 0.030 \times K_f = 0.015 \times K_f$ (two molecules of benzoic acid undergo association) order : $c < a < b$

Sol 6.

108 g of Ag requires 96500 C

$$1 \text{ g of Ag requires } \frac{96500}{108} = 893.52 \text{ C}$$

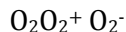
$$Q = I \times t \text{ or } 893.52 = 30 \times t$$

$$t = 893.52/30 = 29.78 \text{ s}$$

Sol 7.

M.O. electronic configuration for

$$O_2 = (\sigma_{1s})^2 (\sigma^*_{1s})^2 (\sigma_{2s})^2 (\sigma^*_{2s})^2 (\sigma_{2pz})^2 (\pi_{2px})^2 (\pi_{2py})^2 (\pi^*_{2px})^1 (\pi^*_{2py})^1$$



Number of bonding electrons:

10 10 10

Number of anti bonding electrons:

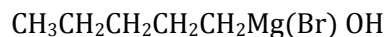
6 5 7

B.O. :

2 2.5 1.5

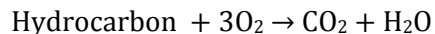
Sol 8.

The reaction of pentyl magnesium bromide with water gives pentane.

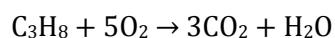
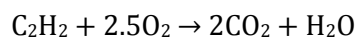
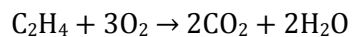
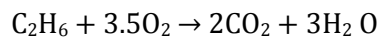


Sol 9.

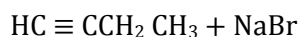
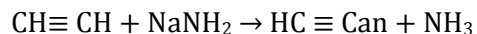
5L of hydrocarbon requires 15L of oxygen for complete combustion or 1 mol of hydrocarbon requires 3 mol of oxygen, therefore the reaction can be represented as:



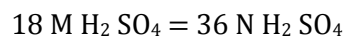
Now let us consider complete combustion of hydrocarbons given in options.



Therefore the hydrocarbon is ethane

Sol 10.

Sod. acetylide ethyl bromide 1- Butyne

Sol 11.

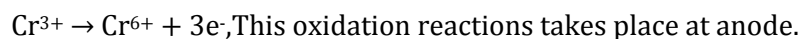
$$N_1V_1 = N_2V_2$$

$$36 \times 10 = N_2 \times 10000$$

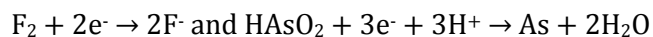
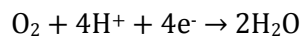
$$N_2 \frac{360}{10000} = 0.036 \text{ N}$$

Sol 12.

Conversion of Cr^{3+} to $\text{Cr}_2\text{O}_7^{2-}$ involves loss of 3 electrons per Cr Atom, i.e.,



Other reactions are reduction reactions which occur at cathode.

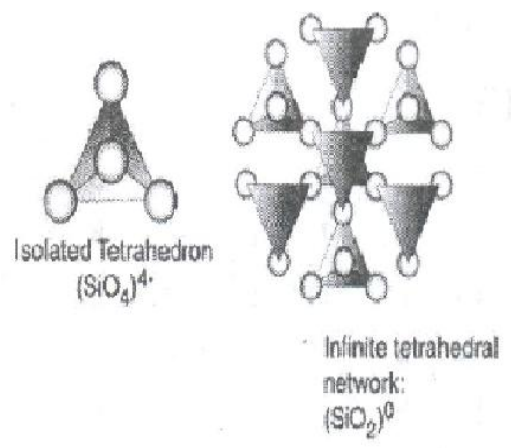


Sol 13.

Zeise's salt, $K[PtCl_3(\eta^2-C_2H_4)]$ is not a 18 e^- species. The 18-electron rule is a rule used primarily for predicting formulae for stable metal complex. The rule is based on the fact that the valence shells of transition metals consist of nine valence orbitals, which collectively can accommodate 18 electrons as either bonding or nonbonding electron pairs. This means that, the combination of these nine atomic orbitals with ligand orbitals creates nice molecular orbitals that are either metal-ligand bonding or non-bonding. When a metal complex has 18 valence electrons, it is said to have achieved the same electron configuration as the noble gas in the period. Zeise's salt $[PtCl_3(C_2H_4)]^-$ violates the 18e rule and is an example of 16e complex.

Sol 14.**Sol 15.**

The basic unit in all silicates is SiO_4^{4-}

**Sol 16.**

$$\text{Average} = \frac{\Delta[A]}{\Delta t} = \frac{(0.10 - 0.050) 0.05}{(20 - 10) 10}$$

$$= 0.005 \text{ or } 5 \times 10^{-3}$$

Sol 17.

Pyroelectricity (from the Greek pyr, fire, and electricity) is the ability of certain materials to generate a temporary voltage when they are heated or cooled. The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes. This polarization change gives rise to a voltage across the crystal.

Sol 18.

Edge length in KF = 537. Pm

In FCC structures the distance between two ions will be half of the edge length of the unit cell, i.e., 268.75 pm.

Sol 19.

Greater is the valency of the oppositely charged ions of the electrolyte being added, the faster is the coagulation.

Sol 20.

In colloidal and surface chemistry, the critical micelle concentration (CMC) is defined as the concentration of surfactants above which micelles formation takes place by association of surfactant molecules.

Sol 21.

Number of moles of $\text{CuCl}_2 = \frac{13.44}{134.4} = 0.1$;

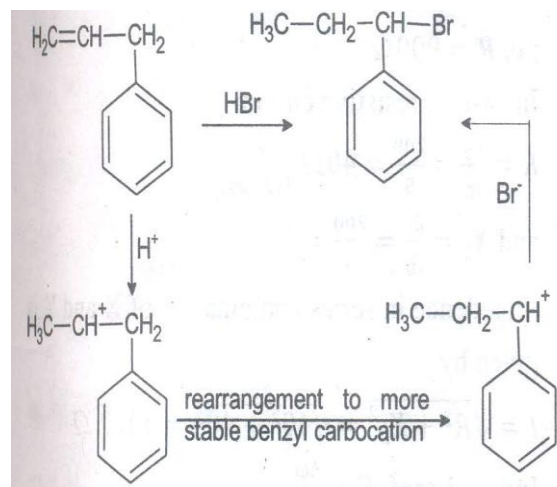
$\text{CuCl}_2 \rightarrow \text{Cu}^{2+} + 2\text{Cl}^-$;

Complete ionization of CuCl_2 gives 3 ions; value of $i = 3$

$\Delta T_b = i \cdot K_b \cdot m = 3 \times 0.52 \times 0.1 = 0.156 = 0.16$

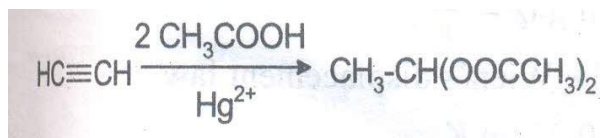
Sol 22.

the reaction involves the formation of a more stable benzyl carbocation by the rearrangement of initially formed secondary carbocation.

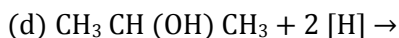
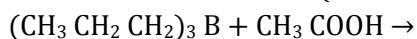
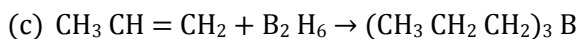
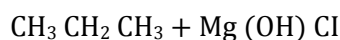
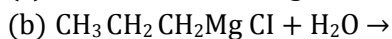
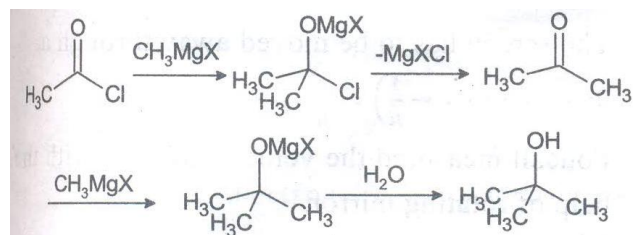


Sol 23.

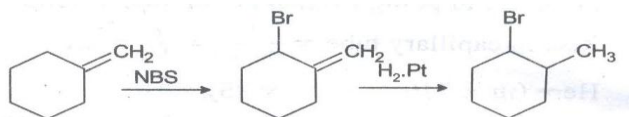
The reaction involves addition of 2 moles of acetic acid to acetylene.

**Sol 24.**

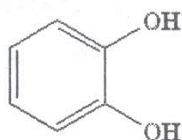
The reaction between acetyl chloride and CH_3MgX results in the formation of a tertiary alcohol.

**Sol 25.**

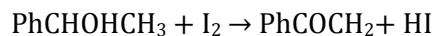
NBS is a specific reagent for allylic bromination

**Sol 26.**

Catechol is

**Sol 27.**

Positive iodoform test is given by PhCHOHCH_3

**Sol 28.**

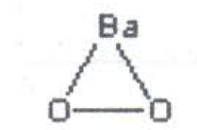
In solution, when stored at temperatures ranging from 30 to 80°C, aspartame is progressively degraded into diketopiperazine. It is therefore not usable in foods heated at higher temperature.

Sol 29.

When H_2S gas is passed in a metal sulphate solution in the presence of NH_4OH , 4th group cations are precipitated as corresponding sulphides. Zn gives white ppt in 4th group analysis.

Sol 30.

BaO₂ is a peroxide.



PHYSICS

Sol 1.

In a Vernier Callipers

Length measured = Reading before zero + number of coinciding Vernier division x Vernier Constant

$$= 9 \times 10^{-3} + 0 \times 1 \times 10^{-4}$$

$$= 0.009 \text{ m} = 9 \text{ mm}$$

Sol 2.

Maximum height of a projectile is given by

$$H_{max} = \frac{u^2}{2g} \text{ for } \theta = 90^\circ$$

$\Rightarrow H_{max}$ is independent of mass

Sol 3.

Only the total momentum of an isolated system remains unchanged.

Both statements are not correct.

Sol 4.

$$\text{Given } p^1 = \frac{3}{2}p \Rightarrow v' = \frac{3}{2}v$$

As K.E. $\propto V^2$

$$\Rightarrow \frac{K.E.}{K.E.} = \frac{9}{4}$$

$$\text{And increase in K.E.} = \frac{E.E.^1 - K.E.}{K.E.} \times 100$$

$$= \frac{5}{4} \times 100 = 125\%$$

Sol 5.

Kinetic energy of rotation

$$K.E._{rot} = \frac{1}{2} I \omega^2$$

$$\Rightarrow I = \frac{2 K.E._{rot}}{\omega^2}$$

$$= \frac{2 \times 360}{30 \times 30} = 0.8 \text{ kgm}^2$$

Sol 6.

The speed of a planet orbiting the sun always increases in going from aphelion to perihelion

Sol 7.

$$\text{Rise in capillary tube} = \frac{h}{\sin 30^\circ} = \frac{h}{1/2} = 2h$$

Sol 8.

$$\text{Here } (m \times 540 + m \times 1 \times 95) \times 10^3$$

$$= 10 \times 10^{-3} \times 80 \times 10^3 + 94 \times 10^{-3} \times 1 \times 5 \times 10^3$$

$$= 2g$$

Sol 9.

Using the relation

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$1930 = \sqrt{\frac{3 \times 8.314 \times 10^{11} \times 300}{m}}$$

$$\Rightarrow M = 2 \Rightarrow \text{H}_2 \text{ gas}$$

Sol 10.

$$\frac{V_H}{V_0} = \sqrt{\frac{32}{2}} = 4$$

Sol 11.

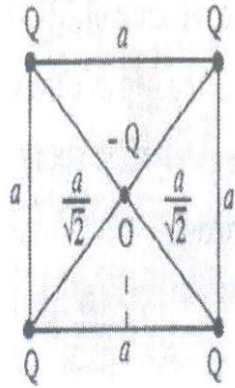
The source of sound can be identified by the overtones present in the sound.

Sol 12.

Potential at O is given by $V = 4 \cdot \frac{1}{4\pi\epsilon_0 r} Q$

$$= 4 \times \frac{1}{4\pi\epsilon_0} \frac{Q}{a/\sqrt{2}}$$

$$= \frac{Q\sqrt{2}}{\pi\epsilon_0 a}$$



and work done = $V \times Q = \frac{Q\sqrt{2}}{\pi\epsilon_0 a} Q = \frac{\sqrt{2}Q^2}{\pi\epsilon_0 a}$

Sol 13.

The resistance will be greater when the lamp is switched on

Sol 14.

Given $E_{mag} = MB$ And $E_{th} = \frac{3}{2}KT_1 \Rightarrow \frac{E_{th}}{E_{mag}} = \frac{\frac{3}{2}KT_1}{MB} = \frac{3KT_1}{2MB}$

Sol 15.

Using $R \frac{V}{I_g} = 100$

$$100 + R = \frac{V}{I_g}$$

Also $1000 + R = \frac{2v}{I_g} \cdot 100 \Rightarrow 1100 + R = \frac{2v}{I_g}$

$$\therefore \frac{1100 + R}{100 + R} = \frac{2v}{I_g} \times \frac{I_g}{V} = 2 \Rightarrow 1100 + R = 200 + 2R$$

i.e. $R = 900 \Omega$

Sol 16.

In case of resistive circuit

$$R = \frac{E_0}{I_0} = \frac{200}{5} = 40\Omega$$

And $X_L = \frac{E_0}{I_0} = \frac{200}{5} = 40\Omega$

Impedence in series combination of X and Y is given by

$$I = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\Omega$$

Sol 17.

$$\text{Induced } emf. E = \frac{\Delta\phi}{\Delta txR}$$

$$\text{current, } I = \frac{Q}{\Delta t} = \frac{E}{R} = \frac{\Delta\phi}{\Delta txR}$$

$$\text{comparing } Q = \frac{\Delta\phi}{R}$$

Sol 18.

Using the Wien's displacement law

$$\lambda_m T = 0.29 \text{ cm K}$$

$$\lambda_m = \frac{0.29}{T} = \frac{0.29}{2.7} = 0.11 \text{ cm}$$

This wavelength belongs to microwave.

Sol 19.

The screen has to be moved away through a distance $t \left(1 - \frac{1}{\mu}\right) = \frac{t}{\mu} (\mu - 1)$

Sol 20.

Foucault measured the velocity of light with the help of rotating mirror.

Sol 21.

$$\text{Deviation } \delta = (\mu - 1) A$$

For glass $\mu = 1.5$

$$\Rightarrow \delta_1 = (1.5 - 1) A = 0.5A$$

When prism is dipped into water, the new refractive index

$$\mu' = \frac{3/2}{4/3} = \frac{9}{8}$$

$$\therefore \delta_2 = \left(\frac{9}{8} - 1\right) A = \frac{1}{8} A$$

$$\text{Dividing } \frac{\delta_2}{\delta_1} = \frac{1}{8 \times 0.5} = \frac{1}{4}$$

Sol 22.

Stopping potential does not depend upon the distance of source from photocell

$$\text{Saturation current} \propto \frac{1}{\text{square of distance of source}}$$

$$\therefore I_1 = 18 \propto \frac{1}{(0.2)^2} \text{ and } I_2 \propto \frac{1}{(0.6)^2} \Rightarrow \frac{I_2}{18} = \frac{(0.2)^2}{(0.6)^2} \times 18 = 2 \text{ mA}$$

Sol 23.

$$\text{Using } \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \left(1 - \frac{3}{4}\right) = \left(\frac{1}{2}\right)^{3/4T}$$

$$\text{or } \frac{1}{4} = \left(\frac{1}{2}\right)^{3/4T}$$

$$\text{Or } 2 = \frac{3}{4T} \Rightarrow T = \frac{3}{8} \text{ S}$$

Sol 24.

$$\text{Using } hv = Rchz^2 \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right] = Rchz^2 \left[\frac{n^2(n-1)^2}{(n-1)^2 n^2} \right]$$

$$= Rchz^2 \left[\frac{2n-1}{n^2(n-1)^2} \right]$$

$$\text{or } hv = \frac{2Rcz^2}{n^3}$$

$$\Rightarrow v \propto \frac{1}{n^3}$$

Sol 25.

$$\text{Band gap } E_g = hv = \frac{hc}{\lambda}$$

$$\text{Given } \lambda = 2480 \text{ nm} = 2480 \times 10^{-9} \text{ m}$$

$$= 248 \times 10^{-8} \text{ m}$$

$$\Rightarrow E_g = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{248 \times 10^{-8}}$$

$$= 7.984 \times 10^{-20} \times 10^{-20} \text{ J}$$

$$= \frac{7.984 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 0.499 \text{ eV}$$

$$= 0.5 \text{ eV}$$

Sol 26.

$$f_c \propto \sqrt{N_{max}}$$

Where

N_{max} = maximum electron density of ionosphere

$$\text{Here } \frac{f_{c1}}{f_{c2}} = \left(\frac{N_{max1}}{N_{max2}} \right)^{1/2}$$

$$\Rightarrow \frac{N_{max1}}{N_{max2}} = \left(\frac{f_{c1}}{f_{c2}} \right)^2 = \left(\frac{10}{8} \right)^2 = \frac{25}{16}$$

Sol 27.

Using the formula

$$f' = f \left(\frac{v+v_0}{v} \right) \text{ where } v = \text{speed of sound}$$

$$\Rightarrow 5.5 = 5 \left(\frac{v+v_A}{v} \right) \text{ where } v_A = \text{speed of train A and } 6.0 = 5 \left(\frac{v+v_B}{v} \right) \text{ where } v_B = \text{speed of train B}$$

$$\text{Dividing I} = \frac{v_B}{v_A} = 2$$

Sol 28.

Using $I = neAv_d$

$$\text{Drift speed } v_d = \frac{1}{neA}$$

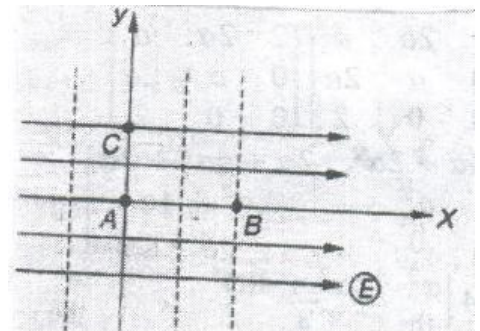
$$\Rightarrow v_d \propto \frac{1}{A}$$

\Rightarrow For non uniform cross section or different values of A, drift speed will be different at different section. Only current or rate of flow of charge will be same.

Sol 29.

Potential decreases in the direction of electric field. The dotted lines represent the equipotential lines

$$\therefore V_A = V_C \text{ and } V_A > V_B$$

**Sol 30.**

X-rays suffer diffraction when the width of slit is of the order of wavelength of x-rays. Here wavelength of x-rays ($1 - 100 \text{ \AA}$) is very much less than slit width (0.6 mm). Therefore no diffraction will occur.

MATHEMATICS

Sol 1.

$$f(\theta) = \sin^4 \theta + \cos^4 \theta$$

$$f(\theta) = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{1}{2}(2 \sin \theta \cos \theta)^2$$

$$= 1 - \frac{1}{2}(\sin 2\theta)^2$$

$$= 1 - \frac{1}{2}\left(\frac{1 - \cos 4\theta}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} \cos 4\theta$$

Period of $\cos 4\theta$ is $\frac{2\pi}{4}$

$\therefore f(\theta)$ is periodic with period $\frac{\pi}{2}$

Sol 2.

$xRy \Rightarrow yRx \therefore R$ is symmetric

Sol 3.

Let n and $(n + 1)$ be the two consecutive roots of

$$x^2 + bx + c = 0$$

$$\therefore n + (n + 1) = -b \text{ and } n(n + 1) = c$$

$$\therefore b^2 - 4c = (-b)^2 - 4c$$

$$= (2n + 1)^2 - 4c(n + 1)$$

$$= 4n^2 + 4n + 1 - 4n^2 - 4n$$

$$= 1$$

Sol 4.

$$\text{Let } z = \frac{i}{1+i}$$

$$z = \frac{i}{i+1} \times \frac{(i-1)}{(i-1)}$$

$$z = \frac{i^1 - i}{-1 - 1} = \frac{-1 - i}{-2}$$

$$\therefore z = \frac{1+i}{2} \Rightarrow z = \frac{1-i}{2}$$

Sol 5.

$$A = \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4\alpha + 2\alpha^2 & 2\alpha + 4\alpha^2 + 2\alpha \\ 0 & \alpha^2 & 2\alpha^2 + 4\alpha \\ 0 & 0 & 4 \end{bmatrix}$$

$$|A^2| = 4 \begin{vmatrix} \alpha^2 & 2\alpha^2 + 4\alpha \\ 0 & 0 \end{vmatrix}$$

$$= 16 \alpha^2 = 16$$

$$\text{Given } |A^2| = 16$$

$$\Rightarrow \alpha^2 = 1$$

$$\Rightarrow |\alpha| = 1$$

Sol 6.

$$\text{Given } A^2 + A - I = 0$$

$$\Rightarrow A^{-1} A^2 + A^{-1} A - A^{-1} I = 0$$

$$\Rightarrow (A^{-1} A) A + I - A^{-1} I = 0$$

$$\Rightarrow IA + I - A^{-1} = 0$$

$$\Rightarrow A + I = A^{-1}$$

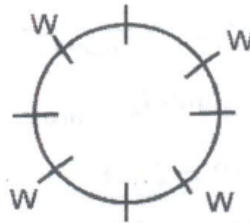
Sol 7.

First fix the position of one woman, the number of ways to sit 3 women is $3!$ and the number of ways to sit 3 men is 4P_3

\therefore Total number of ways

$$= 3! \times {}^4P_3$$

$$= 3! \times 4!$$



Sol 8.

Given that

$$P(A) = 0.20, P(B) = 0.30, P(A \cap B) = 0.10$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.20 + 0.30 - 0.10$$

$$P(A \cup B) = 0.40$$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.40$$

$$= 0.60$$

Sol 9.

$$\text{Let } P(n) = 2^{3n} - 1$$

$$\text{Step I : } P(1) = 2^{3 \times 1} - 1 = 8 - 1 = 7 \text{ which is divisible by 7}$$

So $P(1)$ is true.

Step II : Let $P(m)$ be true. The n

$$2^{3m} - 1 \text{ is divisible by 7} \Rightarrow 2^{3m-1} = 7\lambda \text{ for some } \lambda \in \mathbb{N}$$

$$\text{Step III : } P(m+1) = 2^{3(m+1)} - 1$$

$$= 2^{3m} 2^3 - 1$$

$$= (7\lambda + 1) 2^3 - 1$$

$$= 56\lambda + 8 - 1$$

$$= 7(8\lambda + 1), \text{ which is divisible by 7}$$

$\therefore P(m+1)$ is true Hence $P(n)$ divisible by 7.

Sol 10.

$T_{r+1} = n C_r a^r$ is the $(r+1)$ th term in the expansion of $(x+a)^n$.

\therefore In the expansion of $\left(x^2 + \frac{1}{x}\right)^8$

$$T_4 = T_{3+1} = {}^8C_3 (x^2)^{8-3} \left(\frac{1}{x}\right)^5$$

$$= {}^8C_3 x^{10} \frac{1}{x^5}$$

$$= \frac{8 \times 7 \times 6}{3 \times 2} x^5 = 56 x^5$$

Sol 11.

$$T_{r+1} = {}^{10}C_r (x^3)^r \left(-\frac{1}{x^2}\right)^{10-r}$$

$$= {}^{10}C_r (-1)^{10-r} \frac{x^{3r}}{x^{20-2r}}$$

$$T_{r+1} = {}^{10}C_r (-1)^{10-r} x^{-20+5r}$$

The term is independent of x if

$$-20 + 5r = 0$$

$$5r = 20 \Rightarrow r = 4$$

We get

$$T_5 = {}^{10}C_4 (-1)^{10-4} = {}^{10}C_4 (-1)^6$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

Sol 12.

We know that,

$$= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \dots (i)$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \dots \dots \dots (ii)$$

Adding (i) and (ii), we get

$$e + e^{-1} = 2 + \frac{2}{2!} + \frac{2}{4!} + \dots \dots \dots$$

$$\Rightarrow \frac{e^2 + 1}{e} - 2 = \frac{2}{2!} + \frac{2}{4!} + \dots \dots \dots$$

$$\Rightarrow \frac{e^2 + 1 - 2e}{e} = 2 \left(\frac{1}{2!} + \frac{1}{4!} + \dots \dots \right)$$

$$\Rightarrow \frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \dots \dots \dots$$

Sol 13.

$$\text{Given that } \frac{x^m}{y^n} = (x - y)^{(m-n)}$$

$$m \log x - n \log y = (m - n) \log (x - y)$$

$$\frac{m}{n} - \frac{n}{y} \frac{dy}{dx} = \frac{(m - n)}{(x - y)} \left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} \left[\frac{m - n}{x - y} \right] = -\frac{m}{x} + \frac{m - n}{x - y}$$

$$\frac{dy}{dx} \left[\frac{my - ny - nx + ny}{y(x - y)} \right] = \frac{-mx + my + mx - nx}{x(x - y)}$$

$$\frac{dy}{dx} \frac{x}{y} = \frac{my - nx}{my - nx} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Sol 14.

Using mean value through for $f(x) = x^2 + 3x$ in interval $[2, 4]$

$$f'(c) = \frac{f(4) - f(2)}{4 - 2}$$

$$2c + 3 = \frac{(16+12) - (4+6)}{2}$$

$$2c + 3 = 9 \Rightarrow 2c = 6$$

$$\Rightarrow c = 3$$

Sol 15.

The equations of given curve and line are given

$$y = x^2 \text{ and } y = x$$

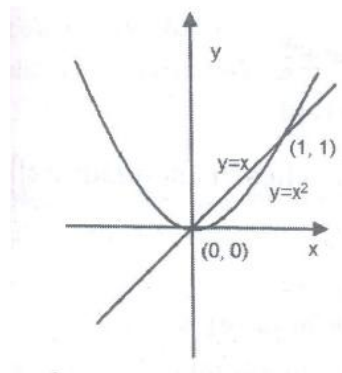
The point of intersection

Are $(0, 0)$ and $(1, 1)$

$$\text{Area} = \int_{x^2}^x dy dx = \int_0^1 [y]_{x^2}^x dx$$

$$= \int_0^1 (x - x^2) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units}$$



Sol 16.

$$I = \int \frac{dx}{x(x^n-1)} = \int \frac{x^{n-1} dx}{x^n(x^n-1)}$$

$$\text{put } x^n - 1 = t \Rightarrow nx^{n-1} dx = dt$$

$$I = \frac{1}{n} \int \frac{dt}{(t+1)t} = \frac{1}{n} \int \left[\frac{1}{t} - \frac{1}{t+1} \right] dt$$

$$= \frac{1}{n} [\log t - \log(t+1)] + c$$

$$= \frac{1}{n} \log \left[\frac{t}{t+1} \right] + c$$

$$= \frac{1}{n} \log \left[\frac{x^n-1}{x^n} \right] + c$$

Sol 17.

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots \dots \dots (i)$$

$$= \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x) + \cos(\frac{\pi}{2}-x)}$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \dots \dots \dots (ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2} 1 \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

Sol 18.

The differential equation is $\frac{d^2y}{dx^2} = e^{2x}$ (i) Integrating (i) both sides, we get

$$\frac{dy}{dx} = \frac{e^{2x}}{2} + c \text{ (ii)}$$

Integrating (ii) on both sides, we get

$$y = \frac{e^{2x}}{2} + cx + d$$

Sol 19.

Given equation is

$$\left(1 + \frac{dy}{dx}\right)^5 = \frac{d^4y}{dx^4}$$

$$\left(1 + \frac{dy}{dx}\right)^4 = \left(\frac{d^4y}{dx^4}\right)^5$$

Sol 20.

Given that $e = \frac{1}{3}$, $ae = 3 \Rightarrow a = 9$

$$\text{Now } b^2 = a^2(1 - e^2) = 81 \left(1 - \frac{1}{9}\right) = 72$$

\therefore Equation of ellipse is $\frac{x^2}{81} + \frac{y^2}{72} = 1$

Sol 21.

Given equations of the circle is

$$x^2 + y^2 + 4x - 4y + 2 = 0 \text{ Centre of circle is } (-2, 2)$$

Radius of circle = $\sqrt{6}$

Let equation of tangent be $x + y = c$

The perpendicular distance from $(-2, 2)$ to the circle is equal to the radius of circle.

$$\therefore \left| \frac{2+2-c}{\sqrt{1+1}} \right| = \sqrt{6} \Rightarrow \frac{c}{\sqrt{2}} = \sqrt{6} \Rightarrow c = \sqrt{12}$$

Hence equation of tangent is $x + y = \sqrt{12}$

Sol 22.

The given of the circle is

$$x^2 + y^2 - 4x - 4y + 4 = 0$$

whose centre is $C(2, 2)$ and radius = 2

$$S_1 = 5^2 + 6^2 - 20 - 24 + 4$$

$$= 25 + 36 - 20 - 24 + 4 = 21$$

\therefore P lies outside the circle

$$PC = \sqrt{(5-2)^2 + (6-2)^2} = \sqrt{9+16} = 5$$

The least distance between circle and the point P = $5 - 2 = 3$

Sol 23.

A parallelepiped is formed by planes drawn through the points (1, 2, 3) and (5, 7, 9) parallel to the coordinate planes.

Let a, b, c be the length of edges, then

$$A = 5 - 1 = 4, b = 7 - 2 = 5, c = 9 - 3 = 6$$

So length of the diagonal of a parallelepiped

$$= \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{16 + 25 + 36} = \sqrt{77}$$

Sol 24.

Given equation of the sphere is

$$x^2 + y^2 + z^2 + 4x - 2y + 6z + 5 = 0$$

and plane is $x + 2y + 3z - 4 = 0$

centre of the sphere is (-2, 1, -3) and

$$\text{radius} = \sqrt{4 + 1 + 9 - 5} = 3$$

$$\text{Length of perpendicular from centre of sphere to the plane} = \frac{|-2+2-9-4|}{\sqrt{1+4+9}} = \frac{13}{\sqrt{14}}$$

Sol 25.

Since vectors \vec{a} , \vec{c} and \vec{b} form a right handed system

$$\therefore \vec{c} = \vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -y & z \\ y & 0 & 0 \end{vmatrix}$$

$$= z\hat{j} + y\hat{k}$$

Sol 26.

By triangle law

$$\hat{c} = \hat{a} + \hat{b}$$

$$\vec{c} \times \vec{c} = (\vec{a} + \vec{b}) \times \vec{c}$$

$$0 = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

Sol 27.

A total of 8 is obtained in following cases

$$\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

\therefore probability of getting a total of is $\frac{5}{36}$

Sol 28.

Probability of getting a score of 6 in a single throw

$$= \frac{6}{36} = \frac{1}{6}$$

Probability of getting score of 6 in throw of four times $= {}^4C_2 \left(\frac{1}{6}\right)^2 \left(1 - \frac{1}{6}\right)^2 = 6 \times \frac{1}{36} \times \frac{25}{36} = \frac{25}{216}$

Sol 29.

$$\begin{aligned} \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) &= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}\right) \\ &= \tan^{-1}\left(\frac{\frac{1}{6}}{\frac{5}{6}}\right) \\ &= \tan^{-1}(1) = \frac{\pi}{4} \end{aligned}$$

Sol 30.

Given that α is root of

$$25 \sin^2 \theta + 5 \sin \theta - 12 = 0$$

$$\therefore 25 \sin^2 \alpha + 5 \sin \alpha - 12 = 0$$

$$\Rightarrow (5 \sin \alpha - 3) (5 \sin \alpha + 4) = 0$$

$$\Rightarrow \sin \alpha = \frac{3}{5}, -\frac{4}{5}$$

$$\Rightarrow \sin \alpha = \frac{3}{5} \text{ [because } \alpha \text{ lies in 1st quadrant]}$$

$$\text{Now } \cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

$$= \pm \sqrt{1 - \frac{9}{25}} = \pm \frac{4}{5}$$

$$\Rightarrow \cos \alpha = \frac{4}{5} \text{ [[because } \alpha \text{ lies in 1st quadrant]]}$$