SOLVED PROBLEMS

OBJECTIVE

1. The angle between two lines whose direction cosines are given by the equation l + m + n = 0, $l^2 + m^2 + n^2 = 0$ is

(A) π/3 *(B)* 2π/3

(C) π/4 *(D) None of these*

hb : Eliminating n between the two relations, we have

$l^2 + m^2 - (l + m)^2 = 0$ or 2lm = 0 => either l = 0 or m = 0

if l = 0, then m + n = 0 i,e. m = – n

=> l/0 = m/1 = n/–1, giving the direction ratios of one line.

If m = 0, then l + n = 0 i.e. l = – n

=> l/0 = m/1 = n/–1, giving direction ratios of the other lines.

The angles between these lines is

$$\cos^{-1}\left\{\pm\frac{0.1+1.0+(-1)(-1)}{\sqrt{0^2+1^2+(-1)^2}\,\sqrt{1^2+0^2+(-1)^2}}\right\} = \cos^{-1}\left(\pm\frac{1}{2}\right) = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}.$$

2. The equation of the plane which contains the line of intersection of the planes x + y + z – 6 = 0 and 2x + 3y + z + 5 = 0 and perpendicular to the xy plane is:

(A) x – 2y + 11 = 0 *(B)* x + 2y + 11 = 0

(C) x + 2y – 11 = 0 *(D)* x – 2y – 11 = 0

hb : Equation of the required plane is (x + y + z – 6) + λ(2x + 3y + z + 5) = 0

i.e. (1 + 2λ)x + (1 + 3λ)y + (1 + λ)z + (–6 + 5λ) = 0

This plane is perpendicular to xy plane whose equation is z = 0 i.e. $0 . x + 0 . y + z = 0$

∴ By condition of perpendicularity

$0.(1 + 2\lambda) + 0. (1 + 3\lambda) + (1 + \lambda) .1 = 0$ i.e. $\lambda = -1$

∴ Equation of required plane is

$(1 - 2)x + (1 - 3)y + (1 - 1)z + (-6 - 5) = 0$ or $x + 2y + 11 = 0$.

3. The coordinates of the foot of the perpendicular drawn from the origin to the plane $3x + 4y - 6z + 1 = 0$ are :

(A) (–3/61, 4/61, 6/61) *(B)* (3/61, –4/61, 6/61)

(C) (–3/61, –4/61, 6/61) *(D)* (3/61, 4/61, 6/61)

hb : The equation of the plane is $3x + 4y - 6z + 1 = 0$ ……(1)

The direction ratios of the normal to the plane (1) are 3, 4, –6. So equation of the line through (0, 0, 0) and perpendicular to the plane (1) are

$x/3 = y/4 = z/-6 = r$ (say) ……(2)

The coordinates of any point P on (2) are (3r, 4r, –6r). If this point lie on the plane (1), then

$3(3r) + 4(4r) - 6(-6r) + 1 = 0$ i.e. $r = -1/61$

Putting the value of r coordinates of the foot of the perpendicular P are (–3/61, –4/61, 6/61).

4. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $x/2 = y/3 = z/-6$ is:

(A) 7 unit *(B)* 4 unit

(C) 1 unit *(D)* 2 unit

hb : Here we are not to find perpendicular distance of the point from the plane but distance measured along with the given line. The method is as follow:

The equation of the line through the point (1, –2, 3) and parallel to given line is

$x-1/2 = y+2/ 3 = z-3/-6 = r$ (say)

The coordinate of any point on it is (2r + 1, 3r – 2, –6r + 3).

If this point lies in the given plane then

2r + 1 – (3r – 2) + (–6r + 3) = 5 Þ –7r = –1 or r = 1/7

∴ point of intersection is (9/7, –11/7, 15/7)

∴ The required distance

= the distance between the points (1, –2, 3) and (9/7, –11/7, 15/7)

= $\sqrt{(1–9/7)^2 + (–2+11/7)^2 + (3–15/7)^2}$ = 1/7 = √49 = 1 unit.

5. The image of the point (1, 3, 4) in the plane 2x – y + z + 3 = 0 is :

(A) (–3, 5, 2) *(B)* (3, 2, 5)

(C) (–5, 3, –2) *(D)* (–2, 5, 3)

hb : As it is clear from the figure that PQ will be perpendicular to the plane and foot of this perpendicular is mid point of PQ i.e. N.

So, direction ratios of line PQ is 2, –1, 1

=> Equation of line PQ = x–1/2 = y–3/–1 = z–4/1 r (say)

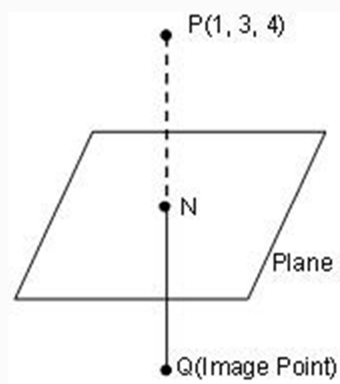
Any point on line PQ is (2r + 1, –r + 3, r + 4)

If this point lies on the plane, then

2(2r + 1) – (–r + 3) + (r + 4) + 3 = 0=> r = –1

∴ coordinate of foot of perpendicular N = (–1, 4, 3)

As N is middle point of PQ



∴ $-1 = 1+x_1/2$, $4 = 3+y_1/2$, $3 = 4+z_1/2$

=> $x_1 = -3$, $y_1 = 5$, $z_1 = 2$

∴ Image of point P (1, 3, 4) is the point Q (–3, 5, 2).

6. The equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible is :

(A) $3(x^2 + y^2 + z^2) + 2(x + y + z) - 1 = 0$

(B) $3(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0$

(C) $(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0$

(D) $(x^2 + y^2 + z^2) + 2(x + y + z) - 1 = 0$

hb : Let equation of sphere be given by

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ……(1)

As sphere passes through points (1, 0, 0), (0, 1, 0) and (0, 0, 1). So we have

$1 + 2u + d = 0$, $1 + 2v + d = 0$, $1 + 2w + d = 0$

On solving $u = v = w = -1/2\ (d + 1)$

If r is the radius of the sphere, then

$r = \sqrt{}u^2 + v^2 + w^2 - d$

$r^2 = 3/4\ (d + 1)^2 - d = m$ (say)

for r to be minimum

$d\mu/dd = 0 \Rightarrow 3/4.2(d + 1) - 1 = 0$ or $d = -1/3$

Also, $d^2\mu/dd_2 = 3/2$ positive at $d = -1/3$

Hence m is minimum at $d = -1/3$

So, substituting value of d we have u = v = w = –1/3

∴ equation of the sphere

$x^2 + y^2 + z^2 – (x + y + z) – 1/3 = 0 \Rightarrow 3(x^2 + y^2 + z^2) – 2(x + y + z) – 1 = 0.$

7. A point moves so that the ratio of its distances from two fixed points is constant. Its locus is a:

(A) plane *(B)* st. lines

(C) circle *(D)* sphere

hb : Let the coordinates of moving point P be (x, y, z). Let A (a, 0, 0) and B (–a, 0, 0) be two fixed points. According to given condition

$AP/BP = \text{constant} = k \text{ (say)} \Rightarrow AP^2 = k^2BP^2$

or, $(x – a)^2 + (y – 0)^2 + (z – 0)^2 = k^2\{(x + a)^2 + (y – 0)^2 + (z – 0)^2\}$

$\Rightarrow (1 – k^2)(x^2 + y^2 + z^2) – 2ax(1 + k^2) + a^2(1 – k^2) = 0$

∴ required locus is $x^2 + y^2 + z^2 – 2a(1+k^2)/(1–k^2) + a^2 = 0$. Which is a sphere.

8. The ratio in which yz–plane divides the line joining (2, 4, 5) and (3, 5, 7)

(A) -2 : 3 *(B) 2 : 3*

(C) 3 : 2 *(D) -3 : 2*

hb : Let the ratio be λ : 1

x-coordinate is $3\lambda+2 / \lambda+1 = 0 \Rightarrow \lambda = –2/3$.

Hence (A) is the correct answer.

9. A line makes angles α, β, γ, δ with the four diagonals of a cube then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta =$

(A) 1 *(B) 4/3*

(C) 3/4 *(D) 4/5*

hb The direction ratios of the diagonal (1, 1, 1)

Direction cosine are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

Similarly direction cosine of AS are $(-1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$

BP are $(1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$

CQ are $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$

Let l, m, n be direction cosines of the line

$\cos\alpha = l+m+n/\sqrt{3}$, $\cos\beta = l-m-n/\sqrt{3}$, $\cos\gamma = l+m-n/\sqrt{3}$, $\cos\delta = l-m+n/\sqrt{3}$

$\cos^2 a + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4(l^2 + m^2 + n^2) / 3 = 4/3$ (since $l^2 + m^2 + n^2 = 1$)

Hence (B) is the correct answer.

10. The points (0, -1, -1), (-4, 4, 4), (4, 5, 1) and (3, 9, 4) are

(A) collinear *(B) coplanar*

(C) forming a square *(D) none of these*

hb : Equation of the plane passing through the points (0, -1, -1), (-4, 4, 4) and (4, 5, 1) is $\begin{vmatrix} x & (y+1) & (z+1) \\ -4 & 5 & 5 \\ 4 & 6 & 2 \end{vmatrix} = 0$ …. (1)

The point (3, 9, 4) satisfies the equation (1).

Hence (B) is the correct answer.

11. A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. The locus of the point common to plane through A, B, C parallel to coordinate planes is

(A) ayz + bzx + cxy = xyz *(B) axy + byz + czx = xyz*

(C) axy + byz + czx = abc *(D) bcx + acy + abz = abc*

hb : Let the equation to the plane be $x/\alpha + y/\beta + z/\gamma = 1$

=> $x/\alpha + y/\beta + z/\gamma = 1$ (the plane passes through a, b, c)

Now the points of intersection of the plane with the coordinate axes are $A(\alpha, 0, 0)$, $B(0, \beta, 0)$ & $C(0, 0, \gamma)$

=> Equation to planes parallel to the coordinate planes and passing through A, B & C are $x = \alpha$, $y = \beta$ and $z = \gamma$.

∴ The locus of the common point is

$a/x + b/y + c/z = 1$ (by eliminating α, β, γ from above equation)

Hence (A) is the correct answer.

12. *Consider the following statements:*

Assertion (A): the plane y + z + 1 = 0 is parallel to x-axis.

Reason (R): normal to the plane is parallel to x-axis.

Of these statements:

(A) both A and R are true and R is the correct explanation of A

(B) both A and R are true and R is not a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

hb : Given plane $y + z + 1 = 0$ is parallel to x-axis as $0.1 + 1.0 + 1.0 = 0$

but normal to the plane will be perpendicular to x-axis.

Hence (C) is the correct answer.

13. *The equation of the plane containing the line x = α/l, y = β/m and z = γ/n is a(x – α) + b(y – β) + c(z – γ) = 0, where al + bm + cn is equal to*

(A) 1 *(B) –1*

(C) 2 *(D) 0*

hb : Since straight line lies in the plane so it will be perpendicular to the normal at the given plane. Since direction cosines of straight line are l, m, n and direction ratios of normal to the plane are a, b, c. So, $al + bm + cn = 0$.

Hence (D) is the correct answer.

14. *The shortest distance between the two straight lines x–4/3 / 2 = y+6/5 / 3 = z–3/2 / 4 and 5y+6/8 = 2z–3/9 = 3x–4/5 is*

(A) √29 *(B) 3*

(C) 0 *(D) 6√10*

hb : Since these two lines are intersecting so shortest distance between the lines will be 0.

Hence (C) is the correct answer.

15. *A straight line passes through the point (2, –1, –1). It is parallel to the plane 4x + y + z + 2 = 0 and is perpendicular to the line x/1 = y/–2 = z–5/1. The equations of the straight line are*

(A) x–2/4 = y+1/1 = z+1/1 *(B) x+2/4 = y–1/1 = z–1/1*

(C) x–2/–1 = y+1/1 = z+1/3 *(D) x+2/–1 = y–1/1 = z–1/3*

hb : Let direction cosines of straight line be l, m, n

∴ $4l + m + n = 0$

$l – 2m + n = 0$

=> $l/3 = m/–3 = n/–9$ => $l/–1 = m/+1 = n/3$

∴ Equation of straight line is $x–2/–1 = y+1/1 = z+1/3$.

Hence (C) is the correct choice.

16. *If centre of a sphere is (1, 4, –3) and radius is 3 units, then the equation of the sphere is*
(A) $x^2 + y^2 + z^2 - 2x - 8y + 6z + 17 = 0$ *(B) $2(x^2 + y^2 + z^2) - 2x - 8y + 6z + 17 = 0$*
(C) $x^2 + y^2 + z^2 - 4x + 16y + 12z + 17 = 0$ *(D) $x^2 + y^2 + z^2 + 2x + 8y - 6z - 17 = 0$*

hb : Equation of sphere will be $(x - 1)^2 + (y - 4)^2 + (z + 3)^2 = 9$
Hence (A) is the correct answer.

17. *If equation of a sphere is $2(x^2 + y^2 + z^2) - 4x - 8y + 12z - 7 = 0$ and one extremity of its diameter is (2, –1, 1), then the other extremity of diameter of the sphere will be*
(A) (2, 9, –13) *(B) (0, 9, 7)*
(C) (0, 5, 7) *(D) (2, 5, –13)*

hb : The centre of the sphere is (1, 2, –3) so if other extremity of diameter is (x_1, y_1, z_1), then
$x_1+2/2 = 1$, $y_1-1/2 = 2$, $z_1+1/2 = -3$
∴ Required point is (0, 5, 7).
Hence (C) is the correct answer.

18. The direction cosines of the line which is perpendicular to the lines with direction cosines proportional to (1, – 2, – 2), (0, 2, 1) is
(A) (2/3, 1/3, 2/3) (B) (2/3, –1/3, –2/3)
(C) (2/3, 1/3, –2/3) (D) (–2/3, –1/3, –2/3)

hb : Let direction ratios of the required line be
Therefore a - 2 b - 2 c = 0
And 2 b + c = 0
=> c = - 2 b
a - 2 b + 4b = 0 => a = - 2 b
Therefore direction ratios of the required line are <- 2b, b, - 2b> = <2, - 1, 2>
direction cosines of the required line

$$= \left(\frac{2}{\sqrt{2^2+1^2+2^2}}, \frac{-1}{\sqrt{2^2+1^2+2^2}}, \frac{2}{\sqrt{2^2+1^2+2^2}}\right) = \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right)$$

19. The points (4, 7, 8), (2, 3, 4), (–1, –2, 1) and (1, 2, 5) are :

(A) the vertices of a parallelogram (B) collinear
(C) the vertices of a trapezium (D) concyclic

hb : Let A Ξ (4, 7, 8), B Ξ (2, 3, 4), C Ξ (- 1, - 2, 1), D Ξ (1, 2, 5)
Direction cosines of AB Ξ (2/6, 4/6, 4/6) = (1/3, 2/3, 2/3)
Direction cosines of CD Ξ (–2/6, –4/6, –4/6)
= (1/3, 2/3, 2/3)
So, AB parallel to CD
Direction cosines of AD Ξ $(3/\sqrt{43}, 5/\sqrt{43}, 3/\sqrt{43})$
Direction cosines of BC Ξ $(-3/\sqrt{43}, -5/\sqrt{43}, -3/\sqrt{43})$
= $(3/\sqrt{43}, 5/\sqrt{43}, 3/\sqrt{43})$
so, AD is parallel to BC.

Therefore ABCD is a parallelogram.

20. The equation of the plane parallel to the plane 4x – 3y + 2z + 1 = 0 and passing through the point (5, 1, – 6) is :

(A) $4x - 3y + 2z - 5 = 0$ (B) $3x - 4y + 2z - 5 = 0$

(C) $4x - 3y + 2z + 5 = 0$ (D) $3x - 4y + 2z + 5 = 0$

hb : Equation of the plane parallel to the plane $4x - 3y + 2z + 1 = 0$ is of the form of $4x – 3y + 2z + k = 0$ again it passes through (5, 1, - 6)

so, $20 - 3 - 12 + k = 0$ Þ $k = - 5$

Therefore required equation is $4x - 3y + 2z - 5 = 0$.

21. A plane is passed through the middle point of the segment A (–2, 5, 1), B (6, 1, 5) and is perpendicular to this line. Its equation is :

(A) $2x - y + z = 4$ (B) $2x - y + z = 4$

(C) $x - 3y + z = 5$ (D) $x - 4y + 2z = 5$

hb : Mid-point of A (- 2, 5, 1) and B (6, 1, 5) is (2, 3, 3)

direction ratios of the line joining A and B is <2, - 1, 1>

Therefore equation of the line perpendicular to AB and passing through (2, 3, 3) is

$2(x – 2) – 1(y – 3) + 1(z – 3) = 0 \Rightarrow 2x - y + z = 4$

22. A plane meets the co-ordinates axes in A, B, C such that the centroid of triangle ABC is (a, b, c). The equation of the plane is :

(A) $x/a + y/b + z/c = 3$ (B) $x/a + y/b + z/c = 1$

(C) $x/a + y/b + z/c = 2$ (D) None of these

hb : The plane meets the co-ordinate axes at A, B, C such that centroid of the triangle ABC is (a, b, c)

so, the plane cuts X-axis at (3 a, 0, 0)

So, X-intercept = 3 a

The plane cuts Y-axis at (0, 3 b, 0)

=> Y-intercept = 3 b

the plane cuts Z-axis at (0, 0, 3 c)

=> Z-intercept = 3 c

Therefore required equation is x/3a + y/3b + z/3c = 1

=> x/a + y/b + z/c = 3.

23. The radius of the sphere (x + 1)(x + 3) + (y – 2)(y – 4) + (z + 1)(z + 3) = 0 is:

(A) √2 (B) 2

(C) √3 (D) 3

hb : (x + 1) (x + 3) + (y - 2) (y - 4) + (z + 1)(z + 3) = 0 is the given equation of sphere.

So, end points of the diameter are

(- 1, 2, - 1) and (- 3, 4, - 3)

radius = $\sqrt{(-2+1)^2 + (3-2)^2 + (-2+1)^2} = \sqrt{3}$

24. The sum of the direction cosines of a straight line is

(A) zero (B) one

(C) constant (D) none of these

hb : cos α = l, cos β = m, cos γ = n

sum of direction cosines cos α + cos β + cos γ

= l + m + n

which is constant.

25. The angle between straight lines whose direction cosines are (1/2, –1/2, 1/√2) and (1/√3, 1/√3, –1/√3) is

(A) $\cos^{-1}$ (2/√3) (B) $\cos^{-1}$ (1/√6)

(C) $\cos^{-1}$ (–1/√6) (D) none of these *hb :* $\theta = \cos^{-1}\left(\frac{\frac{1}{2\sqrt{3}} - \frac{1}{2\sqrt{3}} - \frac{1}{\sqrt{6}}}{\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}}\sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right)$

26. Which one of the following is best condition for the plane ax + by + cz + d = 0 to intersect the x and y axes at equal angle

(A) |a| = |b| (B) a = –b

(C) a = b (D) $a^2 + b^2 = 1$

hb : The plane a x + b y + c z + d = 0 intersects x and y axes at equal angles

Therefore |cos α| = |cos β|

=> |l| = |m|

=> |a| = |b|.

27. The equation of a straight line parallel to the x-axis is given by

(A) x–a/1 = y–b/1 = z–c/1 (B) x–a/0 = y–b/1 = z–c/1

(C) x–a/0 = y–b/0 = z–c/1 (D) x–a/1 = y–b/0 = z–c/0

hb : Equation of straight line parallel to x-axis is

x–a/1 = y–b/0 = z–c/0

because l = cos α= 1, m = cos β = cos π/2 = 0, n = cos γ = 0.

28. If P (2, 3, –6) and Q (3, –4, 5) are two points, the direction cosines of line PQ are

(A) –1/√171, –7/√171, –11/√171 (B) 1/√171, –7/√171, 11/√171

(C) 1/√171, 7/√171, –11/√171 (D) –7/√171, –1/√171, 11/√171

hb : P ≡ (2, 3, - 6), Q ≡ (3, - 4, 5)

direction ratios = <1, - 7, 11>

direction cosines = $(1/\sqrt{1+49+121}, 7/\sqrt{1+49+121}, 11/\sqrt{1+49+121})$

$= (1/\sqrt{171}, -7/\sqrt{171}, 11/\sqrt{171})$

29. The ratio in which yz–plane divide the line joining the points A(3, 1, –5) and B(1, 4, –6) is

(A) –3 : 1 (B) 3 : 1

(C) –1 : 3 (D) 1 : 3

hb : A ≡ (3, 1, - 5), B ≡ (1, 4, - 6)

Therefore $3+\lambda/\lambda+1 = 0 \Rightarrow \lambda = -3$

Therefore required ratio is - 3 : 1

30. A straight line is inclined to the axes of x and z at angels 45^0 and 60^0 respectively, then the inclination of the line to the y-axis is

(A) 30^0 (B) 45^0

(C) 60^0 (D) 90^0

hb : A straight line is inclined to the axes of x and z at an angle 45^0 and 60^0

$l^2 + m^2 + n^2 = 1$

$\Rightarrow m^2 = 1/4$

$\Rightarrow m = 1/2$

=> angle made by 60^0

31. The angle between two diagonals of a cube is

(A) $\cos\theta = \sqrt{3}/2$ (B) $\cos\theta = 1/\sqrt{2}$

(C) $\cos\theta = \sqrt{1/3}$ (D) none of these

hb : $\cos\theta = -a^2 + a^2 + a^2 / \sqrt{a^2 + a^2 + a^2}\ \sqrt{a^2 + a^2 + a^2}$ of side is a

= 1/3.

32. Given that A (3, 2, –4), B (5, 4, –6) and C (9, 8, –10) are collinear. The ratio in which B divides AC

(A) 1 : 2 (B) 2 : 1

(C) –1 : 2 (D) –2 : 1

hb : $9\lambda + 3/\lambda+1 = 5 \Rightarrow 9\alpha - 5\alpha = 2$

$\Rightarrow \lambda = 1/2$.

33. If P_1P_2 is perpendicular to P_2P_3, then the value of k is, where $P_1(k, 1, -1)$, $P_2(2k, 0, 2)$ and $P_3(2 + 2k, k, 1)$

(A) 3 (B) –3

(C) 2 (D) –2

hb : Direction ratios of $P_1 P_2$ = - 1, 3>

direction ratios of $P_2 P_3$ = <2, k, - 1>

Therefore 2 k - k - 3 = 0

=> k = 3.

34. A is the point (3, 7, 5) and B is the point (–3, 2, 6). The projection of AB on the line which joins the points (7, 9, 4) and (4, 5, –8) is

(A) 26 (B) 2

(C) 13 (D) 4

hb : Distances of the line joining (7, 9, 4) and (4, 5, - 8) is < 3/13, 4/13, 12/13 >

Therefore required projection is 26/13 = 2 (B)

35. The shortest distance of the point from the x-axis is equal to(Ref.P.K.Sharma_Three Dimen._P.C6.1Q.1)

(A) $\sqrt{x_1^2 + y_1^2}$ (B) $\sqrt{x_1^2 + z_1^2}$

(C) $\sqrt{y_1^2 + z_1^2}$ (D) None of these

Ans. (C)

hb . Foot of perpendicular drawn from P to x-axis will have its coordinates as (x, 0, 0).

Required distance = $\sqrt{y_1^2 + z_1^2}$

36. The point of intersection of the xy plane and the line passing through the points and is:(Ref.P.K.Sharma_Three Dimen._P.C6.1Q.4)

(A) (–13/5, 23/5, 0) (B) (13/5, 23/5, 0)

(C) (13/5, –23/5, 0) (D) (–13/5, –23/5, 0)

Ans. (B)

hb . Direction ratios of AB are 2, -3, 5.

Thus equation of AB is, x–3/2 = y–4/–3 = z–1/5

For the point of intersection of this line with xy–plane, we have

Z = 0

=> x–3/2 = y–4/–3 = –1/5

=> x=3 –2/5 = 13/5, y = 4 + 3/5 = 23/5

Hence, the required point is (15/5, 23/5, 0)

37. The projections of the line segment AB on the coordinate axes are –9, 12, -8 respectively. The direction cosines of the line segment AB are:(Ref.P.K.Sharma_Three Dimen._P.C6.1Q.5)

(A) –9/17, 12/17, –8/17 (B) –9/289, 12/289, –8/289

(C) –9/√17, 12/√17, –8/√17 (D) None of these

Ans. (A)

Length of segment AB = √81 + 144 + 64 = 17

Thus direction cosines of AB are

.–9/17, 12/17, –8/17

38. The direction cosines of two mutually perpendicular lines are l_1,m_1,n_1 and l_2,m_2,n_2. The direction cosines of the line perpendicular to both the given lines will be:(Ref.P.K.Sharma_Three Dimen._P.C6.1Q.6)

(A) $l_1+ l_1, m_1+m_2, n_1+n_2$ (B) $l_2 – l_2, m_1– m_2, n_1+n_2$

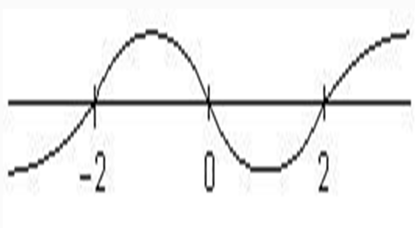
(C) $l_1 l_2, m_1m_2, n_{1n2}$ (D) $m_1n_2 – m_2n_1, n_1 l_2 – n_2, l_1m_2 – l_2m_1$

Ans. (D)

Let the direction cosine of the required line be l, m and n.

We must have,

$ll_1 + mm_1 + nn_1 = 0, ll_2 + mm_2 + nn_2 = 0$



We have $l_1l_2 + m_1m_2 + n_1n_2 = 0$

Thus

$\Sigma(m_1n_2–n_1m_2)^2 = (\Sigma l_2)^2 (\Sigma l_2)^2 – \Sigma l_1l_2 = 1$

$\Rightarrow l = m_1n_2 - n_1m_2, m = n_1 l_2 - n_2 l_1,$

$n = l_1m_2 – l_2m_1$

39. A directed line segment angles α,β,γ with the coordinate axes. The value of Σcos2α is always equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.1Q.7)

(A) -1 (B) 1

(C) -2 (D) 2

Ans. (A)

$\Sigma\cos 2\alpha = \Sigma(2\cos^2\alpha - 1)$

$= 2\Sigma l^2 - 3$

$= -1$

40. The locus represented by xy + yz = 0 is:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.9)

(A) A pair of perpendicular lines (B) A pair of parallel lines

(C) A pair of parallel planes (D) A pair of perpendicular planes

Ans. (D)

$xy + yz = 0$

$\Rightarrow x(y+z) = 0$

$\Rightarrow x=0,\ y+z = 0$

Thus it represents a pair of planes

$X = 0,\ y + z = 0$

that are clearly mutually perpendicular.

41. The plane 2x – 3y + 6z – 11 = 0 makes an angle (a) with x-axis. The value of ‘a’ is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.10)

(A) $\sqrt{3}/2$ (B) $\sqrt{2}/3$

(C) 2/7 (D) 3/7

Ans. (C)

If ‘θ’ be the angle between the plane and x–axis, then

$\sin\theta\ 2/\sqrt{4+9+36} = 2/7$

$\Rightarrow \theta = \sin^{-1}(2/7)$
$\Rightarrow a = (2/7)$

42. The planes x + y = 0, y + z = 0 and x + z = 0:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.12)

(A) meet in a unique point (B) meet in a unique line

(C) are mutually perpendicular (D) none of these

Ans. (B) Clearly, given planes have a common line of intersection namely the z-axis.

43. The equation of a plane passing through (1, 2, -3), (0, 0, 0) and perpendicular to the plane 3x – 5y + 2z = 11, is:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.13)

(A) $3x + y + 5/3\ z = 0$ (B) $4x + y + 2z = 0$

(C) $3x – y + z/3$ (D) $x + y + z = 0$

Ans. (D)

Let the required plane be

$Ax + by + cz = 0$.

We have $3a – 5b + 2c = 0$, $a + 2b – 3c = 0$

=> $a/15–4 = b/2+9 = c/6+5$

=> $a : b : c = 11 : 11 : 11$

Thus plan is $x + y + z = 0$

44. The direction ration of a normal to the plane passing through (1, 0, 0), (0, 1, 0) and making an angle π/4 with the plane x + y = 3 are:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.15)

(A) $(1, \sqrt{2}, 1)$ (B) $(1, 1, \sqrt{2})$

(C) (1,1,2) (D) $(\sqrt{2}, 1, 1)$

Ans. (B)

Let the plane be $x/a + y/b + z/c = 1$

=> $1/a = 1$, $1/b = 1$

.=> $a = b = 1$

Also, $\sin\frac{\pi}{4} = \frac{\left|\frac{1}{a}+\frac{1}{b}\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}\sqrt{1+1}}$

=> c = ± 1/√2

Thus direction rations are

(1,1,√2) or (1,1,–√2)

45. The equation of a plane passing through the line of intersection of the planes x + y + z = 5, 2x – y + 3z = 1 and parallel to the line y = z = 0 is:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.16)

(A) 3x – z = 9 (B) 3y – z = 9

(C) x – 3z = 9 (D) y – 3z = 9

Ans. (B)

Plane will be in the form

(x + y + z – 5) + a(2x – y + 3z) –1) = 0 i.e., x(1 + 2a) + y(1 – a) + z(1 + 3a) = 5 + a

It is parallel to the line y = z = 0.

Since, (1 + 2a) = 0

∴ a = – 1/2

Thus required plane is

3/2y – 1/2 z = 9/2

i.e., 3y – z = 9

46. The angle between lines whose direction cosines are given by l + m + n = 0, $l^2 + m^2 – n^2 = 0$, is:(Ref.P.K.Sharma_Three Dimen._P.C6.2Q.17)

(A) π/2 (B) π/3

(C) π/6 (D) None of these

Ans. (D)

.l + m + n = 0, $l^2 + m^2 - n^2 = 0$

We also have

$l^2 + m^2 + n^2 = 1$

=> $2n^2 = 1$

=> n = 1/√2, – 1/√2

Also, $l^2 + m^2 = n^2 = (-(l+m))^2$

=> lm = 0 and l + m = ± 1/√2

Hence, direction cosines are lines are

(1/√2, 0, –1/√2), (–1/√2, 0, 1/√2)

(0, 1/√2, –1/√2), (0, –1/√2, 1/√2)

Angle between these lines in both cases is zero.

47. Centroid of the tetrahedron OABC, where , , and O is the origin is (1, 2, 3). The value of is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.3Q.19)

(A) 75 (B) 80

(C) 121 (D) None of these

Ans. (A)

We have

4 = a + 1 + 2 + 0,

=> a = 1,

8 = 2 + b + 1 + 0

=> b = 5,

12 = 3 + 2 + c + 0.

=> c = 7,

$\therefore a^2 + b^2 + c^2 = 1 + 25 + 49 = 75$

48. The equation of the plane passing through the points (2, -1, 0), (3, -4, 5) and parallel to the line 2x = 3y = 4z is:(Ref.P.K.Sharma_Three Dimen._P.C6.3Q.20)

(A) $125x - 90y - 79z = 340$ (B) $32x - 21y - 36z = 85$

(C) $73x + 61y - 22z = 85$ (D) $29x - 27y - 22z = 85$

Ans. (D)

Give line is $2x = 3y = 4z$

i.e., $x/6 = y/4 = z/3$

Let the plane be

$Ax + by + cz = 1.$

We have

$6a + 4b + 3c = 0$

$2a - b = 1$

$3a - 4b + 5c = 1.$

$\Rightarrow a = 29/85,\ b = 27/85,\ c = -22/85$

Thus equation of plane is

$29x - 27y - 22z = 85.$

49. A plane passes through the point . If the distance of this plane from the origin is maximum, then it's equation is:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.1)

(A) $x + 2y - 3z + 4 = 0$ (B) $x + 2y + 3z = 0$

(C) $2y - x + 3z = 0$ (D) $x - 2y + 3z = 0$

Ans. (B) Clearly in this case OA will be a normal to the plane.

Direction cosine of segment OA are

1/√14, 2/√14, 3/√14

and .OA = √1 + 4 = 0 = √14

Thus the equation of plane is

x + 2y + 3z = 14.

50. The shortest distance of the plane 12 + 4y + 3z = 327, from the sphere , is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.3)

(A) 39 units (B) 26 sq. units

(C) 13 units (D) None of these

Ans. (C)

Center and radius of given sphere are (–2, 1, 3) and 13 unit respectively.

Now, distance of center of the sphere from the given plane

= |–24 + 4 + 9 – 327| / $\sqrt{12^2 + 4^2 + 3^2}$ = 26 units

∴Shortest distance = (26 – 13) = 13 units.

51. The lines x = ay + b, z = cy + d and x = a'y + b', z = c'y + d' will be mutually perpendicular provided:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.5)

(A) (a + a')(b + b')(c + c') (B) aa' + cc' + 1 = 0

(C) aa' + bb' + cc' + 1 = 0 (D) (a + a') (b + b') (c + c') + 1 = 0

Ans. (B)

Give lines are

x–b/a = y = z – d/c and x–b' / a' = y = z–d'/c'

These lines will be mutually perpendicular, provided

a.a' + 1.1' + c.c' = 0

.=> a.a' + c.c' + 1= 0

52. The straight lines x–2/1 = y–3/1 = z–4/–k and x–1/k = y–4/2 = z–5/1, will intersect provided:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.6)

(A) k = {3, -3} (B) k = {0, -1}

(C) k = {-1, 1} (D) k = {0, -3}

Ans. (D)

Any point on the first line can be takes as

$P_1 = (r_1 + 2, r_1 + 3, -kr_1+4)$

These lines will intersect if for some r_1 and r_2 we have

$r_1 + 2 = kr_2 + 1,$

$r_1 + 3 = 2r_2 + 1,$

$-kr_1 + 4 = r_2 + 5,$

$\therefore r_1 + kr_2 + 1 = 0, r_1 = 2r_2 + 1,$

=> $r_2 = 2/k–2, r_1 = k+2 / k–2k$

putting these values in the last condition, we get

$k^2 + 3k = 0$

.=> k = {–3,0}

53. The plane ax + by + cz = d, meets the coordinate axes at the points, A, B and C respectively. Area of triangle ABC is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.7)

(A) $d^2\sqrt{a^2 + b^2 + c^2} / |abc|$ (B) $d^2\sqrt{a^2 + b^2 + c^2} / 2|abc|$

(C) $d^2\sqrt{a^2 + b^2 + c^2} / 4 |abc|$ (D) None of these

Ans. (B)

A = (d/a, 0, 0), B = (0, d/a, 0), c = (0,0, d/a)

Area of triangle OAB = $\Delta_1 = 1/2\ d^2/|ab|$

Area of triangle OBC = Δ_2 = 1/2 d^2/|bc|

Area of triangle OAC = Δ_3 = 1/2 d^2/|ac|

If area of triangle ABC be Δ, then

$$\Delta^2 = \Delta_1^2 + \Delta_2^2 + \Delta_3^2 = \frac{d^4}{4}\left(\frac{a^2+b^2+c^2}{a^2 b^2 c^2}\right)$$

$$\Rightarrow \quad \Delta = \frac{d^2\sqrt{a^2+b^2+c^2}}{2|abc|}$$

54. Equation of the plane passing through (-1, 1, 4) and containing the line x–1/3 = y–2/1 = z/5, is:(Ref.P.K.Sharma_Three Dimen._P.C6.4Q.9)

(A) 9x – 22y + 2z + 23 = 0 (B) x + 22y + z = 25

(C) 9x + 22y - 3z = 1 (D) 22y – 9x + z = 35

Ans. (D)

Equation of any plane containing the line x–1/3 = y–2/1 = z/5 will be

a(x–1) + b (y–z) + cz = 0

where, 3a + b + 4c = 0 …. (i)

It is given that plane passes through (–1, 1, 4).

∴ –2a – b + 4c = 0 …. (ii)

From (i) and (ii), we get

a/–9 = b/22 = c/1.

Thus the equation of required plane is,

–9(x – 1) + 22(y – 2) + z = 0

i.e., 22y – 9x + z = 35

55. Equation of the plane containing the lines x/1 = y–2/3 = z+4/–1 and x–4/2 = y/3 = z/1is,:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.10)

(A) x + y – 4z = 6 (B) x - y + 4z = 6

(C) x + y + 4z = 6 (D) None of these

Ans. (D)

Equation of any plane containing the line x/1 = y–2/3 = z+4/–1 is ax + b (y–2) + c (z+4) = 0

where a + 3b – c = 0

This plane will also contain the second line if

2a – 3b + c = 0

and 4a b(0 – 2) + c(0 + 4) = 0

Solving these equation, we get

a = 0, b = 0, c = 0.

That means the given lines are non–coplanar.

56. Equation of the plane such that foot of altitude drawn from (-1, 1, 1) to the plane has the coordinate (3, -2, -1), is:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.11)

(A) x + y + z = 0 (B) 4x - 3y – 2z = 20

(C) 3x + y – z = 8 (D) 4x + 3y – z = 7

Ans. (B)

Clearly, the direction ratio of the plane are

4, –3, –2

Thus equation of plane will be

4x – 3y – 2z = d

It will necessarily pass through (3, –2, –1)

i.e,, d = 12 + 6 + 2 = 20

Thus the equation of plane is

$4x - 3y - 2z = 20.$

57. The distance of the point (-1, 2, 6) from the line x–2/6 = y–3/3 = z+ 4/–4, is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.13)

(A) 7 units (B) 9 units

(C) 10 units (D) 12 units

Ans. (A)

Any point on the line is

$P = (6r_1 + 2, 3r_1 + 3, -4r_1 - 4).$

Direction ration of the line segment PQ, where Q = (–1, 2, 6), are

$6r_1 + 3, 3r_1 + 1, - 4r, - 10.$

If 'P' be the foot of altitude drawn from Q to the given line, then

$6(6r_1 + 3) + 3(3r_1 + 1) + 4(4r_1 + 10) = 0.$

=> $r_1 = -1.$

Thus, P = (–4, 0, 0)

∴ Required distance = $\sqrt{9 + 4 + 36}$

= 7 units.

58. The point of intersection of the lines x+1 / 3 = y + 3/5 = z + 5 / 7 and x–2/1 = y–4/3 = z–6/5 is:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.14)

(A) (1/3, –1/3, –2/3) (B) (1/2, –1/2, –3/2)

(C) (1/2, 1/2, 1/2) (D) (1/3, 1/3, 1/3)

Ans. (B)

Any point on the first line is

$(3r_1 - 1, 5r_1 - 3, 7r_1 - 5)$

and any point on the second line is

$(r_2 + 2, 3r_2 + 4, 5r_2 + 6)$.

At the point of intersection, we must have

$3r_1 – 1 = r_2 + 2$,

$5r_1 – 3 = 3r_2 + 4$.

$7r_1 – 5 = 5r_1 + 6$.

Thus, $r_1 = 1/2$, $r_2 = –3/2$

Hence required point is i.e. (1/2, –1/2, –3/2)

59. The shortest distance between the line x + y + 2z – 3 = 2x + 3y + 4z – 4 = 0 and the z-axis is:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.15)

(A) 1 unit (B) 2 units

(C) 3 units (D) 4 units

Ans. (B)

We have,

x + y + 2z – 3 = 0, x + 2z – 2 + 3/2 y = 0

Solving these equations, we get

Y = –2.

Thus required shortest distance is 2 units.

60. The length of projection, of the line segment joining the points (1, -1, 0) and (-1, 0, 1), to the plane 2x + y + 6z = 1, is equal to:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.17)

(A) $\sqrt{255/61}$ (B) $\sqrt{237/61}$

(C) $\sqrt{137/61}$ (D) $\sqrt{155/61}$

Ans. (B)

Let A = (1, –1, 0), B = (–1, 0, 1).

Direction rations of segment AB are

2, –1, –1.

If ‘θ’ be the acute angle between segment AB and normal to plane,

$$\cos\theta = \frac{|2.2 - 1.1 - 1.6|}{\sqrt{4+1+36}\,.\sqrt{4+1+1}} = \frac{3}{\sqrt{246}}.$$

Length of projection

= (AB) sin θ

= √6.√1–9/246 = √237/61 units

61. Reflection of the line x–1/–1 = y–2/3 = z–3/1 in the plane x + y + z = 7 is:(Ref.P.K.Sharma_Three Dimen._P.C6.5Q.18)

(A) x–1/3 = y–2/1 = z–4/1 (B) x–1/–3 = y–2/–1 = z–4/1

(C) x–1/–3 = y–2/1 = z–4/–1 (D) x–1/3 = y–2/1 = z–4/1

Ans. (C)

Given line passes through (1, 2, 4) and this point also lies on the given plane.

Thus required line will be in the form of x–1/l = y–2/m = z–4/n.

Any point on the given line is

$(-r_1 + 1, 3r_1 + 2, r_1 + 4)$.

If $r_1 = 1$, this point becomes P = (0, 5, 5).

Let Q = (a, b, c) be the reflection of ‘P’ in the given plane, then

a/2.1 + b+5/2.1 + 5+c/2.1 = 7

i.e, a + b + c = 4,

and a/1 = b–5/1 = c–5/1 = λ(say)

=> $a = \lambda$, $b = 5 + \lambda$, $c = 5 + \lambda$

=> $10 + 3\lambda = 4$

=> $\lambda = -2$

Thus, Q = (–2, 3, 3)

Hence direction rations of reflected line are

–3, 1, –1

Thus it's equation is

$x-1/-3 = y-2/1 = z-4/-1$