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Subject: **CHEMISTRY, MATHEMATICS & PHYSICS**

Paper Code: **JEE_ Main_ Sample Paper - III**

Part – A – Chemistry

1) b

Exp: In an fcc system, number of atoms in the unit cell is,

At the Corners $8X\frac{1}{8}=1$

At the face centers $6X\frac{1}{2}=3$

Total = 4 = n

For each atom there is one octahedral (n) void and two tetrahedral voids. 2n

2) b

Exp: $\overset{+6}{Mo}O_3 \rightarrow \overset{0}{Mo} \quad x = 6$

$H_2 \longrightarrow 2H^+ \quad x = 2$

∴ 1 mole MoO_3 will react with 3 moles of H_2

125/144 moles MoO_3 will react with 3x125/144 x 22.4lit. of H_2 at NTP = 58.33lit. at NTP.

3) c

Exp: (b) and (d) are ruled out on the basis that at the initial point of 273K, 1 atm, for 1.0 mole volume must be 22.4L, and it should increase with rise in temperature. By solving Pv = nRT & Solving for v, we get our answer.

4) c

Exp: Rate of effusion ∝ p_i; p_i = Particle pressure of i^{th} component

$\propto \sqrt{(1/M)}$

5) a

Exp: Temperature and volume are related for adiabatic process as: $T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

∴ $T_2 = T_1V_1^{\gamma-1} = T_1(V_1/V_2)^{\gamma-1} = 300(1/8)^{\gamma-1}$

= 300 x ½ = 150K

$C_P/C_v = \gamma = 4/3$ ∴$C_v = 3R$ & $C_P = 4R$

Adiabatic work done

$\Delta w = - C_V \Delta T = - 3 \times 2 \times (150-300) = 900cal$

6) c

Exp: PV = RT at temp T for one mol

P(V + ΔV) = R(T+1) at temp.(T + 1) for one mol

∴ P ΔV = R

7) a

Exp: The energy of an electron in a Bohr atom is expressed as

$E_n = - kZ^2/n^2$ where, k = constant,

Z = atomic number,

n = orbit number

= - 13.6eV for H (n = 1)

When n = 2,

$E_2 = -13.6/2^2 eV$

= - 3.40eV

(n can have only integral value 1, 2, 3, ……….. ∞)

8) d

Exp: Mg^{2+}: $1s^2 2s^2 2p^6$: no unpaired electron.

Ti^{3+}: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^1$: one unpaired electron

V^{3+}: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^2$: two unpaired electron

Fe^{2+}: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6$: four unpaired electrons

9) d

Exp: The minimum $[OH^-] = 1.34 \times 10^{-5}$ M will be no precipitation of $Mg(OH)_2$ can be obtained by

$K_{sp} = [Mg^{2+}][OH^-]^2$

$\Rightarrow 9.0 \times 10^{-12} = (0.05) \times [OH^-]^2$

$\therefore [OH^-] = 1.34 \times 10^{-5}M$

Thus, solution having $[OH^-] = 1.34 \times 10^{-5}$ M will not show precipitation of $Mg(OH)_2$ in 0.05M Mg^{2+}. These hydroxyl ions are to be derived by basic buffer of NH_4Cl and NH_4OH.

$pOH = pK_b + \log [salt]/[base]$

$pOH = pK_b + \log [NH_4^+]/[NH_4OH]$

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

In pressure of $[NH_4Cl]$, all the NH_4^+ are provided by NH_4Cl as due to common ion effect, dissociation of NH_4OH is suppressed.

$-\log [OH^-] = -\log 1.8 \times 10^{-5} \log [NH^+_4]/[0.05]$

$\therefore NH_4^+ = 0.67M$ or $[NH_4Cl] = 0.67M$.

10) a

Exp: One can calculate ionic product from given data and for precipitation ionic product >K_{sp}.

11) b

Exp: The rate increase with temperature but not directly.

12) d

Exp:

$$\text{Rate}(S_N 2) = 5.0x10^{-5}\ x10^{-2}[R-X] = 5.0x10^{-7}[R-X]$$

$$\text{Rate}(S_N 1) = 0.20x10^{-5}[R-X]$$

$$\%\ \text{of}\ S_N 2 = \frac{5x10^{-7}[R-X]x100}{5x10^{-7}[R-X]+0.20x10^{-5}[R-X]} = 20$$

13) a

Exp: Addition of HgI_2 to KI solution establishes the following equilibrium:

$$HgI_2 + 2KI \rightleftharpoons K_2[HgI_4]$$

The above equilibrium decreases the number of ions (4 ions on left side of reactions becomes three ions on right side), hence rises the freezing point.

14) b

Exp: The balanced nuclear reaction is

$_0n^1 + {}_{92}U^{235} \rightarrow {}_{54}Xe^{139} + {}_{38}Sr^{94} + {}_0n^3$

15) b

Exp: b)

$HA \rightleftharpoons H^+ A^-$

$C(l-\alpha) \qquad C\alpha \qquad C\alpha$

$C\alpha = H^+]=10^{-2}, \alpha = 10^{-1} \qquad [A^-]=10^{-2}$

$[HA]=0.1(1 - 10^{-1}) = 0.09$

Total moles = $C(1 + \alpha)$

$$\therefore \pi = C(1+\alpha)RT = 0.1(1.1)RT = 0.11RT$$

16) b $\Rightarrow$

Exp: $H^+ + e^- \quad (1/2)H_2(g) \frac{1}{(H^+)_1}$

$E_1 = 0 - .0591 \log$

$$E_1 = 0 + .0591\log[H^+]_1 = -.0591pH_1$$

$$E_2 = -.0591pH_2$$

$pH_1 = pk_a + \log\frac{salt}{acid}$; $pH_1 = pk_a + \log\frac{a}{b}$ ....(1)

$pH_2 = pk_a + \log\frac{b}{a}$; $pH_2 = pk_a - \log\frac{a}{b}$ ....(2)

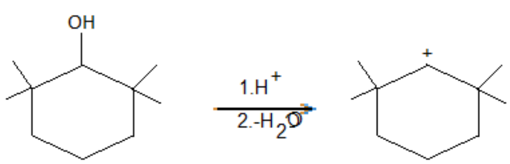
Add (1) & (2) $pH_1 + pH_2 = 2pk_a$

$2pk_a = -\frac{E_1}{.0591} - \frac{E_2}{.0591}; pk_a = -\left[\frac{E_1 + E_2}{0.118}\right]$

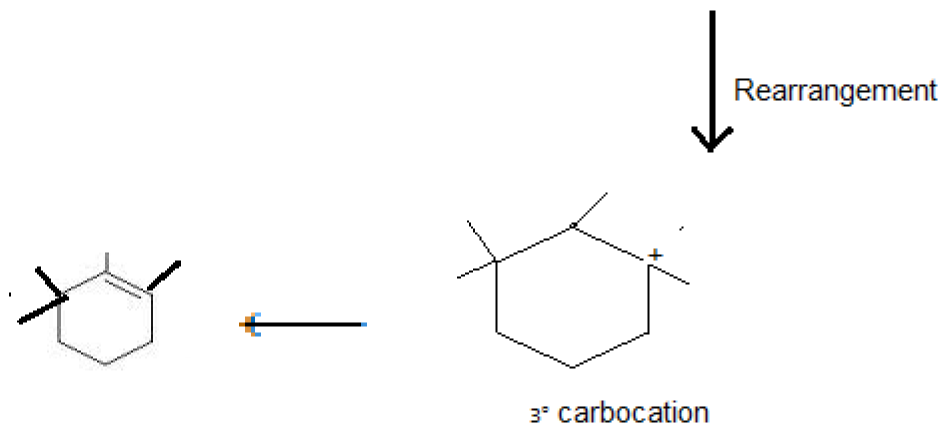
17) b

Exp: $NaBH_4$ reduces only carbonyl group, protecting the double bond and as well as acid.

18) a

Exp: 

2,2,6,6-tetramethyl cyclohexanol gives 2° carbocation, here it undergoes 1, 2 - methyl shift to yield 3° carlocation.



19) a

Exp:

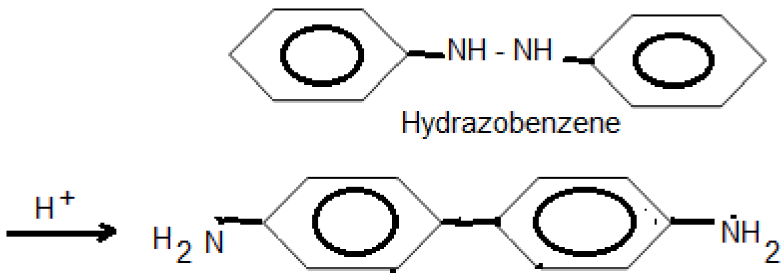
$$C_2H_5OH + Cl_2 \longrightarrow CH_3CHO + 2HCl$$
$$CH_3CHO + 3Cl_2 \longrightarrow CCl_3CHO + 3HCl$$
$$\underset{A}{CCl_3CHO} + 2CuO \longrightarrow \underset{\text{Red ppt}}{Cu_2O} + CCl_3COOH$$

Hence, compound 'A' is chloral.

20) c

Exp: By oxidation of tertiary alcohols with stronger oxidizing agents, ketones may be formed along with carboxylic acids.

21) d



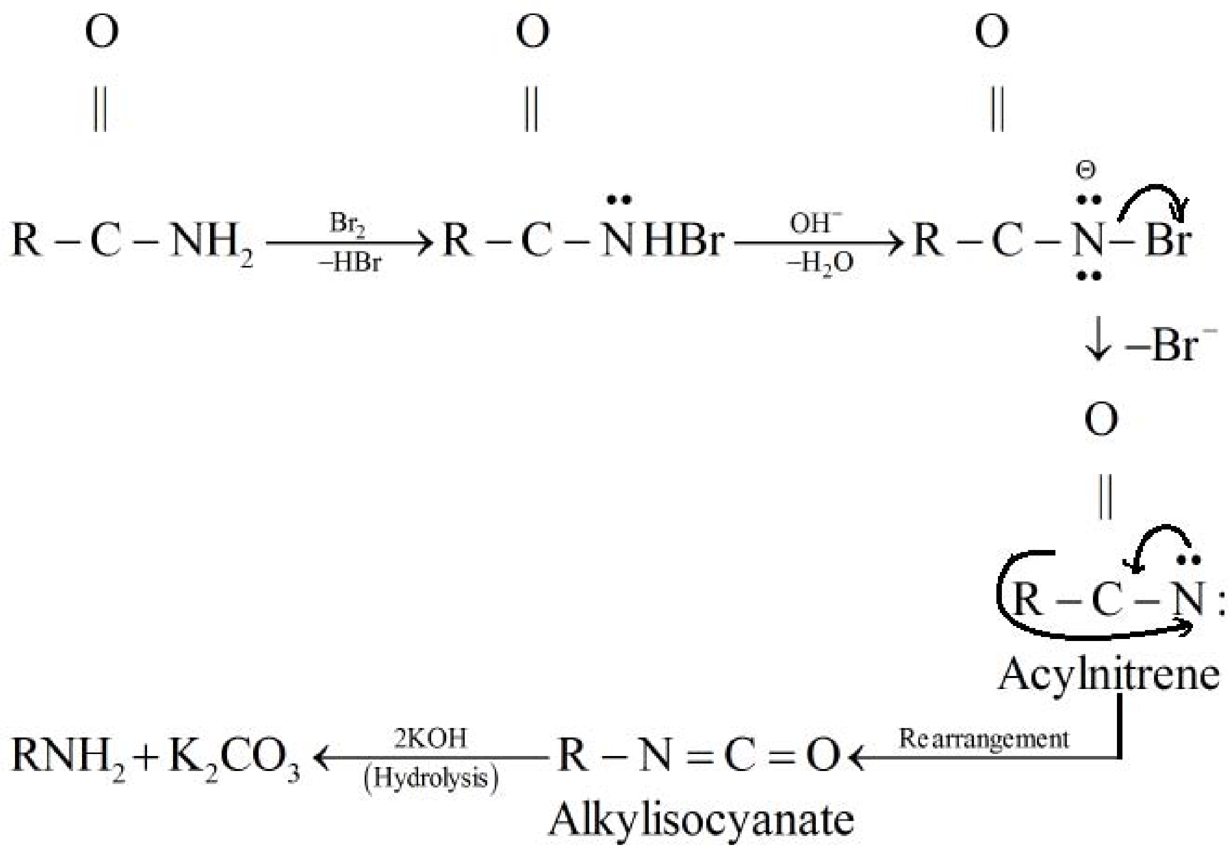
(Benzidine)

Exp:

Hydrazobenzene undergo rearrangement in strongly acidic solution (i.e., H^+) to form 4,4' – diaminobiphyenly commonly called benzidine. This reaction is also known as benzidine rearrangement

22) a

Exp:



23) c

Exp: Azurite is a basic carbonate ore of copper.

$2CuCO_3.Cu(OH)_2$

24) b

Exp: The colour exhibited by transition metal ions is due to the pressure of unpaired electrons in d-orbitals which permits d – d excitation of electrons.

In TiF_6^{2-} - Ti is in + 4 O.S.; = $3d^0$ = colourless

In CoF_6^{3-} - Co is in +1 O.S.; = $3d^5$ = coloured

In Cu_2Cl_2 – Cu is in + 1 O.S.; $3d^{10}$ – colourless

In $NiCl_4^{2-}$ - Ni is in +2 O.S.; $3d^8$ – coloured

25) a

Exp: $Cr_2O_7^{2-} + 2OH^- \rightarrow 2CrO_4^{2-} + H_2O$

Hence CrO_4^{2-} ion is obtained.

26) a

Exp: The green colour appears due to the formation of Cr^{+++}ion

$Cr_2O_7^{2-} + 3SO_3^{2-} + 8H^+ \rightarrow 3SO_4^{2-} + 2Cr^{3+} + 4H_2O$

27) d

Exp: Amides reacts with solutions of chlorine of bromine in excess sodium hydroxide (NaOH) to give primary amines containing one carbon atom less than the original amide. This is Hoffman's bromamide reaction.

28) b

Exp: Rate of a reaction is always determined by the slowest step. Hence, rate determining step is:

$P + Q \rightarrow R + S(slow).\quad Rate = K[P][Q]$

29) b

Exp: Enthalpy of neutralization of NaOH and CH_3COOH is less than 57.1 kJ mol^{-1} because CH_3COOH uses some heat energy to get partially dissociated.

30) a

Exp: Methane cannot be prepared by Wurtz reaction because in this reaction we use equimolecular amounts of two halides, alkyl groups of which join together.

RX + 2Na + XR $\rightarrow$ R – R + 2 NaX.

Part – B – Physics

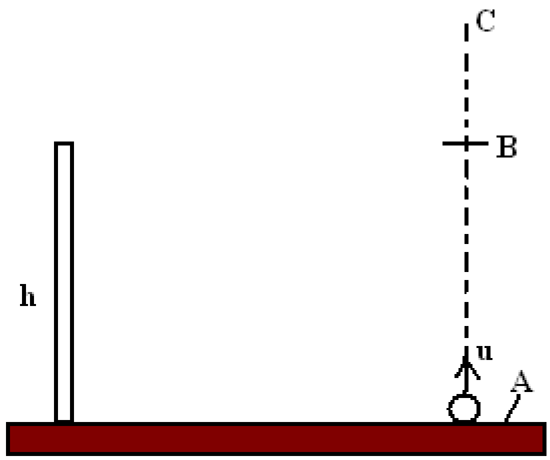
31) d

Exp: Density ρ =m/πr^{2}L

∴ Δρ/ρ x 100 = (Δm/m + 2 Δr/r + ΔL/L) x 100

After substituting the values, we get the maximum percentage error in density = 4%

32) b



$t_{AC} = 6/2 = 3s$

$t_{AC} = 12/2 = 6s$

$\therefore \quad t_{AB} = 3s$

$\therefore \quad 0 = u - (10)6$

Exp: or $\quad u = 60m/s$

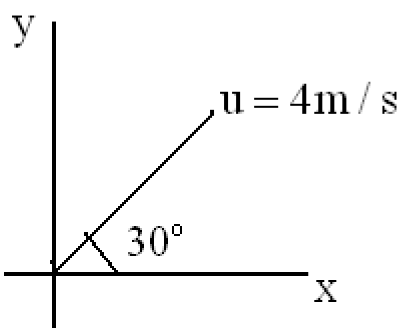
Further $\quad h = ut_{AB} - \frac{1}{2}gt_{AB}^2$

$= (60)(3) - \frac{1}{2}(10)(3)^2$

$= 135m$

33) b

Exp: Components of velocity of ball relative to lift are:



$U_X = 4\cos 30^o = 2\sqrt{3}$ m/s

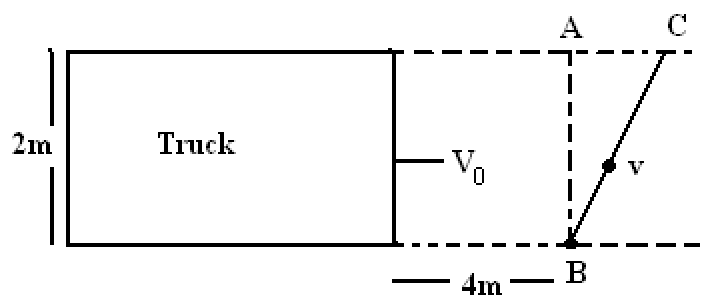
And $u_Y = 4 \sin 30^o = 2m/s$

And acceleration of ball relative to lift is $12 m/s^2$ in negative y-direction or vertically downwards. Hence time of flight

$T = 2u_y/12 = u_y/6 = 2/6 = 1/3s$

34) c

Exp: Let the man starts crossing the road at an angle θ with horizontal as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describe the distance 4+AC or 4+2cotθ.

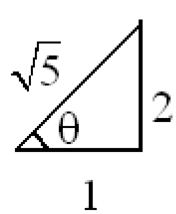


∴ $(4+2\cot\theta)/8 = 2/v\sin\theta$

Or $v=8/(2\sin\theta+\cos\theta)$ ....(1)

For minimum v, $dv/d\theta = 0$

Or $-8(2\cos\theta-\sin\theta)/(2\sin\theta+\cos\theta)^2= 0$



Or $2\cos\theta - \sin\theta = 0$

Or $\tan\theta = 2$

From Eq. (1)

$V_{min} = 8/(2(2/\sqrt{5}) + 1/\sqrt{5}) = 8/\sqrt{5} = 3.57 m/s$

35) d

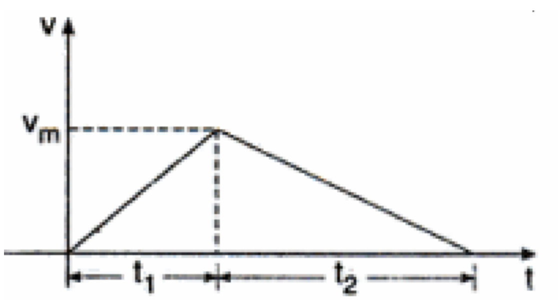
Exp: Weight of lift = Mg

Maximum tension = n Mg

∴ Maximum acceleration = (nMg-Mg)/M = (n-1)g

And maximum retardation = g

Corresponding velocity –time graph for shortest time will be as shown.



Here $(n-1) g = v_m/t_1$ ....(1)

Or $t_1=v_m/(n-1)g$

And $g = v_m/t_2$ ....(2)

Or $t_2 = v_m/g$

Area under v-t graph is total displacement h.

Henceh = ½ $(t_1 + t_2)v_m$

From Eqs.(1), (2) and (3) we get,

$V_m = \sqrt{2gh\left(\frac{n-1}{n}\right)}$

36) c

Exp: N sin θ provides the force responsible for motion of the wedge,

N sin θ=Ma

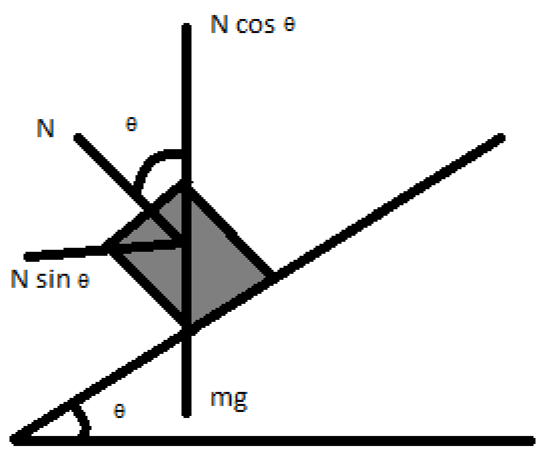
Let A be the acceleration of the block w.r.t the wedge. When wedge moves with acc. a then the acc. of the block is (Acos θ- a) in the opposite direction.

N is the normal reaction b/w the two, for both of them,

N sin θ= Ma = m (Acos θ- a)

mAcos 60= Ma+ma

A=(M+m)2a/m



37) d

Exp: Extension in the spring is

$x = AB - R = 2R \cos 30° - R$

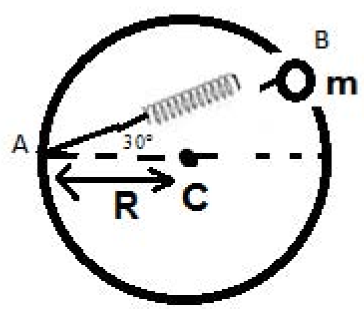
$= (\sqrt{3} - 1) R$

∴Spring force

$F = kx = (\sqrt{3} + 1)mg/R\ (\sqrt{3} - 1) R$

$= 2mg$

Free body diagram of bead in shown in figure



$N = (F + mg) \cos 30^{o} - mg \sin 30^{o} = (2mg + mg)\sqrt{3}/2$

$= 3\sqrt{3}/2$

38) d

Exp: At the highest point, from conservation of energy,

$½ mv^2 = ½ mu^2 + 2mgl$, also $mg = mu^2/l$ (at highest point)

$v^2 = u^2 + 4gl = 5gl$

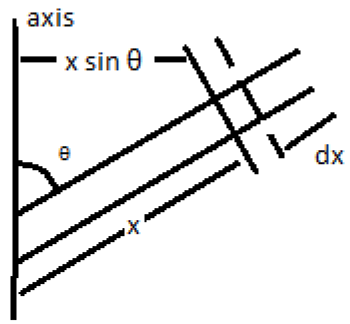
Tension at lowest point, $T = mg + mv^2/l = 6mg$

39) c

Exp: Mass of the element = (m/l) dx.

Moment of inertia of the element about the axis = (m/l dx) $(x \sin\theta)^2$.

$I = m/l \sin^2\theta. \int^1_0 x^2 dx = ml^2/3 \sin^2\theta$.



40) b

Exp: $y = 4\cos^2(1/2\ t) \sin(1000t) = 2(1+\cos t)\sin(1000t)$

$= 2\sin(1000t) + 2\sin(1000t)\cos t$

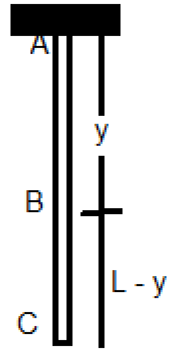
$= 2\sin(1000t) + \sin(1001t) + \sin(999t)$.

41) b

Exp: $V_P = -GM/\surd(r^2+a^2)$, $V_C = -GM/a$, $½\ mv^2 = m[V_P - V_C]$

42) b

Exp: Mass of section BC = m/L (L-y).



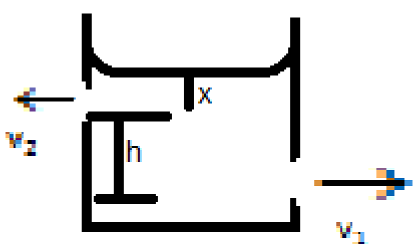
∴tension at B = T m/L (L-y)g.

∴elongation of element dy at B

= dx = (dy) T/AY = m/L (L-y) g dy/AY

Total elongation = ∫dx = mg/LAY $\int_0^L$(L-y)dy = MgL/2YA

43) b



Exp: $v_1 = \sqrt{2g(h+x)}, v_2 = \sqrt{2gx}$

Let p = density of the liquid

Let α = area of cross-section of each hole.

Volume of liquid discharged per second at a hole = αv. $(v = v_1 or\ v_2)$

Mass of liquid discharged per second = αvp.

Momentum of liquid discharged per second = $\alpha v^2 p$. the force exerted at the upper hole (to the right) = $\alpha p v_2^2$ and the force exerted at the lower hole (to the left) = $\alpha p v_1^2$.

Net force on the tank = $\alpha p\ [v_1^2 - v_2^2] = \alpha p\ [2g(h+x) - 2gx] = 2\alpha pgh.$

44) a

Exp: γ(mixture)=(n1Cp1 +n2Cp2)/(n1Cv1+n2Cv2)=1.5

Cp1=5R/2, Cv1 =3R/2

Cp2=7R/2, Cv2=5R/2, therefore , n1=n2

45) b

Exp: Problems like this are best solved by using their electrical analogues.

For a rod of lengths l, area of cross-section A and thermal conductivity k, we define the thermal resistance as

R = l/kA

The given situation is like two resistances in series. We define the heat current $I = (\theta_1-\theta_2)/R$

As the resistances are in series, they carry the same current. Let θ be the temperature of their junction.

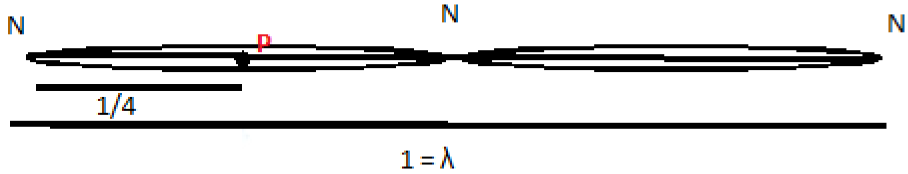
$I = \theta_1-\theta/R_1 = \theta-\theta_2/R_2$, where $R_1 = l/K_1A$ and $R_2 = l/k_2A$

Or $\theta_1-\theta/\theta-\theta_2 = R_1/R_2 = k_2/k_1$.

Solve for θ.

46) d

Exp: In the first case



Point P is an anti node i.e., the string is vibrating in its second harmonic. Let f_0 be the fundamental frequency.

Then

$2f_0=100Hz$

$\therefore$ $f_0 = 50Hz$

Now P is an anti node (at length l/4 from one end) so centre should be a node. So, next higher frequency will be sixth harmonic or $6f_0$ which is equal to 300Hz as shown below:

47) c

Exp: Since current I is independent of R_6, it follows that the resistances R_1, R_2, R_3 and R_4 must form a balanced Wheatstone bridge.

48) d

Exp: 2Ω resistance is short circuited i.e., potential difference across it is zero. Or current passing through 2Ω resistance is zero. Further, potential difference across 4 Ω and 5 Ω resistance is 20 V. Therefore, current passing through them will be 5 A and 4A respectively.

49) d

Exp: Voltage across bulb B_2 will be less than across B_3. $\Rightarrow W_2 < W_3$.

50) b

Exp: V = E – ir = E- $\frac{Er}{R+r}$

$$= E\left[\frac{R+r-r}{R+r}\right]$$

$$V = \frac{ER}{(R+r)}$$

$$\Rightarrow V = 0 at R = 0$$

$$R = \infty, V = E$$

51) Ans: b

Exp: Q = $Q_0 e^{-t/\tau}$ = Q_0/η

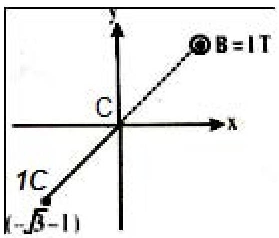
Or $e^{-t/\tau} = \frac{1}{\eta}$ or $t/\tau = \ln \eta$

52) Ans: c

Exp: $\frac{\vec{p}_m}{\vec{L}} = \frac{q}{2m}$

53) Ans: c

Exp: The centre will be at C as shown :



Coordinates of the centre are (r cos 60°, r sin 60°)

Where r = radius of circle

$= \frac{mv}{Bq} = \frac{1X1}{1X1} = 1 i.e., \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

54) Ans: d

Exp: $F = q[v(-\hat{i})]XB(\hat{i}) = 0$

Because B as well as v is are along axis of circular CN

55) Ans: d

Exp: The output D for the given combination

$$D = \overline{(A+B).C} = \overline{(A+B)} + \overline{C}$$

If A = B = C = 0 then

$$D = \overline{(0+0)} + \overline{0} = \overline{0} + \overline{0} = 1+1 = 1$$

If A = B = 1, C = 0 then

$$D = \overline{(1+1)} + \overline{0} = \overline{0} + \overline{0} = 1+1 = 1$$

56) Ans: b

Exp: We have,

$$6\pi\eta rv = \frac{4}{3}\pi r^3 pg - \frac{4}{3}\pi r^3 g\ \sigma$$

Where $p \to p_{water}$ *and* $\sigma \to p_{air}$

$$\Rightarrow \qquad \eta = \frac{2gr^2(p-\sigma)}{9v}$$

$$= \frac{2\times 9.81\times(0.2\times 10^{-2})^2\times 999}{9\times 8.7}$$

$$= 1\times 10^{-3}\ poise$$

57)

Ans: c

Exp: Work done

$= = \int_0^{x_0} p_0 A dx + \frac{k{x_0}^2}{2}$

$= p_0 A x_0 + \frac{k x_0^2}{2} = 80 + 40 = 120$ J.

where p = pressure of gas,

p_0 = atmospheric pressure

k = spring constant

x_0 = compression in spring

58)

Ans: b

Exp: $v \propto \frac{1}{n}$, so the curve is rectangular hyperbola.

59) b

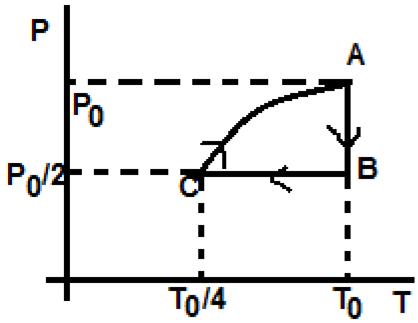
Exp: Clearly the coordinates of A are (2f, 2f)

$\therefore \quad f = \frac{40}{2}$

= 20cm

60) c

Exp: Process AB is isothermal expansion, BC is isobaric compression and in process CA , $P \propto V$



Part – C – Math

61) b)

Exp: $\tan^{-1}((x-1)/(x+1)) + \tan^{-1}((2x-1)/(2x+1))$

$= \tan^{-1} ((x-1)/(x+1) + (2x-1)/(2x+1)/1-(x-1))/((x+1)(2x-1)/(x+1)(2x+1))$

$= \tan^{-1} (2x^2-2x+x-1+2x^2+2x-x-1)/(2x^2+2x+x+1-2x^2+2x+x-1)$

$= \tan^{-1} (4x^2-2)/6x = \tan^{-1} (2x^2-1)/3x$

But,

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} (x+y)/(1-xy)$, only where, $xy< 1$

i.e., $((x-1)(2x-1))/((x+1)(2x+1)) < 1$

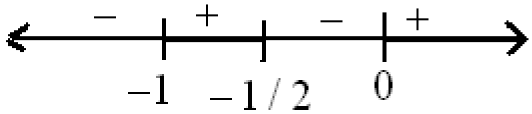
$((x-1)(2x-1))/((x+1)(2x+1)) – 1 <0$

$((x-1)(2x-1)-(x+1)(2x+1))/((x+1)(2x+1)) < 0$

$(2x^2-2x-x+1-2x^2-2x-x-1)/((x+1)(2x+1)) < 0$

$-6x/((x+1)(2x+1) < 0$

$x((x+1)(2x+1)) > 0$



Plotting on the number line,

$\therefore x\in(-1, -1/2)\cup (0, \infty)$

$\tan^{-1} 2x^2-1/3x = 23/36$

$2x^2-1/3x = \tan^{-1} 23/36$

$12(2x^2-1) = 23x$

$24x^2-12 = 23x$

$24x^2 – 23x – 12 = 0$

$24x^2-32x+9x – 12 = 0$

$8x(3x-4)+3(3x-4) = 0$

$(8x+3)(3x-4) = 0$

$x = 4/3, -3/8.$

But $x = - 3/8$ does not lie in the specified interval.

$\therefore x = 4/3.$

62) b

Exp: The determinant is

p(qr-bc) + b (ca – ar) + c(ab – aq) = 0

∴pqr + 2abc – pbc – qca – rab = 0

Add the following to both sides

2pqr – 2abc + 2 pbc + 2 qca + 2qca + 2rab – 2aqr – 2bqr – 2cpq

Then

LHS = 3pqr + pbc + qca + rab – 2aqr – 2bqr – 2cpq

= p(q-b) (r-c) + q(p-q)(r-c)+ r(p-a)(q-b)

RHS = 2(p-a)(q-b)(r-c)

∴ p(q-b)(r-c)+q(p-a)+r(p-a)(q-b) = 2(p-q)(q-b)(r-c)

Divide by (p – a) (q – b) (r-c)

p/p-a + q/q-b + r/r-c = 2.

∴ the value of the required expression is 2.

63) b

Exp:

$\alpha = e^{2\pi i/7} \Rightarrow a^7 = e^{2\pi i} = 1, \alpha \neq 1$

or $\alpha^7 - 1 = 0$

or $(\alpha - 1)(\alpha^6 + \alpha^5 + \alpha^4 + ... + 1) = 0$

$\Rightarrow 1 + \alpha + \alpha^2 + ...\alpha^6 = 0$

$= \dfrac{1-(\alpha^k)^7}{1-\alpha^k} = \dfrac{1-\alpha^7 k}{1-\alpha^k} = \dfrac{1-1}{1-\alpha^k} = 0$

where $k \neq 7m$

$f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$ (given)

Replace x by $x, \alpha x, \alpha^2 x, ..., \alpha^6 x$ in the above and add the seven results thus obtained.

L.H.S. $= (A_0 + A_0 + ...) + \sum_{k=1}^{20} A_K x^k x (1 + \alpha^k + \alpha^{2k} + ...\alpha^{6k})$

$= 7A_0 + 0$ as shown above

$\therefore$ Given expression $= 7A_0$ which is independent of α

64) B

Exp: $(1+x+2x^3)\,(3x^2/2 - 1/3x)^9 = (1+x+2x^3)$

$\sum_{r=0}^{9} {}^9C_r \left(\dfrac{3x^2}{2}\right)^{9-r} \left(\dfrac{-1}{3x}\right)^r$

$$= \sum_{r=0}^{9} (-1)^r \, {}^9C_r \frac{3^{9-2r}}{2^{9-2r}} x^{18-3r} + \sum_{r=0}^{9} (-1)^r \, {}^9C_2 \frac{3^{9-2r}}{2^{9-r}} x^{19-3r}$$

$$+2\sum_{r=0}^{9} (-1)^r \, {}^9C_r \frac{3^{9-2r}}{2^{9-r}} x^{21-3r}$$

Making the power of the variable x zero one by one, we get 18 – 3r = 0 (or) r = 6 in the first series and the corresponding term

$= {}^9C_6 (3^{-3}/2^3) = {}^9C_6 (1/6^3) = 7/18$

$19\text{-}3r = 0 \Rightarrow r = 19/3$

Not an integer in the second series and hence there is no term independent of x in the second series.

$21\text{-}3r=0 \Rightarrow r = 7$ in the third series and the corresponding term

$= 2(-1)^{7\,9}C_7 (3/2)^{9-7} (1/3)^7 = -29.8/1.2 = 1/2^2 \; 1/3^5 = - 2/27$

Hence the term independent of x = 7/18 – 2/27 = 21-4/54 = 17/54

65) D

Exp: If (-10, 2) is an interior point, the circle is more likely to touch x – y = 0 at P(-4, -4) in the 3rd quadrant since OP = 4√2.

The line x + y = 0 meets the circle at A and B such that AB = 6√2; CM = OP = 4√2 and AM = ½ AB = 3√2.

CA = $\sqrt{(CM^2+AM^2)} = 5\sqrt{2}$ = radius of the circle = CP.

∴ C lines on PC whose equation is x+y+8=0 and on CM whose equation is x – y + 10 = 0.

This is for the reason that it M be (-x, x), then $\sqrt{(x^2 + x^2)} = 5\sqrt{2}$ giving x = 5 i.e., M is (-5, 5).

x + y + 8= 0 and x – y + 10 = 0 when solved simultaneously fixes the centre at (-9, 1).

∴the equation to the circle is $(x+9)^2 + (y-1)^2 = (5\sqrt{2})^2$

i.e., $\boxed{x^2 + y^2 + 18x - 2y + 32 = 0}$ ....(i)

Substituting x = - 10, y = 2 on the L.H.S of the above equations, we get

100 +4 – 180 – 4 + 32 = - 48 < 0.

∴ (-10, 2) lies inside the circle. Hence the circle (i) is the required circle since it satisfies all the conditions stated in the problem.

66) A

Exp: Any tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ is $y = mx \pm\sqrt{(a^2m^2+b^2)}$

Let P be(h, k)

$k = mh\pm\sqrt{(a^2m^2+b^2)}$

$(k - mh)^2 = a^2m^2 + b^2$

$m^2(h^2-a^2) – 2hkm + (k^2-b^2)= 0$ …….(i)

$m_1 + m_2 = 2hk/(h^2-a^2)$ and $m_1m_2 = k^2-b^2/h^2-a^2$ ………(ii)

If θ_1 and θ_2 are the angles of inclination of tangents to the x-axis, then

$\tan\theta_1 + \tan\theta_2 = 2hk/h^2-a^2$ and $\tan\theta_1 \tan\theta_2 = k^2-b^2/h^2-a^2$ …..(iii)

given that $\tan^2\theta_1 + \tan^2\theta_2 = \lambda$

$(\tan\theta_1 + \tan\theta_2)^2 – 2 \tan\theta_1 \tan\theta_2 = \lambda$

$(2hk/h^2-a^2) – 2(k^2-b^2)/h^2-a^2) = \lambda$ (from (ii))

$4h^2k^2 – 2(k^2 – b^2) (h^2 –a^2) = \lambda (h^2 – a^2)^2$

$2(h^2k^2 + k^2a^2 + h^2b^2 – a^2b^2) = \lambda (h^2 – a^2)^2$

required locus is $2(x^2y^2+a^2y^2+x^2b^2-a^2b^2) = \lambda(x^2-a^2)^2$

67) a

Exp: Let the required line by method $P+\lambda Q=0$ be

$\therefore$ Perpendicular from (0, 0) = $\sqrt{5}$ gives

$$\frac{1}{\sqrt{(1+2\lambda)^2+(5-3\lambda)^2}}=\sqrt{5}$$

Squaring and simplifying

$$(8\lambda-7)^2=0 \Rightarrow \lambda=7/8$$

Hence the required line is

$(x-3y+1)+7/8(2x+5y-9)=0$
$or\, 22x+11y-55=0 \Rightarrow 2x+y-5=0$

68) c

Exp: $f(x) = \left[\frac{2\sin x\,[1+\cos x](1-\cos x)}{2\cos x\,[1+\sin x]\,(1-\sin x)}\right]^{\frac{2}{3}} = \left(\frac{\sin^3 x}{\cos^3 x}\right)^{\frac{2}{3}} = \frac{\sin^2 x}{\cos^2 x}$

$\Rightarrow \cos^2 x \neq 0$

Hence $X \neq \frac{\pi}{2}$ and $x \neq \frac{3\pi}{2}$

69) b

Exp: Let($(\alpha, 3-\alpha)$ be any point on x+y=3

$\therefore$ Equation of chord of contact is $\alpha x+(3-\alpha)y=9$
$i.e., \alpha(x-y)+3y-9=0$

$\therefore$ The chord passes through the point (3,3) for all values of α .

70) a

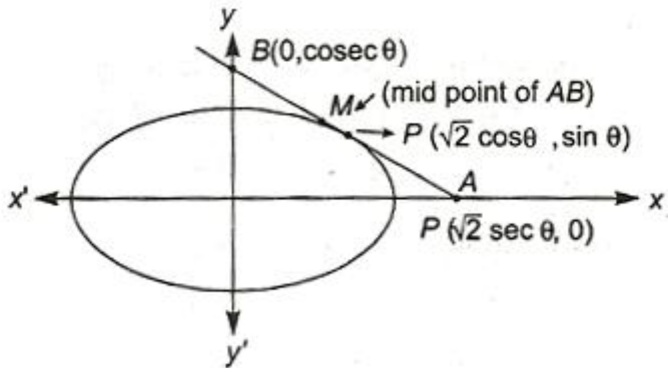
Exp: Let any point be P ($\sqrt{2}\cos\theta$, $\sin\theta$) on

$\frac{x^2}{2}+\frac{y^2}{1}=1.,$

Equation of tangent is,

$$\frac{x\sqrt{2}}{2}\cos\theta+\frac{y}{1}\sin\theta=1$$

Whose intercept on coordinate axes are A ($\sqrt{2}\sec\theta$, 0) and B (0, cosec θ)



∴ Mid-point of its intercept between axes is

$$\left(\frac{\sqrt{2}}{2}\sec\theta, \frac{1}{2}cosec\theta\right) = (\mathrm{h}, \mathrm{k}) \qquad \text{(say)}$$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}h}$

And $\sin\theta = \frac{1}{2k}$

Now, $\cos^2\theta + \sin^2\theta = \frac{1}{2h^2} + \frac{1}{4k^2}$

$\Rightarrow \quad \frac{1}{2h^2} + \frac{1}{4k^2} = 1$

Thus, locus of mid-point M is

$$\frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

71) b

Exp: $g(x) = \int_0^x f(t)\,dt \Rightarrow g(2) = \int_0^2 f(t)\,dt$

$\Rightarrow \quad g(2)=\int_0^1 f(t)\,dt+\int_1^2 f(t)\,dt$ .... (i)

Now, $\frac{1}{2}\le f(t)\le 1 \; for\; t\in[0,1], we\; get$

$\int_0^1 \frac{1}{2}dt \le \int_0^1 f(t)\,dt \le \int_0^1 1dt$

(apply line integral inequality)

$\frac{1}{2}\le \int_0^1 f(t)\,dt \le 1$ .... (ii)

Again, $0\le f(t)\le \frac{1}{2} \; for \; t\in[1,2]$

$\int_1^2 0 \text{ dt} \le \int_1^2 \text{f}(\text{t})\text{dt} \le \int_1^2 \frac{1}{2}dt$

(apply line integral inequality)

$\Rightarrow \quad 0\le \int_1^2 f(t)\; dt \le \frac{1}{2}$ .... (iii)

From Eqs. (ii) and (iii), we get

$\frac{1}{2}\le \int_0^1 f(t)\,dt+\int_1^2 f(t)\; dt \le \frac{3}{2}$

$\Rightarrow \quad \frac{1}{2}\le g(2)\le \frac{3}{2}$ [From Eq. (i)]

$\Rightarrow \quad 0\le g(2)\le \frac{3}{2}$

72) b

Exp: Given,

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $C_3 \to C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ sin^2\theta & cos^2\theta & 1+sin^2\theta+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow \quad 1 + \sin^2\theta + 4\sin 4\theta + \cos^2\theta = 0$

$$\Rightarrow \quad 2+4\sin 4\theta = 0 \Rightarrow \sin 4\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \quad 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$

$$\Rightarrow \quad \theta = \frac{n\pi}{4} - (-1)^n\frac{\pi}{24}$$

For $0 \le \theta \le \frac{\pi}{2}$, we have $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$.

73) a

Exp: Here, sin C $= \frac{1-\cos A\cos B}{\sin A\sin B} \le 1$ .... (i)

$$\Rightarrow \quad \frac{1-\cos A\cos B}{\sin A\sin B} - 1 \le 0$$

$\Rightarrow$ 1 - cos A cos B - sin A sin B $\leq$ 0

$\Rightarrow$ 1 - cos (A - B) $\leq$ 0

$\Rightarrow$ cos (A - B) $\geq$ 1

But cos (A - B) cannot be greater than 1

$\Rightarrow$ cos (A - B) = 1

$\Rightarrow$ A – B = 0

$\Rightarrow$ A = B

$\Rightarrow$ $\sin C = \frac{1-cos^2 A}{sin^2 A}[fromEq.(i)]$

$= \frac{sin^2 A}{sin^2 A} = 1$

$\Rightarrow$ C = 90^0

$\Rightarrow$ A = B = 45^0

Using sine rule,

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow$ $\frac{1}{\sqrt{2}a} = \frac{1}{\sqrt{2}b}. = \frac{1}{c}$

$\Rightarrow$ a: b: c = 1:1: $\sqrt{2}$.

74) C

Exp: $\Rightarrow$ S_1S_2 = 2ae

$\therefore$ ordinate of P $\equiv \frac{a}{e}$ (1 - e^2)

$$\Delta PS_1S_2 = \frac{1}{2} \times 2ae \times \frac{a}{e}\left(1-e^2\right)$$

$$\Rightarrow \quad a^2\left(1-e^2\right) \le \frac{a^2}{2}.$$

75) b

Exp: The lines given by $ax^2 + 5xy + 2y^2 = 0$ are mutually perpendicular if a + 2 = 0

$\therefore$ a = -2.

76) c

Exp: Let (h, k) be the coordinates of the midpoint of a chord which subtends a right angle at the origin. Then equation of the chord is

$$kx + ky - 4 = h^2 + k^2 - 4 (u\sin g\ T = S')$$
$$or \quad hx + ky = h^2 + k^2$$

The combined equation of the pair of lines joining the origin to the points of intersection of $x^2 + y^2 = 4$ *and* $hx + ky = h^2 + k^2$ *is*

$$x^2 + y^2 - 4\left(\frac{hx+ky}{h^2+k^2}\right)^2 = 0$$

Lines given by the above equations of the pair of liens joining the origin to therefore coeff.of $x^2 + coeff.of\ y^2 = 0$

$$\Rightarrow 2(h^2+k^2) - (4h^2 + 4k^2) = 0 \Rightarrow h^2 + k^2 = 2$$
$$\therefore Locus\ of\ (h,k) is\ x^2 + y^2 = 2$$

77) a

Exp: since the line passing through the focus and perpendicular to the directrix is x-axis, therefore axis of the required parabola is x-axis. Let the coordinates of the focus be(a, 0). Since the vertex is the midpoint of the line joining the focus and the point (-5, 0) where the directrix x + 5 = 0 meets the axis. Therefore

$$-3 = \frac{a-5}{2} \qquad \Rightarrow a = -1$$

Thus, the coordinates of the focus are (-1, 0).

Let P (x, y) be a point on the parabola. Then by definition

$$\sqrt{(x+1)^2 + y^2} = \left(\frac{x+5}{\sqrt{1}}\right)^2 \qquad \Rightarrow y^2 = 8(x+3)$$

78) b

$$Let\ D=\begin{bmatrix} d_1 & 0 & 0 & .... & 0 \\ 0 & d_2 & 0 & .... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & .... & d_n \end{bmatrix}$$

$$Then |D| = d_1, d_2, ...d_n$$

$$and\ cofactor\ of\ D_{11} = d_2 d_3 ....d_n$$

$$cofactor\ of\ D_{22} = d_1 d_3 ....d_n\ etc$$

$$\text{co}\ factor\ of\ D_{ij} = 0\ when\ i \neq j : \therefore D^{-1} = \frac{1}{|D|} adjD$$

$$= \frac{1}{d_1 d_2 ....d_n}\begin{bmatrix} d_2 d_3 ....d_n & 0 & 0 & .... & 0 \\ 0 & d_2 d_3 ....d_n & 0 & .... & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & .... & d_1 d_2 ...d_{n-1} \end{bmatrix}$$

Exp:

$$= \begin{bmatrix} \frac{1}{d_1} & 0 & 0 & ..... & 0 \\ 0 & \frac{1}{d_2} & 0 & ..... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & ... & \frac{1}{d_n} \end{bmatrix} = diag(d_1^{-1} d_2^{-1} ......d_n^{-1})$$

79) c

For the non-trival solution. we must have

$$\begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0 \text{ or } \begin{vmatrix} 1-a & 0 & a \\ b-1 & 1-b & b \\ 0 & c-1 & 1 \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2;\ C_2 \to C_2 - C_3$

$or (1-a)[(1-b)-b(c-1)] + a(b-1)(c-1), we\ get$

$$\frac{1}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 0$$

Adding 1 *on both sides, we get*

$$\left(\frac{1}{c-1} + 1\right) + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

$$or\ \frac{c}{c-1} + \frac{b}{b-1} + \frac{a}{a-1} = 1$$

Exp: $$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1.$$

80) b

Exp: Clearly AB = $\sqrt{2}$. The given set of lines represent square of side = $\sqrt{2}$. So its area = $(\sqrt{2})^2$ = 2.

81) d

Exp: $$\int_0^1 \tan^{-1} x\, dx \quad = x \tan^{-1} x \frac{-1}{2} \int \frac{2x}{1+x^2}\, dx$$

$$= \left[x \tan^{-1} x - \frac{1}{2} \log\ (1+x)^2\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}\ \ln 2$$

82) b

Exp: S = $Lt_{n\to\infty}\Sigma_{r=0}^{2n-1}\frac{1}{n}\sec^2\left(\frac{r}{n}\right)$

= $\int_0^2 \sec^2 x dx = [tan\ x]_0^2$

= tan2

83) b

Exp: If the given the plane contains the given line then the normal to the plane, must be perpendicular to the line and the condition for the same is $a\ell + bm + cn = 0$

84) a

Exp: n (S) = 36

Let E be the event of getting the sum of digit on the dice equal to 7, then n(E) = 6.

P(E) = $\frac{6}{36} = \frac{1}{6} = p, then P(E') = q = \frac{5}{6}$

Probability of not throwing the sum 7 in first m trails = q^m

$\therefore P($at least one 7 in m throws) = 1 - q^m = 1-$\left(\frac{5}{6}\right)^m$

$$1-\left(\frac{5}{6}\right)^m > 0.95$$

According to the question $\Rightarrow \left(\frac{5}{6}\right)^m < 0.05$

$$\Rightarrow m\{\log_{10} 5 - \log_{10} 6\} < \log_{10} 1 - \log_{10} 20$$

$$\therefore m > 16.44$$

Hence, the least number of trails = 17.

85) b

Exp: The equation of given curve is

$$x + y = e^{xy} \qquad \text{.............. (i)}$$

On differentiating w.r.t. x, we get

$$1+\frac{dy}{dx} = e^{xy}\left(y + x\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{ye^{xy}-1}{1-xe^{xy}}$$

Since, tangent is parallel to the y-axis, then

$$\frac{dy}{dx} = \frac{1}{0}$$

$$\Rightarrow \quad 1 - xe^{xy} = 0$$

$$\Rightarrow \quad 1 - x(x+y) = 0 \qquad \text{[from Eq. (i)]}$$

This holds for x = 1, y = 0.

86) c

Exp: Since, sum of the coefficients in the expansion of $(1 + 2x)^n$ is 6561.

$\therefore \quad 3^n = 6561$

$\Rightarrow \quad 3^n = 3^8$

$\Rightarrow \quad n = 8$

$$\therefore \quad \frac{T_{r+1}}{T_r} = \frac{8+1-r}{r} \times 2x$$

$$= \frac{9-r}{r} \times 2 \qquad (\because x = 1)$$

Now, $\frac{T_{r+1}}{T_r} > 1$

$\Rightarrow \quad 18 - 2r > r \Rightarrow r < 6$

Thus, 6th and 7th terms are greatest and equal in magnitude.

Now, $T_6 = {}^8C_5 (1)^{8-5} (2)^5$

$= \quad 1792$

87) b

$$P(E_2/E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

$$\frac{1}{2} = \frac{P(E_1 \cap E_2)}{1/4}$$

Exp: $\Rightarrow P(E_1 \cap E_2) = \frac{1}{8} = P(E_2).P(E_1/E_2)$

$$= P(E_2).\frac{1}{4} = \frac{1}{8} \Rightarrow P(E_2) = \frac{1}{2}$$

$$Since P(E_1 \cap E_2) = \frac{1}{8} = P(E_1).P(E_2)$$

$\Rightarrow$ Events are independent

Also $P(E_1 \cup E_2) = \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = \frac{5}{8}$

$\Rightarrow$ E_1& E_2 are non exhaustive

88) b

Exp: Let an equation of the required plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

The meets the coordinates axes in

$A(a,0,0), B(0,b,0) and C(0,0,c).$

So that the coordinates of the centroid of the triangle ABC are $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ =

$\left(1, r, r^2\right)\left(given\right) \Rightarrow a = 3, b = 3r, c = 3r^2$

Hence the required equation of the plane is

$\frac{x}{3}+\frac{y}{3r}+\frac{z}{3r^2}=1$ or $r^2x+ry+z=3r^2$

89) c

$$\int \frac{e^x(1+nx^{n-1}-x^{2n})}{(1-x^n)\sqrt{1-x^{2n}}}dx$$

Exp: $$=\int e^x\left(\sqrt{\frac{1+x^n}{1-x^n}}+\frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}\right)dx$$

$$=\frac{e^x\sqrt{1-x^{2n}}}{1-x^n}+C$$

90) d

Exp: Number of ways = Arrangement of (m -1) things of one kind and (n-1) things of the other kind = $\frac{(m+n-2)!}{(m-1)!(n-1)}$