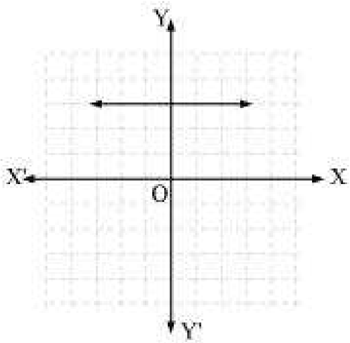
Exercise 2.1

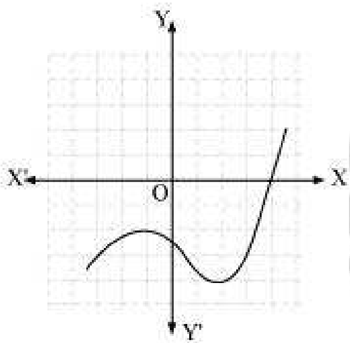
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

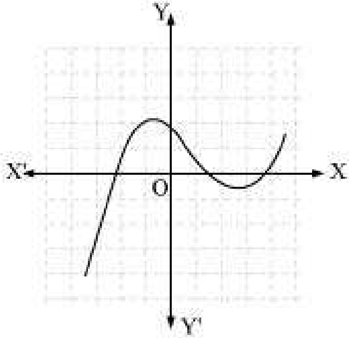
(i)



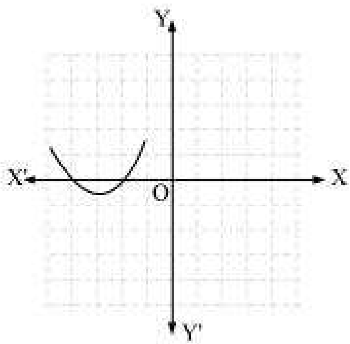
(ii)



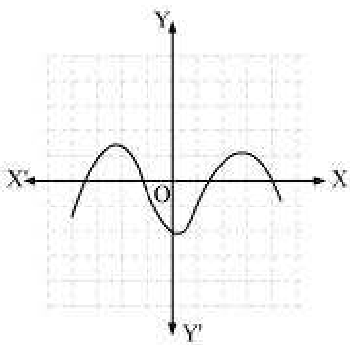
(iii)



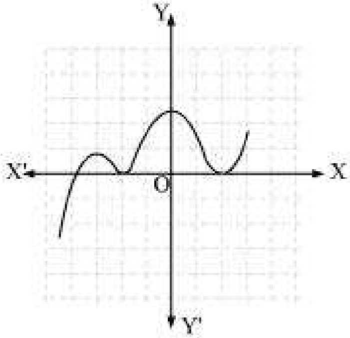
(iv)



(v)



(v)



Answer:

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Answer:

[illegible] [illegible]

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and −2.

Sum of zeroes = [illegible]

Product of zeroes [illegible]

[illegible] [illegible]

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = [illegible]

Product of zeroes [illegible]

[illegible]

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e.,

$x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = [illegible]

Product of zeroes = [illegible]

[illegible]

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes = [illegible]

Product of zeroes = [illegible]

[illegible]

The value of $t^2 - 15$ is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when

$t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = [illegible]

Product of zeroes = [illegible]

[illegible]

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e.,

when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

Sum of zeroes = [illegible]

Product of zeroes [illegible]

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$

(iv) $1, 1$ (v) $-\frac{1}{4}, \frac{1}{4}$ (vi) $4, 1$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

[illegible]

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

[illegible]

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

[illegible]

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

Therefore, the quadratic polynomial is x^2-4x+1.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

Answer:

[illegible]

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad - 2x \\
 & - \qquad\quad + \\
\hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \qquad + 6 \\
 & + \qquad\quad - \\
\hline
 & 7x - 9 \\
\hline
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

[illegible]

[illegible]

Quotient = $x^2 + x - 3$

Remainder = 8

[illegible]

$$
\begin{array}{r}
-x^2 - 2 \\
-x^2 + 2 \enclose{longdiv}{x^4 + 0.x^2 - 5x + 6} \\
x^4 - 2x^2 \\
- \quad + \\
\hline
2x^2 - 5x + 6 \\
2x^2 \qquad - 4 \\
- \qquad + \\
\hline
-5x + 10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

[illegible]

[illegible]

[illegible]

Answer:

[illegible]

$t^2 - 3 = t^2 + 0.t - 3$

[illegible]

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

[illegible]

[illegible]

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

[illegible]

$$
\begin{array}{r|l}
 & x^2-1 \\
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & -\quad +\quad - \\
\hline
 & -x^3 \qquad +3x+1 \\
 & -x^3 \qquad +3x-1 \\
 & + \qquad\quad -\quad + \\
\hline
 & 2 \\
\hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are [illegible].

Answer:

[illegible]

Since the two zeroes are [illegible],

[illegible] is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

[illegible]

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are [illegible], -1 and -1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer:

[illegible]

$g(x)$ = ? (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

[illegible]

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

[illegible]

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$

$= 6x^2 + 2x + 2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

[illegible]

[illegible]

Answer:

(i) [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 2$, $b = 1$, $c = -5$, $d = 2$

[illegible]

[illegible]

[illegible]

[illegible]

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 1, *b* = −4, *c* = 5, *d* = −2.

Verification of the relationship between zeroes and coefficient of the given polynomial

[illegible]

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

=2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

[illegible]

[illegible]

[illegible]

If a = 1, then b = −2, c = −7, d = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Answer:

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are a − b, a + a + b

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The zeroes are $1-b, 1, 1+b$.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

]It two zeroes of the polynomial [illegible] are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, [illegible] $= x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing [illegible] by $x^2 - 4x + 1$.

[illegible]

Clearly, [illegible] = [illegible]

It can be observed that $\left(x^2 - 2x - 35\right)$ is also a factor of the given polynomial.

And $\left(x^2 - 2x - 35\right)$ = $(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or x = 7 or −5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial [illegible] is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

[illegible] will be perfectly divisible by $x^2 - 2x + k$.

Let us divide [illegible] by $x^2 - 2x + k$

[illegible]

It can be observed that [illegible] will be 0.

Therefore, $(-10+2k)$ = 0 and $(10-a-8k+k^2)$ = 0

For $(-10+2k)$ = 0,

2 k =10

And thus, k = 5

For $(10-a-8k+k^2)$ = 0

10 − a − 8 × 5 + 25 = 0

10 − a − 40 + 25 = 0

− 5 − a = 0

Therefore, a = −5

Hence, k = 5 and a = −5