Exercise 8.1

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer:

Applying Pythagoras theorem for ΔABC, we obtain

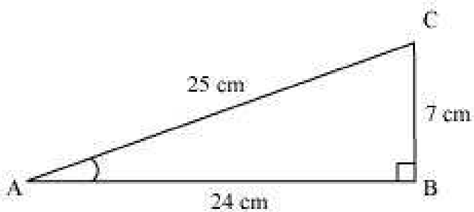
$AC^2 = AB^2 + BC^2$

$= (24\text{ cm})^2 + (7\text{ cm})^2$

$= (576 + 49)\text{ cm}^2$

$= 625\text{ cm}^2$

$\therefore AC = \sqrt{625}\text{ cm} = 25\text{ cm}$

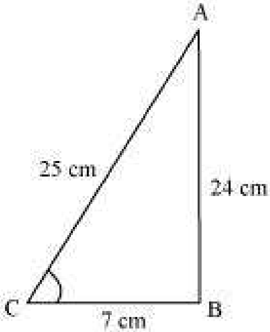


(i) sin A = [illegible]

$= \frac{7}{25}$

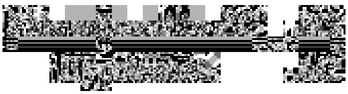
cos A = [illegible]

(ii)



sin C = 

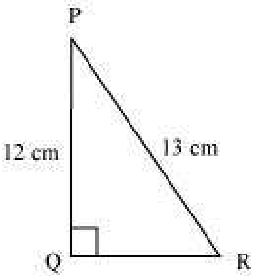
$= \frac{24}{25}$

cos C = 

$= \frac{7}{25}$

Question 2:

In the given figure find tan P − cot R



Answer:

Applying Pythagoras theorem for ΔPQR, we obtain

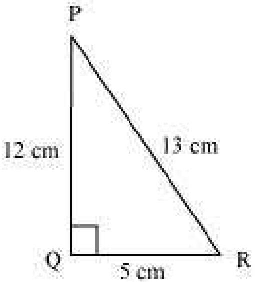
$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\ cm^2 = 144\ cm^2 + QR^2$

$25\ cm^2 = QR^2$

$QR = 5$ cm



[illegible]

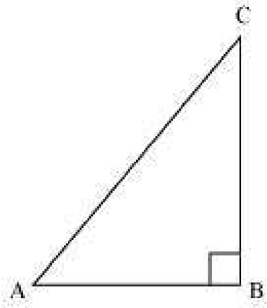
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer:

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

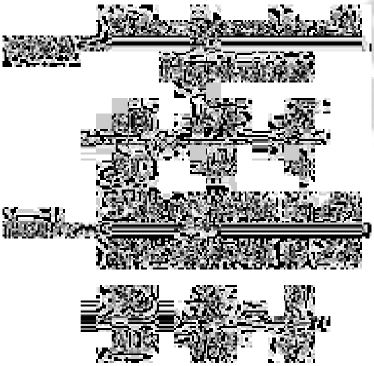
$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

$AB = \sqrt{7}k$

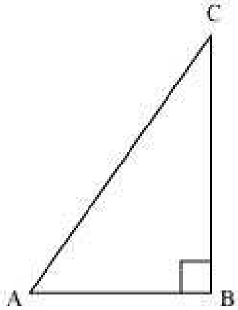


Question 4:

Given 15 cot A = 8. Find sin A and sec A

Answer:

Consider a right-angled triangle, right-angled at B.



[illegible]

[illegible]

It is given that,

cot A = $\frac{8}{15}$

$\frac{AB}{BC} = \frac{8}{15}$

Let AB be 8*k*.Therefore, BC will be 15*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

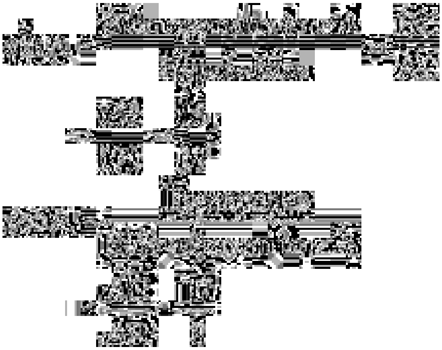
$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17*k*

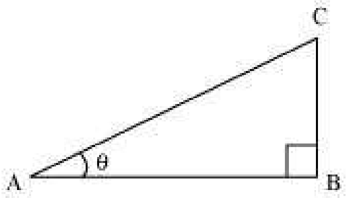


Question 5:

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer:

Consider a right-angle triangle ΔABC, right-angled at point B.



If AC is 13k, AB will be 12k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5k

[illegible]

[illegible]

[illegible]

[illegible]

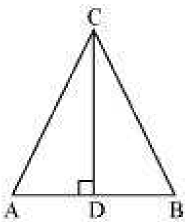
[illegible]

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer:

Let us consider a triangle ABC in which CD ⊥ AB.

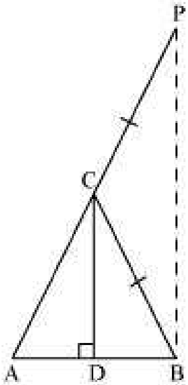


It is given that

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC} \quad \ldots (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

[illegible]

[illegible]

By using the converse of B.P.T,

CD||BP

⇒∠ACD = ∠CPB (Corresponding angles) … (3)

And, ∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

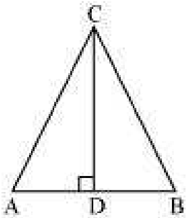
Therefore, the remaining angles should be equal.

∴∠CAD = ∠CBD

⇒ ∠A = ∠B

Alternatively,

Let us consider a triangle ABC in which CD ⊥ AB.



It is given that,

cos A = cos B

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow$ AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

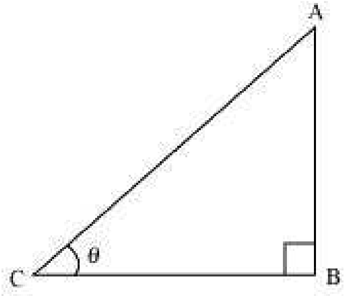
Question 7:

If $\cot\theta = \frac{7}{8}$, evaluate

(i) [illegible] (ii) $\cot^2\theta$

Answer:

Let us consider a right triangle ABC, right-angled at point B.



[illegible]

[illegible]

If BC is $7k$, then AB will be $8k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$AC = \sqrt{113}k$

[illegible]

(i) [illegible]

[illegible]

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2 \theta = (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Question 8:

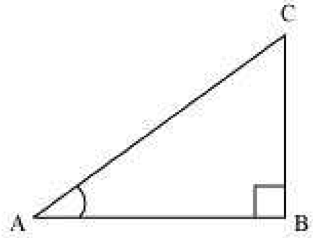
If 3 cot A = 4, Check whether [illegible]

Answer:

It is given that 3cot A = 4

Or, $\cot A = \frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



[illegible]

[illegible]

If AB is $4k$, then BC will be $3k$, where k is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$AC = 5k$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\therefore$

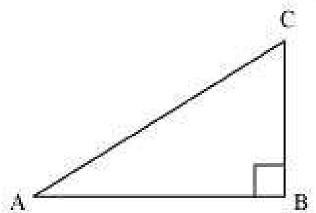
Question 9:

In ΔABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Answer:



[illegible]

If BC is k, then AB will be $\sqrt{3}k$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

= [illegible]

$= 3k^2 + k^2 = 4k^2$

$\therefore AC = 2k$

[illegible]

[illegible]

[illegible]

[illegible]

(i) sin A cos C + cos A sin C

[illegible]

[illegible]

(ii) cos A cos C − sin A sin C

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

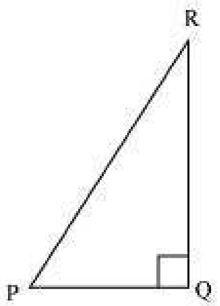
Answer:

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

[illegible]

[illegible]

[illegible]

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ for some value of angle A.

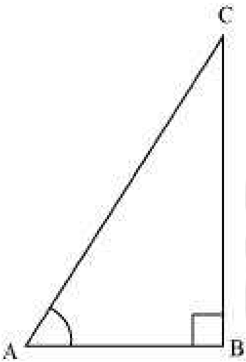
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

Answer:

(i) Consider a ΔABC, right-angled at B.



[illegible]

[illegible]

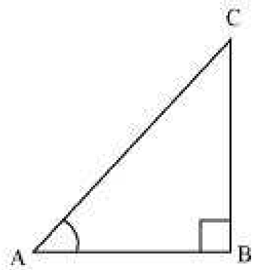
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



[illegible]

[illegible]

Let AC be 12k, AB will be 5k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides AC = 12k and AB = 5k,

BC should be such that,

AC − AB < BC < AC + AB

12k − 5k < BC < 12k + 5k

7k < BC < 17 k

However, BC = 10.9*k*. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

[illegible]

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Question 1:

Evaluate the following

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) [illegible]

(iv) [illegible]

(v) [illegible]

Answer:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

[illegible]

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

[illegible]

(iii) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(iv) [illegible]

(v)

Question 2:

Choose the correct option and justify your choice.

(i) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A). sin60°

(B). cos60°

(C). tan60°

(D). sin30°

(ii) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A). tan90°

(B). 1

(C). sin45°

(D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0°

(B). 30°

(C). 45°

(D). 60°

(iv) [illegible] =

(A). cos60°

(B). sin60°

(C). tan60°

(D). sin30°

Answer:

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

[illegible]

[illegible]

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

[illegible]

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

[illegible]

$= \sqrt{3}$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$;

0° < A + B ≤ 90°, A > B find A and B.

Answer:

$\tan(A+B) = \sqrt{3}$

⇒ [illegible]

⇒ A + B = 60 … (1)

$\tan(A-B) = \frac{1}{\sqrt{3}}$

⇒ tan (A − B) = tan30

⇒ A − B = 30 … (2)

On adding both equations, we obtain

2A = 90

⇒ A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Question 4:

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of sinθ increases as θ increases

(iii) The value of cos θ increases as θ increases

(iv) sinθ = cos θ for all values of θ

(v) cot A is not defined for A = 0°

Answer:

(i) sin (A + B) = sin A + sin B

Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90°

= 1

sin A + sin B = sin 30° + sin 60°

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Clearly, sin (A + B) ≠ sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as

sin 0° = 0

[illegible]

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

[illegible]

[illegible]

[illegible]

$\cos 90° = 0$

It can be observed that the value of cos θ does not increase in the interval of $0° < \theta < 90°$.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

[illegible] = undefined

Hence, the given statement is true.

Question 1:

Evaluate

(I) $\frac{\sin 18°}{\cos 72°}$

(II) $\frac{\tan 26°}{\cot 64°}$

(III) cos 48° − sin 42°

(IV)cosec 31° − sec 59°

Answer:

(I) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°}$

$= \frac{\cos 72°}{\cos 72°} = 1$

(II) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°}$

$= \frac{\cot 64°}{\cot 64°} = 1$

(III)cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(IV) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2:

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II)cos 38° cos 52° − sin 38° sin 52° = 0

Answer:

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$

$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

$= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ)$

$= (1) (1)$

$= 1$

(II) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

$= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ$

$= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$

$= 0$

Question 3:

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A.

Answer:

Given that,

$\tan 2A = \cot (A - 18^\circ)$

$\cot (90^\circ - 2A) = \cot (A - 18^\circ)$

$90^\circ - 2A = A - 18^\circ$

$108^\circ = 3A$

$A = 36^\circ$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90^\circ$

Answer:

Given that,

$\tan A = \cot B$

$\tan A = \tan (90^\circ - B)$

$A = 90^\circ - B$

A + B = 90°

Question 5:

If sec 4A = cosec (A− 20°), where 4A is an acute angle, find the value of A.

Answer:

Given that,

sec 4A = cosec (A − 20°)

cosec (90° − 4A) = cosec (A − 20°)

90° − 4A= A− 20°

110° = 5A

A = 22°

Question 6:

If A, Band C are interior angles of a triangle ABC then show that

[illegible]

Answer:

We know that for a triangle ABC,

∠ A + ∠B + ∠C = 180°

∠B + ∠C= 180° − ∠A

[illegible]

[illegible]

Question 7:

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer:

sin 67° + cos 75°

= $\sin(90° - 23°) + \cos(90° - 15°)$

= $\cos 23° + \sin 15°$

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer:

We know that,

[illegible]

[illegible]

[illegible]

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \dfrac{\sin A}{\cos A}$

However, $\cot A = \dfrac{\cos A}{\sin A}$

Therefore, $\tan A = \dfrac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

[illegible]

[illegible]

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

[illegible]

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

[illegible]

[illegible]

[illegible]

[illegible]

Question 3:

Evaluate

(i) [illegible]

(ii) sin25° cos65° + cos25° sin65°

Answer:

(i) [illegible]

[illegible]

[illegible]

[illegible]

$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

[illegible]

[illegible]

$= \sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0

(B) 1

(C) 2

(D) −1

(iii) (secA + tanA) (1 − sinA) =

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) −1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer:

(i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ [As $\sec^2 A - \tan^2 A = 1$]

= 9

Hence, alternative (B) is correct.

(ii)

(1 + tan θ + sec θ) (1 + cot θ − cosec θ)

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

[illegible]

[illegible]

[illegible]

= cosA

Hence, alternative (D) is correct.

(iv) [illegible]

[illegible]

[illegible]

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer:

(i) [illegible]

[illegible]

(ii) [illegible]

[illegible]

(iii) [illegible]

[illegible]

= $\sec\theta \operatorname{cosec}\theta$ +

= R.H.S.

(iv) [illegible]

[illegible]

= R.H.S

(v) [illegible]

Using the identity cosec^2 A = 1 + cot^2 A ,

L.H.S = [illegible]

[illegible]

= cosec A + cot A

= R.H.S

(vi) [illegible]

[illegible]

(vii) [illegible]

[illegible]

(viii) [illegible]

[illegible]

(ix) [illegible]

[illegible]

Hence, L.H.S = R.H.S

(x) [illegible]