ANSWERS XV

CHEMISTRY

1.c	2.a	3.d	4.c	5.d	6.b	7.c	8.c	9.b	10.d	11.c	12.d	13.a
14.a	15.c	16.d	14.a	15.c	16.d	17.a	18.c	19.a	20.c	21.b	22.a	23.a
24.b	25.d	26.b	27.d	28.b	29.b	30.c						

PHYSICS

1.b	2.c	3.b	4.b	5.a	6.b	7.b	8.d	9.a	10.d	11.a	12.b	13.b
14.b	15.a	16.b	17.d	18.b	19.a	20.c	21.c	22.b	23.d	24.a	25.b	26.d
27.b	28.d	29.d	30.a									

MATHEMATICS

1.b	2.b	3.d	4.c	5.d	6.a	7.b	8.b	9.a	10.b	11.b	12.a	13.b
14.a	15.c	16.d	17.c	18.b	19.b	20.c	21.c	22.b	23.c	24.b	25.b	26.b
27.b	28.a	29.b	30.a									

HINTS AND EXPLANATIONS XV

CHEMISTRY

Sol.1

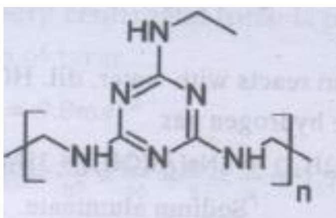
$P + 3H + 4O \rightarrow H_3PO_4$

H_3PO_4 + Magnesia powder → $MgNH_4PO_4\backslash$

$2MgNH_4PO_4 \rightarrow Mg_2P_2O_7 + 2NH_3 + H_2O$
White ppt

Sol.2

Melamine resin or melamine formaldehyde (also shortened to melamine) is a hard, thermosetting plastic material made from melamine and formaldehyde by polymerization



Sol.3

(4)

Sol.4

The standard heat of formation of a compound is the enthalpy change for formation of 1 mol of a substance at 298 K and 1 atm. Pressure from its elements.

Sol.5

Ψ^2 gives probability of finding an electron around the nucleus.

Sol.6

Weight of NaCl (w^2) = 5.8 g

Weight of water (w_1) = 105.85 – 5.85 = 100; Kf for water = 1.86 Km^{-1}

$\Delta T_f = i \text{ x } K_f \text{x } m = i \text{ x } k_f \frac{w^2}{M_2} x \frac{1000}{w_1} = 2 \text{ x } 1.86 \text{ x } \frac{5.85}{58.5} x \frac{1000}{100} = 3.72$

Sol.7

Paper chromatography is a technique that is used to separate and to identify components of a mixture. In this technique, the sample mixture is applied to a piece of filter paper, the edge of the paper is immersed in a solvent, and the solvent moves up the paper by capillary action. This is a partition chromatography in which the substances are distributed or partitioned between two liquid phases. One phase is the water which is held in pores of filter paper used and other phase is that of mobile phase which moves over the paper.

Sol.8

An antigen is a substance that evokes the production of one or more antibodies.

Sol.9

The term vat dye is used to describe a chemical class of dyes that are applied to cellulosic fibre (ie cotton) using a redox reaction. The dye is soluble only in its reduced (oxygen-free) form. The fibre is immersed repeatedly in this oxygen-free dyebath, and then exposed to the air, whereupon the water-soluble reduced form changes colour as oxygen turns it to the water-insoluble form.

Sol.10

When two solutions are mixed and there is a rise in temperature, this means that the molecules of two solutions have more attraction after mixing. This resultant solution which boils at a constant temperature is an azeotrope and due to greater intermolecular attractions it give rise to negative deviation from Raoult's law and has higher boiling point than that of individual solutions.

Sol.11

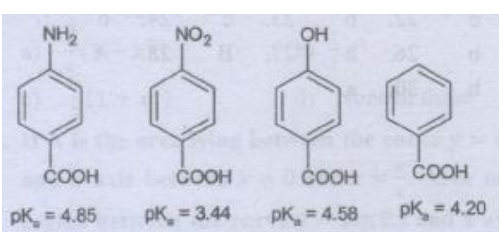
In bcc structures, radius r = $\frac{\sqrt{3}}{4}a$; where a is the edge length

a = $\frac{4\,r}{\sqrt{3}} = \frac{4\ x\ 75}{1.732} = 173.21$

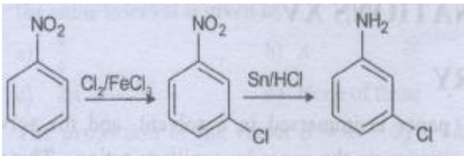
Sol.12

$HCl \rightarrow H^{+} + Cl^{-}; NaCl \rightarrow Na^{+} + Cl$

Sol.13



Sol.14



Sol.15

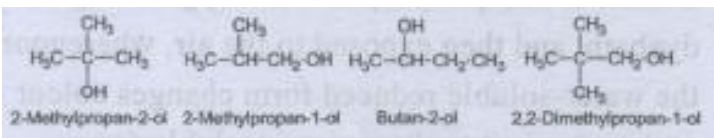
Gold number is a term used in colloidal chemistry. It may be defined as ‘least quantity of protective colloid in milligrams, which is just sufficient to prevent the coagulation of 10 ml of standard gold sol (containing 0.0053% gold) by the rapid addition of 1 ml of 10% NaCl solution. The Coagulation of gold sol is indicated by change in colour from red to violet. Thus smaller the gold number, the greater is protective power of lyophilic colloid.

Sol.16

Alkali metals are good reducing agents.

Sol.17

s Tertiary alcohols are more easily dehydrated.



Sol.18

56 g of N_2 = 2 mol: 44 g of CO_2 = 1 mol;

Total moles = 3: Total pressure

= 3 atm; Partial pressure of N_2 = 2 atm

Sol.19

pH of a buffer solution is given as:

$pH = pK_a + \log\frac{[salt]}{[acid]} = 5.0 + \log\frac{0.4}{0.4} = 5.0$ $\quad (pK_a = -\log K_a = -\log 10^{-5})$

Sol.20

Aqua regia is a mixture of 3 part of HCl and 1 parts of HNO_3 acid.

Sol.21

Carbon monoxide is a colorless, odorless, tasteless and toxic gas produced as a by-product of combustion. Carbon monoxide inhibits the blood's ability to carry oxygen to body tissues including vital organs such as the heart and brain. When CO is inhaled, it combines with the oxygen carrying hemoglobin of the blood to form carboxyhemeglobin (COHb). Once combined with the hemoglobin, that hemoglobin is no longer available for transporting oxygen.

Sol.22

$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

Sol.23

After each half life, period the amount of substance reduces to half, if we start form 100 g of a substance, after first $t_{1/2}$ it becomes 50, after second $t_{1/2}$ it becomes 25 and after third $t_{1/2}$ it becomes 12.5, i.e., 87.5 of initial amount is consumed. So three half lives (or 3 x 20 min) are required for 87.5% disintegration. $t_{1/2}$ = 20 min.

Sol.24

First I.E. of 5d transition elements is higher than first I.E. of both 3d & 4d elements because in 5d transition elements 4f orbitals are filled and these have very poor shielding effect due to which outer electrons experience greater effective nuclear charge resulting in higher ionization energies of elements.

Sol.25

Aluminium reacts with water, dil. HCl & NaOH to liberate hydrogen gas

$2Al + 2NaOH + 2H_2O \rightarrow 2Na[Al(OH)_4] + 3H_2(g)$

Sodium aluminate

$2Al + 6HCl + 12H_2O \rightarrow 2[Al(H_2O)_6]Cl_3 + 3H_2(g)$

$2Al + 6H_2O \rightarrow 2Al(OH)_3 + 3H_2(g)$

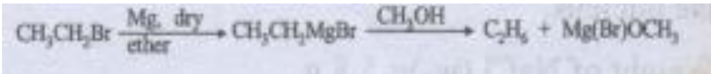
Sol.26

$2\ NO_2 \rightleftharpoons N_2O_4$

Sol.27

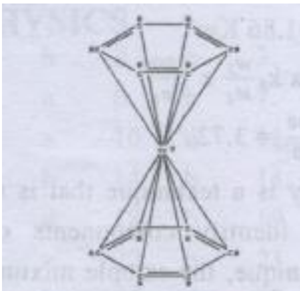
NBS is a specific reagent for allylic bromination.

Sol.28



Sol.29

$Cr(h6\text{-}C_6H_6)_2$



Sol.30

$CH_3CH_2OCH_3 + 2HI \longrightarrow CH_3CH_2I + CH_2I$

PHYSICS

Sol.1

$\frac{ML^2}{Q^2} = ML^2A^{-2}T^{-2}$

$= inductance = Henry\ in\ S.I.units$

Sol.2

For ascending, $h = -ut_1 + \frac{1}{2}g{t_1}^2$

For descending, $h = -ut_2 + \frac{1}{2}g{t_2}^2$

From these equations $h(t_1 - t_2) = \frac{1}{2}gt_1t_2(t_1 - t_2) \Rightarrow h = \frac{1}{2}gt_1t_2$

But $h = \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \Rightarrow t = \sqrt{t_1t_2}$

Sol.3

Centre of mass lies towards heavier mass.

Sol.4

As $W = \frac{1}{2}k(x_2^2 - {x_1}^2)$

$= \frac{1}{2} \times 800 \times (15^2 - 5^2) \times 10^{-4}$

$= 400 \times 200 \times 10^{-4} 8J$

Sol.5

The necessary centripetal force is provided by the friction of tyres.

Sol.6

As $g = \frac{GM}{R^2} = 9.8ms^{-2}$

$\therefore g' = \frac{GM/80}{\left(\frac{R}{4}\right)^2} = \frac{GM}{R^2} \times \frac{16}{80} = \frac{g}{5} = \frac{98}{5} = 1.96ms^{-2}$

Sol.7

Given stress $= \frac{F}{A} = 3.5 \times 10^8$

$\Rightarrow A = \frac{F}{3.5 \times 10^8} = 1.43 \times 10^{-6} \ \Rightarrow \pi r^2 = 1.43 \times 10^{-6} or \frac{\pi D^2}{4} = 1.43 \times 10^{-6}$

$\therefore D^2 = \frac{4 \times 1.43 \times 10^{-6}}{\pi} = 1.82 \times 10^{-6}$

$\Rightarrow D = 1.35 \times 10^{-3} m = 1.35m$

Sol.8

As $\frac{Q}{T} \propto \frac{A}{l} \propto \frac{r^2}{l}$ then ratio of heat flow in A and B is $\frac{\left(\frac{1}{2}\right)}{\left(\frac{2}{1}\right)} = \frac{1}{8}$

Sol.9

As $P \propto mc_2$

$\therefore \frac{P_1}{P_2} = \frac{m_1}{m_2}\left(\frac{c_1}{c_2}\right)^2 = \frac{m_1}{\frac{m_1}{2}} = \frac{1}{2}$

Sol.10

Given $2kx - f = ma$ and accelaration$a = R\alpha$

Solving $f = \frac{2}{3}kx \ \therefore Net\ force = ma = -\frac{4}{3}kx$

Sol.11

The frequency of sound produced by organ pipe will increase with increase in temperature.

Sol.12

The force will be halved as the electric field is halved when one plate is removed.

Sol.13

We know $V = IR \Rightarrow V = 5R \; and \; V' = 4(R+2)$ Given $V' = V \Rightarrow 5R = 4(R+2) \Rightarrow R = 8\Omega$

Sol.14

As $R = \frac{V}{I_g} - G \Rightarrow G = -R + \frac{V}{I_g} = -3.96 \times 10^3 + \frac{20}{5\times10^{-3}}$

$\therefore G = 40\Omega$

Sol.15

Gauss's law is invalid for magnetic monopoles

Sol.16

The self -inductance varies as N^2

Sol.17

Inverse current is induced in the nearby coil by the rising current or switching on whereas switching off or diminishing current also induces similar current in the nearby coil.

Sol.18

Both statements are simply correct.

Sol.19

Converging lens will be used.

Sol.20

The pattern will disappear slowly.

Sol.21

Diffraction of light imposes a limit on the performance of a resolving instrument.

Sol.22

$\bar{v} = \frac{1}{5896\times10^{-8}} = 16961 \; per \; cm$

Sol.23

Given $n = \frac{3240}{1620} = 2$

$\Rightarrow N = (2.68 \times 10^{18}) \left(\frac{1}{2}\right)^2 = 0.67 \times 10^{18}$

$\therefore$ Disintegrated nuclei =$(2.68 - 0.67) \times 10^{18} = 2.01 \times 10^{18}$

Sol.24

Kinetic energy $= \frac{13.6Z^2}{n^2} eV$

Potential $= -\frac{27.2 \times Z^2}{n^2} eV$

Obviously Kinetic energy > Potential energy

Sol.25

Given $V_{cc} = 8V, R_c = 800\Omega, R_{In} = 200\Omega\beta = \frac{I_c}{I_b} = \frac{25}{26}$

As $I_c R_L = 0.8V \Rightarrow I_c = \frac{0.8}{R_L} = \frac{0.8}{800} = 10^{-3}A = 1mA$ Power gain $= \beta^2 \frac{R_L}{R_{in}} = \left(\frac{25}{26}\right)^2 \times \frac{800}{200} = 3.69$

Sol.26

Although charge is induced in the conducting sphere, but net charge on it will be zero.

Sol.27

For second excited state, $n = 3$

$\therefore I_H = I_{Li} = 3\left(\frac{h}{2\pi}\right)$

As $E \propto Z^2$ and $Z_H = 1, Z_{Li} = 3 \Rightarrow |E_{Li}| = a|E_H| or |E_H| < |E_{Li}|$

Sol.28

During adiabatic expansion

$T_1 V_1^{\gamma-1} = T_2 V_1^{\gamma-1}$

For a mono atomic gas, $\gamma = \frac{5}{3} \Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_2}\right)^{\gamma-1} = \left(\frac{AL_2}{AL_1}\right)^{\left(\frac{5}{3}-1\right)} = \left(\frac{L_2}{L_1}\right)^{2/3}$

Here A is area of cross section of piston.

Sol.29

From lens maker formula $\frac{1}{f}=(\frac{\eta_L}{\eta_M}-1)(\frac{1}{R_1}-\frac{1}{R_2})$

η_L and η_M are refractive indices of lens and medium respectively. In case of double convex lens, R_1 is negative and R_2 is positive implying that $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is negative. For the lens to be diverging in nature, focal length f should be negative or $\left(\frac{\eta_L}{\eta_M}-1\right)$ must be positive thereby implying that $\eta_L>\eta_M$. Given is $n_2>n_1$ thus lens should be filled with L_2 and immersed in L_1.

Sol.30

Mass of disc $=4M$

M.O.I. of the disc about the given axis $=\frac{1}{2}(4M)R^2$

M.O.I. of quarter section of disc $=\frac{1}{2}(2MR^2)=\frac{1}{2}MR^2$.

MATHEMATICS

Sol.1

Since $x^2+1=0\Rightarrow x=\pm i$

Sol.2

The relation is not reflexive and transitive but it is symmetric, because$x^2+y^2\Rightarrow y^2+x^2=1$

Sol.3

We have $f(x^2)=|x^2-1|$

$f|x|=\left||x|-1\right|\neq|x-1|=|f(x)|$ and $f(x+y)=|x+y-1|\neq|x-1|+|y-1|\neq f(x)+f(y)$

Hence none of the given option is true.

Sol.4

Let $Z=\frac{1+i\sqrt{3}}{\sqrt{3}+1}=\frac{1}{\sqrt{3}+1}=i\frac{\sqrt{3}}{\sqrt{3}+1}$

Let $\tan\alpha=\left|\frac{Im(z)}{Re(z)}\right|=\sqrt{3}$

Then $\alpha=\frac{\pi}{3}$ since z lies in the first quadrant, therefore are $(z)=\alpha=\frac{\pi}{3}$

Sol.5

Given that sin α, $sin^2\alpha, 1 are\ in\ A.P.$

$\Rightarrow 2sin^2\alpha = \sin\alpha + 1$

$\Rightarrow 2sin^2\alpha - \sin\alpha\, 1 = 0$

$\Rightarrow (2\sin\alpha + 1)(\sin\alpha - 1) = 0 \Rightarrow \sin\alpha = -\frac{1}{2} or \sin\alpha = 1$

$\Rightarrow \alpha = -\frac{\pi}{6} or\ \alpha = \frac{\pi}{2} \quad [\because -\pi < \alpha < \pi]$

For $\alpha = -\frac{\pi}{6}$, the given sequence becomes $-\frac{1}{2}, \frac{1}{4}, 1$

Clearly it is not an A.P.

For $\alpha = \frac{\pi}{2}$, the given sequence becomes 1, 1, 1, which is an A.P. But $\alpha = \frac{\pi}{2}$ does not lies in any intervals.

Sol.6

Let $y = \frac{x^2-2x+4}{x^2+2x+4}$

$\Rightarrow x^2(y-1) + 2x(y+1) + 4(y-1) = 0$ Since x is real, therefore

$4(y+1)^2 - 16(y-1)^2 \geq 0 \Rightarrow (y+1)^2 - (2(y-1))^2 \geq 0 = (3-y)(3y-1) \geq 0 \Rightarrow \frac{1}{3} \leq y \leq 3$

Sol.7

Since each digit of a 10 digit number can be written as either 1 or 2, therefore, required number of 10 digits number is 2^{10}

Sol.8

Let $\left(\sqrt{2} - i\right)^6 = I + F$ when I is an integer and $0 \leq F \leq 1$

Let $G = \left(\sqrt{2} - 1\right)^6$ Then

$I + F + G = \left(\sqrt{2} - 1\right)^6 + \left(\sqrt{2} - 1\right)^6$

$= 2[{}^6C_0(\sqrt{2})^6 + \ldots] = an\ integer$ (i)

$\therefore F + G = 1$

Substituting $F + G = 1$ in (i), we get $I + 1 = 2[{}^6c_0(\sqrt{2})^6 + {}^6c_2(\sqrt{2})^4 + {}^6c_4(\sqrt{2})^2 + {}^6c_6(\sqrt{2})^0]$

$I + 1 = 2[8 + 60 + 30 + 1] \Rightarrow I = 197$

Sol.9

Given series

$$=\left(\frac{1}{2},\frac{1}{4}+\frac{1}{4},\frac{1}{4^2}+\frac{1}{6},\frac{1}{4^3}+\cdots\right)+\left(1+\frac{1}{3},\frac{1}{4}+\frac{1}{5},\frac{1}{4^2}+\cdots\right)$$

$$=\frac{1}{2}\left(\frac{1}{4}+\frac{1}{2},\frac{1}{4^2}+\frac{1}{3},\frac{1}{4^3}+\cdots\right)+2\left\{\frac{1}{2}+\frac{1}{3},\left(\frac{1}{2}\right)^3+\frac{1}{5}\left(\frac{1}{2}\right)^5+\cdots\right\}$$

$$=\frac{1}{2}\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)+2\left(y+\frac{y^3}{3}+\frac{y^5}{5}+\cdots\right)$$

Where $x=\frac{1}{4}$ and $y=\frac{1}{2}$

$$=-\frac{1}{2}\log(1-x)+\log\left(\frac{1+y}{1-y}\right)$$

$$=-\frac{1}{2}\log\left(1-\frac{1}{4}\right)+\log\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)$$

$$=-\frac{1}{2}\log\frac{3}{4}+\log 3$$

$$=\log\left(\frac{3}{4}\right)^{-\frac{1}{2}}+\log 3$$

$$=\log\left(3\sqrt{\frac{4}{3}}\right)=\log\ \sqrt{12}$$

Sol.10

Since A is orthogonal matrix. Therefore

$$AA^T=I=A^TA$$

$$\Rightarrow |AA^T|=|I|=|A^TA|$$

$$\Rightarrow |A||A^T|=1=|A^T||A|$$

$$\Rightarrow |A|^2=1\Rightarrow |A|=\pm 1$$

Sol.11

We have

$$\frac{d}{dx}(\Delta_1)=\begin{vmatrix}1&0&0\\a&x&b\\a&a&x\end{vmatrix}+\begin{vmatrix}x&b&b\\0&1&0\\a&a&x\end{vmatrix}+\begin{vmatrix}x&b&b\\a&x&b\\0&0&1\end{vmatrix}=\begin{vmatrix}x&b\\a&x\end{vmatrix}+\begin{vmatrix}x&b\\a&x\end{vmatrix}+\begin{vmatrix}x&b\\a&x\end{vmatrix}=3\Delta_2$$

Sol.12

Let the image of the point P(−8, 12) in the line mirror AB be Q(a,b) then PQ is perpendicularly bisected at R

The coordinates of R are $\left(\frac{a-8}{2}, \frac{b+12}{2}\right)$

Since R lies on 4x + 7y + 13 = 0

$\therefore 2a - 16 + \frac{7b+84}{2} + 13 = 0$

$\Rightarrow 4a + 7b + 78 = 0$ (i)

Since PQ is perpendicular to AB, therefore (Slope of AB) × (Slope of PQ) = −1

$\Rightarrow \left(-\frac{4}{7}\right) \times \left(\frac{b-12}{a+8}\right) = -1$

$\Rightarrow 7a - 4b + 104 = 0$ (ii)

Solving (i) and (ii), we get a = −16, b = −2

Hence image is (−16, −2)

Sol.13

Let P (h, k) be the coordinates of a point from which equal tangents are drawn to the circles $x^2 + y^2 - 5x - 3 = 0$ and $3x^2 + 3y^2 + 2x + 4y - 6 = 0$ Then $h^2 + k^2 - 3 = h^2 + k^2 + \frac{2}{3}h + \frac{4}{3}k - 2$

$\Rightarrow 17h + 4k + 3 = 0$

Hence locus of (h,k) is 17x 4k + 3 = 0

Sol.14

Let (h,k) be the midpoint of the chord 2x + y – 4 = 0 of the parabola $y^2 = 4x$

Then its equation is $ky - 2(x + h) = k^2 - 4h$ [Using T = S']

$\Rightarrow 2x - ky + k^2 - 2h = 0$ (i)

Equations (i) and 2x + y – 4 = 0 represents the same line

$\therefore -k = 1$ and $k^2 - 2h = -4$

$\Rightarrow k = -1, h = -\frac{5}{2}$

Sol.15

If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line y = mx + c intersect in real points, then the quadratic equation $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ has real roots

Therefore Discriminant ≥ 0

$\Rightarrow c^2 \le a^2m^2 + b^2$

Sol.16

Period of $\sin\frac{\pi x}{2} = \frac{2\pi}{\pi/2} = 4$ Period of $\cos\frac{\pi x}{3} = \frac{2\pi}{\pi/3} = 6$ Period of $\tan\frac{\pi x}{4} = \frac{2\pi}{\pi/3} = 4$

Period of f(x) = L.C.M. of (4,6,4) = 12

Sol.17

We have f(x) = $\begin{cases} [\cos \pi x], & x < 1 \\ |x-2|, & 1 \le x < 2 \end{cases}$

$$= \begin{cases} 2-x, & 1 \le x \le 2 \\ -1 & \frac{1}{2} < x < 1 \\ 0, & 0 < x \le \frac{1}{2} \\ 1, & x = 0 \\ 0, & \frac{1}{2} \le x < 0 \\ -1, & -\frac{3}{2} < x < -\frac{1}{2} \end{cases}$$

It is evident from the definition that f(x) is discontinuous ar x = $\frac{1}{2}$

Sol.18

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ (i)

and z = $\tan^{-1}(\frac{2x\sqrt{1-x^2}}{1-2x^2})$ (ii)

Put x = tan θ in (i), we get $y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\tan^{-} x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$ Put x = sin θ in (ii), we get

z = $\tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right) = \tan^{-1}(\tan 2\theta) = 2\theta = 2\sin^{-1} x$

$\Rightarrow \frac{dz}{dx} = \frac{2}{\sqrt{1-x^2}}$ Thus $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{\sqrt{1-x^2}}{4(1+x^2)} \Rightarrow \left(\frac{dy}{dz}\right)_{x=0} = \frac{1}{4}$

Sol.19

We have $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dz}{dt}} = \frac{4t-2}{2t+3}$$

For x = 2 and y = - 1, we have

$t^2 + 3t - 8 = 2$ and $2t^2 - 2t - 5 = 1$

solving the two equations, we get t = 2

$$\therefore \left(\frac{dy}{dx}\right)_{(2,-1)} = \frac{4\times2-2}{2\times2+3} = \frac{6}{7}$$

Sol.20

$f(x) = \cot^{-1} x + x$

$$\Rightarrow f'(x) = -\frac{1}{1+x^2} + 1 = \frac{x^2}{1+x^2}$$

Clearly $f'(x) > 0$ for all x so, f(x) increases in $(-\infty, \infty)$

Sol.21

Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-1} = e^{-x\log x}$

Then $f'(x) = -\left(\frac{1}{2}\right)^x (\log x + 1) = -x^{-x}(\log x + 1)$

$\therefore f'(x) = 0 \Rightarrow -x^{-x}(\log x + 1) = 0$

$\Rightarrow \log x + 1 = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1}$

Clearly f''(x) < 0 at $x = e^{-1}$

Hence $f(x) = x^{-x}$ is maximum for $x = e^{-1}$

The maximum value is $e^{\frac{1}{e}}$

Sol.22

$\int \cos^3 x\, e^{\log \sin x} dx = \int \cos^3 \sin x\, dx$

$= -\int t^3 dt$ where t = cos x

$$= -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

Sol.23

$$f(x) = \begin{cases} \int_{-1}^{x} -t\,dt & -1 \le x \le 0 \\ \int_{-1}^{0} -t\,dt + \int_{0}^{x} t\,dt & c \ge 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2}\,(1 - x^2) & -1 \le x \le 0 \\ \frac{1}{2}\,(1 + x^2) & x \ge 0 \end{cases}$$

Sol.24

We have A = $\int_{0}^{\frac{\pi}{2}} \sin 2x\,dx$

$= -\frac{1}{2}[\cos 2x]_{0}^{\frac{\pi}{2}}$

$= -\frac{1}{2}[\cos\pi - 1] = 1 = A$

Sol.25

The direction cosines of a vector making equal angles with the coordinate axes is $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$. therefore the unit vector along the vector making equal angles with the coordinate axes is

$\vec{b} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

The projection of $\vec{a}$ on $\vec{b}$ = $\vec{a}.\vec{b}$

$= \left(4\hat{i} - 3\hat{j} + 2\hat{k}\right).\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = \frac{4-3+2}{\sqrt{3}} = \sqrt{3}$

Sol.26

The equation of plane passing through (2, 2, 1) is $a(x - 2)+b(y - 2) + c(z - 1) = 0$

This passes through (9, 3, 6) and in perpendicular to plane 2x + 6y + 6z −1 = 0

$\therefore\ 7a + 1.b + 5c = 0$ and $2a + 6b + 6c = 0$

$\Rightarrow \frac{a}{-24} = \frac{b}{-32} = \frac{c}{40} \Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$

So the required plane is $3(x - 2) + 4(y - 2) - 5(z - 1) = 0$ or $3x + 4y - 5z = 9$

Sol.27

There are 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The last two digits can be dialed in $^{10}P_2$ = 90 ways out of which only one way in favourable, these the required probability = $\frac{1}{90}$

Sol.28

$y = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$

$= \sqrt{2}\cos(x + \frac{\pi}{6} - \frac{\pi}{4})$

$= \sqrt{2}\cos\left(x - \frac{\pi}{12}\right)$

$\Rightarrow$ y is maximum for $x - \frac{\pi}{12} = 0$ i.e. $x = \frac{\pi}{12}$

Sol.29

We have $\sin^{-1} - \cos^{-1} x = \cos^{-1}\frac{\sqrt{2}}{2}$

$\Rightarrow \sin^{-1} x + \cos^{-1} x = \frac{\pi}{6}$ Put $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. therefore

$\sin^{-1} x = \frac{\pi}{3}$ and $\cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{2}}{2}$ is the unique solution.

Sol.30

$1 + \sin\theta + sin^2\theta + \cdots = 4 + 2\sqrt{3}$

$\Rightarrow \frac{1}{1-\sin\,\theta} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin\theta = \frac{1}{2(2+\sqrt{3})}$

$\Rightarrow 1 - \sin\theta = \frac{1}{2}(2 - \sqrt{3})$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$