Solved Examples of Indefinite Integral

1. $\int \frac{dx}{\sqrt{(x-a)(b-x)}} =$

(A) $2\sin^{-1}\sqrt{(x-a/b-a)} + c$ (B) $\sin^{-1}\sqrt{(x-a/b-a)} + c$

(C) $2\sin^{-1}\sqrt{(x+a/b-a)} + c$ (D) none of these

Solution: Put $x = a\cos^2 q + b\sin^2 q$ the given integral becomes.

$$I = \int \frac{2(b-a)\sin\theta\cos\theta\, d\theta}{\{(a\cos^2\theta + b\sin^2\theta - a)(a\cos^2\theta + b\sin^2\theta - b\}^{\frac{1}{2}}}$$

$$= \frac{2(b-a)\sin\theta\cos\theta d\theta}{(b-a)\sin\theta\cos\theta} = \left(\frac{b-a}{b-a}\right)\int 2d\theta = 2\theta + c = 2\sin^{-1}\sqrt{\left(\frac{x-a}{b-a}\right)} + c$$

Hence (A) is the correct answer.

2. If $\int xe^x \cos x dx = f(x)+c$, then f (x) is equal to

(A) $e^x/2\ ((1-x)\sin x - x\cos x)$ (B) $e^x/2\ ((1-x)\sin x + x\cos x)$
(C) $e^x/2\ ((1+x)\sin x - x\cos x)$ (D) none of these

Solution: $I = $ real part of $\int e^{x(1+i)x} dx$

$= \int xe^{(1+i)x} / 1+i - \int e^{(1+i)x} / 1 + i\ dx = xe^{(1+i)x} / 1+ i - e^{(1+i)x} / (1+i)^2$

$= e^{(1+i)x}[x(1+i-1 / (1+i)^2)]$

$= e^x (\cos x + i \sin x) [(x-1) + ix / 1 + i - 1]$

$= e^x/-2\ [i\cos x - \sin x][(x - 1) + ix]$

$I = e^x/2\ [(1 - x)\sin x - x\cos x] + c.$

Trans Web Educational Services Pvt. Ltd
B – 147,1st Floor, Sec-6, NOIDA, UP-201301
Website:www.askiitians.com Email. info@askiitians.com
Tel:0120-4616500 Ext - 204

Hence (A) is the correct answer.

3. I = ∫ dx / $e^x + 4e^{-x}$ = f (x) + c then f (x) is equal to

(A) 2 $\tan^{-1}(2e^x)$ (B) 1/2 $\tan^{-1}(e^x/2)$
(C) 2 $\tan^{-1} e^x/2$ (D) 1/2 $\tan^{-1}(2e^{2x})$

Solution: I = ∫ dx/ $e^x + 4e^{-x}$ = f (x) + c
=> I = ∫ e^x dx/ e^{2x} + 4
Let e^x = t => e^x dx = dt
=> I = ∫ dt / t^2 + 4 = 1/2 $\tan^{-1}$ (t/2) + c = 1/2 $\tan^{-1}(e^x/2)$ + c.

Hence (B) is the correct answer.

4. ∫ dx / (2x+1) (1+√(2x+1)) is equal to :

(A) $\tan^{-1}$ √2x+1 / 1 + √2x+1 + c (B) $\log_e$ √2x+1 / 1 + √2x+1 + c
(C) $\log_e$ (1+√2x+1 / √2x+1) + c (D) $\tan^{-1}$ 1 + √2x+1 / √2x+1 + c

Solution: Substitute √2x+1 = p and solve by partial fractions.

Hence (A) is the correct answer.

5. ∫ sin x / sinx – cos x is equal to :

(A) x/2 – 1/2 log (sinx – cos x) + c (B) x/2 + 1/2 log (sinx – cos x) + c

(C) x/2 – 1/2 log (sinx + cos x) + c (D) none of these

Solution: Let sin x = A(sin x – cos x) + B. d.c of (sin x – cos x)

or sin x = A (sin x – cos x) + B (cos x + sin x)

or sin x= (A + B) sin x + (B –A) cos x equating the coefficient of sin x and cos x, we get

A + B = 1 and B – A = 0

A = 1/2, B = 1/2

$$I = \int \frac{\frac{1}{2}(\sin x - \cos x) + \frac{1}{2}(\cos x + \sin x)}{\sin x - \cos x}$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\sin x - \cos x} dx = \frac{x}{2} + \frac{1}{2}\log(\sin x - \cos x) + c.$$

Hence (B) is the correct answer.

6. $I = \int (1 + x) / x(1+xe^x)^2$ is equal to :

(A) ln $(xe^x / 1 + xe^x) - (1 / 1 + xe^x) + c$

(B) ln $(xe^x / 1 + xe^x) + (1 / 1 - xe^x) + c$

(C) ln $(xe^x / 1 - xe^x) + (1 / 1 + xe^x) + c$

(D) ln $(xe^x / 1 + xe^x) + (1 / 1 + xe^x) + c$

Solution: Let $I = \int \frac{(1+x)}{x(1+xe^x)^2} dx = \int \frac{(1+x)e^x}{(xe^x)(1+xe^x)^2} dx$ = $(1 + xe^x = p, e^x (1+x) dx = dp)$

$I = \int dp / (p - 1)p^2$

$1/(p-1)p^2 = A / (p-1) + B/p + C / p^2$

$1 = Ap^2 + B (p) (p-1) + C (p - 1)$

For p = 1, p = 0, and p = -1, A = 1, C= -1 and B= -1.

$$\Rightarrow I = \int \frac{1}{(p-1)} dp - \int \frac{dp}{p} - \int \frac{dp}{p^2} = \ln\frac{(p-1)}{p} + \frac{1}{p} + c = \ln\left(\frac{xe^x}{1+xe^x}\right) + \left(\frac{1}{1+xe^x}\right) + c$$

Hence (D) is the correct answer.

7. $\int \cos 5x + \cos 4x / 1 - 2\cos 3x \, dx$ is equal to :

(A) $\sin 2x / 2 - \sin x + c$

(B) $-\sin 2x / 2 + \sin x + c$

(C) $-\sin 2x / 2 - \sin x + c$

(D) $\sin 2x / 2 + \sin x + c$

Solution: $\int \frac{2\cos\frac{9x}{2}\cos\frac{x}{2}\cos\frac{3x}{2}}{\left[1-2\left(2\cos^2\frac{3x}{2}-1\right)\right]\cos\frac{3x}{2}}dx = \int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{3\cos\frac{3x}{2}-4\cos^3\frac{3x}{2}}dx$

$$= \int \frac{2\cos\frac{9x}{2}\cos\frac{3x}{2}\cos\frac{x}{2}}{-\cos\frac{9x}{2}}dx = -\int(\cos 2x + \cos x)dx$$

$$= -\frac{\sin 2x}{2} - \sin x + c \ .$$

Hence (C) is the correct answer.

8. $\int$ ex (1+x) / $\cos^2(xe^x)$dx =

(A) – $\cot(xe^x)$ (B) $\tan(xe^x)$

(C) $\tan(e^x)$ (D) none of these

Solution: Put x e^x = t = $\int \sec^2 t dt = \tan t + c = \tan(xe^x) + c$;

Hence (B) is the correct answer.

9. $\int \tan^3 2x \sec^2 x \, dx$ =

(A) $\sec^3 2x + 3 \sec 2x$ (B) $1/6 [\sec^3 2x - 3 \sec 2x]$

(C) $[\sec^3 2x - 3 \sec 2x]$ (D) none of these

Solution: $I = \int \tan^2 2x \tan 2x \sec 2x \, dx = \int (\sec^2 2x - 1) \sec 2x \tan 2dx$

Put sec 2x = t 2 sec 2x tan 2x dx = dt

$I = 1/2 \int (t^2 - 1) dt = 1/2 (t^3 / 3 - 1) = 1/6 (\sec^3 2x - 3 \sec 2x)$

Hence (B) is the correct answer.

10. $\int$ sec x cosec x / log tan x dx

(A) log (tan x) (B) cot (log x)

(C) log log (tan x) (D) tan (log x)

Solution: put log tan x = t

1 / tan x sec^2 x dx = dt

sec x cosec x dx = dt

∫ dt/t = log t + c = log (log tan x)

Hence (C) is the correct answer.

11. ∫ $\cos^3 x e^{\log(\sin x)}$ dx is equal to

(A) $-\sin^4 x / 4 + c$ (B) $-\cos^4 x / 4 + c$

(C) $e^{\sin x} / 4 + c$ (D) none of these

Solution: $e^{\log \sin x} = \sin x$

∴ $\int \cos^3 x \sin x\, dx$ Put cosx = t

=> $-\int t^3 dt = -t4 / 4 + c = -\cos^4 x / 4 + c$

Hence (B) is the correct answer.

12. $\int (x^4 - x)^{1/4} / x^5$ dx is equal to

(A) $4/15 (1- 1/x^3)^{5/4} + c$ (B) $4/5 (1- 1/x^3)^{5/4} + c$

(C) $4/15 (1+ 1/x^3)^{5/4} + c$ (D) none of these

Solution: $I = \int \frac{(x^4 - x)^{\frac{1}{4}}}{x^5} dx$. Put $1 - \frac{1}{x^3} = t$

$\therefore \frac{3}{x^4} dx = dt, \therefore I = \frac{1}{3}\int t^{\frac{1}{4}} dt$

$= \frac{1}{3} \cdot \frac{t^{5/4}}{5/4} + c = \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$

Hence (A) is the correct answer.