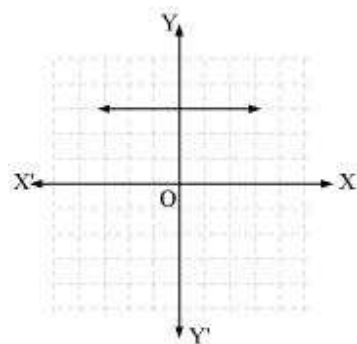


## Exercise 2.1

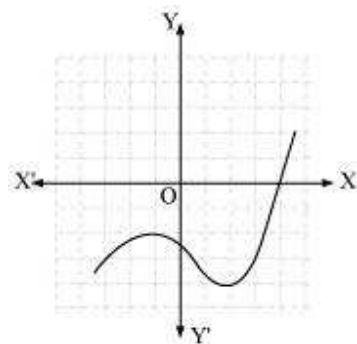
## Question 1:

The graphs of  $y = p(x)$  are given in following figure, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$ , in each case.

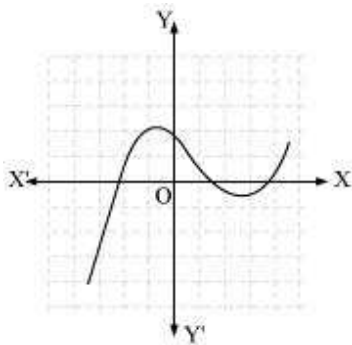
(i)



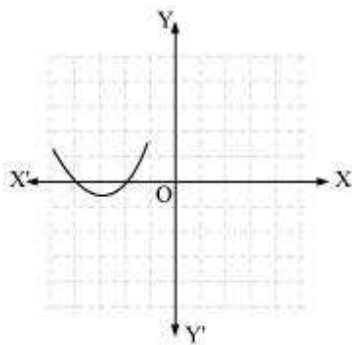
(ii)



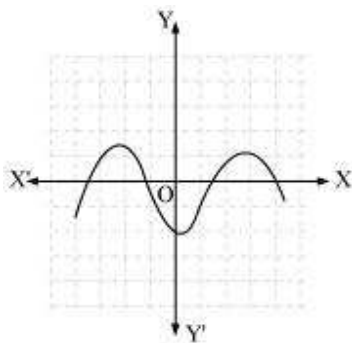
(iii)



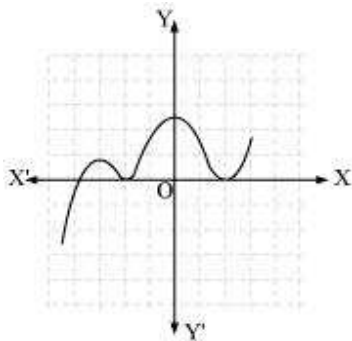
(iv)



(v)



(v)



Answer:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

## Exercise 2.2

### Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i)  $x^2 - 2x - 8$  (ii)  $4s^2 - 4s + 1$  (iii)  $6x^2 - 3 - 7x$

(iv)  $4u^2 + 8u$  (v)  $t^2 - 15$  (vi)  $3x^2 - x - 4$

Answer:



The value of  $x^2 - 2x - 8$  is zero when  $x - 4 = 0$  or  $x + 2 = 0$ , i.e., when  $x = 4$  or  $x = -2$

Therefore, the zeroes of  $x^2 - 2x - 8$  are 4 and  $-2$ .

Sum of zeroes =



Product of zeroes



The value of  $4s^2 - 4s + 1$  is zero when  $2s - 1 = 0$ , i.e.,  $s = \frac{1}{2}$

Therefore, the zeroes of  $4s^2 - 4s + 1$  are  $\frac{1}{2}$  and  $\frac{1}{2}$ .

Sum of zeroes =



Product of zeroes

$$\left(-\frac{1}{3}\right) \times \left(\frac{3}{2}\right) = -\frac{1}{2}$$

The value of  $6x^2 - 3 - 7x$  is zero when  $3x + 1 = 0$  or  $2x - 3 = 0$ , i.e.,

$$x = -\frac{1}{3} \text{ or } x = \frac{3}{2}$$

Therefore, the zeroes of  $6x^2 - 3 - 7x$  are  $-\frac{1}{3}$  and  $\frac{3}{2}$ .

Sum of zeroes =

Product of zeroes =

The value of  $4u^2 + 8u$  is zero when  $4u = 0$  or  $u + 2 = 0$ , i.e.,  $u = 0$  or  $u = -2$

Therefore, the zeroes of  $4u^2 + 8u$  are 0 and  $-2$ .

Sum of zeroes =

Product of zeroes =

The value of  $t^2 - 15$  is zero when  $t - \sqrt{15} = 0$  or  $t + \sqrt{15} = 0$ , i.e., when  $t = \sqrt{15}$  or  $t = -\sqrt{15}$

Therefore, the zeroes of  $t^2 - 15$  are  $\sqrt{15}$  and  $-\sqrt{15}$ .

Sum of zeroes =

Product of zeroes =

The value of  $3x^2 - x - 4$  is zero when  $3x - 4 = 0$  or  $x + 1 = 0$ , i.e., when  $x = \frac{4}{3}$  or  $x = -1$

Therefore, the zeroes of  $3x^2 - x - 4$  are  $\frac{4}{3}$  and  $-1$ .

Sum of zeroes =

Product of zeroes

### Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

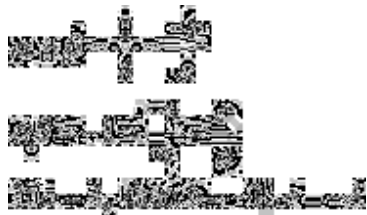
(i)  $\frac{1}{4}, -1$  (ii)  $\sqrt{2}, \frac{1}{3}$  (iii)  $0, \sqrt{5}$

(iv)  $1, 1$  (v)  $\frac{1}{4}, \frac{1}{4}$

Answer:

(i)  $\frac{1}{4}, -1$

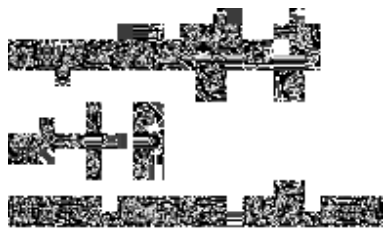
Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .



Therefore, the quadratic polynomial is  $4x^2 - x - 4$ .

(ii)  $\sqrt{2}, \frac{1}{3}$

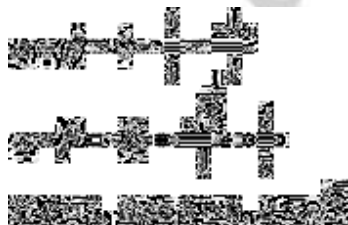
Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .



Therefore, the quadratic polynomial is  $3x^2 - 3\sqrt{2}x + 1$ .

(iii)  $0, \sqrt{5}$

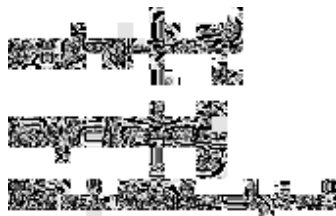
Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .



Therefore, the quadratic polynomial is  $x^2 + \sqrt{5}$ .

(iv) 1, 1

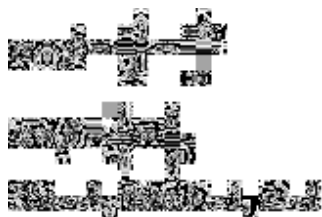
Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .



Therefore, the quadratic polynomial is  $x^2 - x + 1$ .

(v)  $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be  $ax^2 + bx + c$ , and its zeroes be  $\alpha$  and  $\beta$ .



Therefore, the quadratic polynomial is  $4x^2 + x + 1$ .

(vi) 4, 1

Let the polynomial be  $ax^2 + bx + c$ .



Therefore, the quadratic polynomial is  $x^2 - 4x + 1$ .



## Exercise 2.3

### Question 1:

Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following:

- (i)  $\frac{x^3 - 3x^2 + 5x - 3}{x^2 - 2}$
- (ii)  $\frac{x^3 - 3x^2 + 5x - 3}{x^2 - 2}$
- (iii)  $\frac{x^3 - 3x^2 + 5x - 3}{x^2 - 2}$

Answer:

$$\frac{x^3 - 3x^2 + 5x - 3}{x^2 - 2}$$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3 \phantom{-3x^2} -2x} \phantom{-3} \\ -3x^2+7x-3 \\ \underline{-3x^2 \phantom{+7x} +6} \\ 7x-9 \end{array}$$

Quotient =  $x - 3$

Remainder =  $7x - 9$

$$\frac{x^3 - 3x^2 + 5x - 3}{x^2 - 2}$$

$$\begin{array}{r}
 x^2 + x - 3 \\
 x^2 + x - 3 \overline{) x^3 + x^2 - 3x + 8} \\
 \underline{-(x^2 + x - 3)} \phantom{+ 8} \\
 2x + 11 \\
 \underline{-(2x^2 + 2x - 6)} \\
 -2x^2 - x + 17 \\
 \underline{-(2x^2 + 2x - 6)} \\
 -3x + 23 \\
 \underline{-(3x^2 + 3x - 9)} \\
 6x^2 - 6x + 32 \\
 \underline{-(6x^2 + 6x - 18)} \\
 -12x + 50 \\
 \underline{-(12x^2 + 12x - 36)} \\
 12x^2 - 18x + 86 \\
 \underline{-(12x^2 + 12x - 36)} \\
 -30x + 122 \\
 \underline{-(30x^2 + 30x - 90)} \\
 30x^2 - 60x + 212 \\
 \underline{-(30x^2 + 30x - 90)} \\
 -60x + 302 \\
 \underline{-(60x^2 + 60x - 180)} \\
 60x^2 - 120x + 482 \\
 \underline{-(60x^2 + 60x - 180)} \\
 -120x + 662 \\
 \underline{-(120x^2 + 120x - 360)} \\
 120x^2 - 240x + 1022 \\
 \underline{-(120x^2 + 120x - 360)} \\
 -240x + 1382 \\
 \underline{-(240x^2 + 240x - 720)} \\
 240x^2 - 480x + 2102 \\
 \underline{-(240x^2 + 240x - 720)} \\
 -480x + 2822 \\
 \underline{-(480x^2 + 480x - 1440)} \\
 480x^2 - 960x + 4262 \\
 \underline{-(480x^2 + 480x - 1440)} \\
 -960x + 5702 \\
 \underline{-(960x^2 + 960x - 2880)} \\
 960x^2 - 1920x + 8582 \\
 \underline{-(960x^2 + 960x - 2880)} \\
 -1920x + 11462 \\
 \underline{-(1920x^2 + 1920x - 5760)} \\
 1920x^2 - 3840x + 17222 \\
 \underline{-(1920x^2 + 1920x - 5760)} \\
 -3840x + 22982 \\
 \underline{-(3840x^2 + 3840x - 11520)} \\
 3840x^2 - 7680x + 34502 \\
 \underline{-(3840x^2 + 3840x - 11520)} \\
 -7680x + 46022 \\
 \underline{-(7680x^2 + 7680x - 38400)} \\
 7680x^2 - 15360x + 84422 \\
 \underline{-(7680x^2 + 7680x - 38400)} \\
 -15360x + 122822 \\
 \underline{-(15360x^2 + 15360x - 76800)} \\
 15360x^2 - 30720x + 209622 \\
 \underline{-(15360x^2 + 15360x - 76800)} \\
 -30720x + 286422 \\
 \underline{-(30720x^2 + 30720x - 153600)} \\
 30720x^2 - 61440x + 440022 \\
 \underline{-(30720x^2 + 30720x - 153600)} \\
 -61440x + 593622 \\
 \underline{-(61440x^2 + 61440x - 307200)} \\
 61440x^2 - 122880x + 900822 \\
 \underline{-(61440x^2 + 61440x - 307200)} \\
 -122880x + 1208022 \\
 \underline{-(122880x^2 + 122880x - 614400)} \\
 122880x^2 - 245760x + 1822422 \\
 \underline{-(122880x^2 + 122880x - 614400)} \\
 -245760x + 2436822 \\
 \underline{-(245760x^2 + 245760x - 1228800)} \\
 245760x^2 - 491520x + 3665622 \\
 \underline{-(245760x^2 + 245760x - 1228800)} \\
 -491520x + 4894422 \\
 \underline{-(491520x^2 + 491520x - 2457600)} \\
 491520x^2 - 983040x + 7352022 \\
 \underline{-(491520x^2 + 491520x - 2457600)} \\
 -983040x + 9809622 \\
 \underline{-(983040x^2 + 983040x - 4915200)} \\
 983040x^2 - 1966080x + 14724822 \\
 \underline{-(983040x^2 + 983040x - 4915200)} \\
 -1966080x + 19640022 \\
 \underline{-(1966080x^2 + 1966080x - 9830400)} \\
 1966080x^2 - 3932160x + 29470422 \\
 \underline{-(1966080x^2 + 1966080x - 9830400)} \\
 -3932160x + 39300822 \\
 \underline{-(3932160x^2 + 3932160x - 19660800)} \\
 3932160x^2 - 7864320x + 58961622 \\
 \underline{-(3932160x^2 + 3932160x - 19660800)} \\
 -7864320x + 78622422 \\
 \underline{-(7864320x^2 + 7864320x - 39321600)} \\
 7864320x^2 - 15728640x + 117944022 \\
 \underline{-(7864320x^2 + 7864320x - 39321600)} \\
 -15728640x + 157265622 \\
 \underline{-(15728640x^2 + 15728640x - 78643200)} \\
 15728640x^2 - 31457280x + 315708822 \\
 \underline{-(15728640x^2 + 15728640x - 78643200)} \\
 -31457280x + 394352022 \\
 \underline{-(31457280x^2 + 31457280x - 157286400)} \\
 31457280x^2 - 62914560x + 551638422 \\
 \underline{-(31457280x^2 + 31457280x - 157286400)} \\
 -62914560x + 708924822 \\
 \underline{-(62914560x^2 + 62914560x - 314572800)} \\
 62914560x^2 - 125829120x + 1023507622 \\
 \underline{-(62914560x^2 + 62914560x - 314572800)} \\
 -125829120x + 1338080422 \\
 \underline{-(125829120x^2 + 125829120x - 629145600)} \\
 125829120x^2 - 251658240x + 1967226022 \\
 \underline{-(125829120x^2 + 125829120x - 629145600)} \\
 -251658240x + 2596371622 \\
 \underline{-(251658240x^2 + 251658240x - 1258291200)} \\
 251658240x^2 - 503316480x + 3854662822 \\
 \underline{-(251658240x^2 + 251658240x - 1258291200)} \\
 -503316480x + 5112954022 \\
 \underline{-(503316480x^2 + 503316480x - 2516582400)} \\
 503316480x^2 - 1006632960x + 7629536422 \\
 \underline{-(503316480x^2 + 503316480x - 2516582400)} \\
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 \underline{-(1006632960x^2 + 1006632960x - 5033164800)} \\
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 \underline{-(1006632960x^2 + 1006632960x - 5033164800)} \\
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 \underline{-(2013265920x^2 + 2013265920x - 10066329600)} \\
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 \underline{-(4026531840x^2 + 4026531840x - 20132659200)} \\
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 \underline{-(4026531840x^2 + 4026531840x - 20132659200)} \\
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 8053063680x^2 - 16106127360x + 120868754422 \\
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 \underline{-(128849018880x^2 + 128849018880x - 644245094400)} \\
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 \underline{-(257698037760x^2 + 257698037760x - 1288490188800)} \\
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 \underline{-(257698037760x^2 + 257698037760x - 1288490188800)} \\
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 \underline{-(515396075520x^2 + 515396075520x - 2576980377600)} \\
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 \underline{-(515396075520x^2 + 515396075520x - 2576980377600)} \\
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 \underline{-(1030792151040x^2 + 1030792151040x - 5153960755200)} \\
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 \underline{-(2061584302080x^2 + 2061584302080x - 10307921510400)} \\
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 \underline{-(8246337208320x^2 + 8246337208320x - 41231686041600)} \\
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 \underline{-(65970697666560x^2 + 65970697666560x - 329853488332800)} \\
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 \underline{-(263882790666240x^2 + 263882790666240x - 1319413953331200)} \\
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 \underline{-(263882790666240x^2 + 263882790666240x - 1319413953331200)} \\
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 \underline{-(527765581332480x^2 + 527765581332480x - 2638827906662400)} \\
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 \underline{-(4222124650659840x^2 + 4222124650659840x - 21110623253299200)} \\
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 \underline{-(4222124650659840x^2 + 4222124650659840x - 21110623253299200)} \\
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 \underline{-(8444249301319680x^2 + 8444249301319680x - 42221246506598400)} \\
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 \underline{-(8444249301319680x^2 + 8444249301319680x - 42221246506598400)} \\
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 \underline{-(16888498602639360x^2 + 16888498602639360x - 84442493013196800)} \\
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 \underline{-(16888498602639360x^2 + 16888498602639360x - 84442493013196800)} \\
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 \underline{-(33776997205278720x^2 + 33776997205278720x - 168884986026393600)} \\
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 \underline{-(33776997205278720x^2 + 33776997205278720x - 168884986026393600)} \\
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 \underline{-(67553994410557440x^2 + 67553994410557440x - 337769972052787200)} \\
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 \underline{-(67553994410557440x^2 + 67553994410557440x - 337769972052787200)} \\
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 \underline{-(135107988821114880x^2 + 135107988821114880x - 675539944105574400)} \\
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 \underline{-(135107988821114880x^2 + 135107988821114880x - 675539944105574400)} \\
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 \underline{-(270215977642229760x^2 + 270215977642229760x - 1351079888211148800)} \\
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 \underline{-(270215977642229760x^2 + 270215977642229760x - 1351079888211148800)} \\
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 \underline{-(540431955284459520x^2 + 540431955284459520x - 2702159776422297600)} \\
 540431955284459520x^2 - 1080863910568919040x + 8108698052761920022 \\
 \underline{-(540431955284459520x^2 + 540431955284459520x - 2702159776422297600)} \\
 -1080863910568919040x + 10808724378017920022 \\
 \underline{-(1080863910568919040x^2 + 1080863910568919040x - 5404319552844595200)} \\
 1080863910568919040x^2 - 2161727821137838080x + 16217448756035840022 \\
 \underline{-(1080863910568919040x^2 + 1080863910568919040x - 5404319552844595200)} \\
 -2161727821137838080x + 21617461189188800022 \\
 \underline{-(2161727821137838080x^2 + 2161727821137838080x - 10808639105689190400)} \\
 2161727821137838080x^2 - 4323455642275676160x + 32434922378377600022 \\
 \underline{-(2161727821137838080x^2 + 2161727821137838080x - 10808639105689190400)} \\
 -4323455642275676160x + 43234735769434400022 \\
 \underline{-(4323455642275676160x^2 + 4323455642275676160x - 21617278211378380800)} \\
 4323455642275676160x^2 - 8646911284551352320x + 86469471538868800022 \\
 \underline{-(4323455642275676160x^2 + 4323455642275676160x - 21617278211378380800)} \\
 -8646911284551352320x + 86469687750240000022 \\
 \underline{-(8646911284551352320x^2 + 8646911284551352320x - 43234556422756761600)} \\
 8646911284551352320x^2 - 17293822569102704640x + 172938365500480000022 \\
 \underline{-(8646911284551352320x^2 + 8646911284551352320x - 43234556422756761600)} \\
 -17293822569102704640x + 172938797827907200022 \\
 \underline{-(17293822569102704640x^2 + 17293822569102704640x - 86469112845513523200)} \\
 17293822569102704640x^2 - 34587645138205409280x + 345877165655814400022 \\
 \underline{-(17293822569102704640x^2 + 17293822569102704640x - 86469112845513523200)} \\
 -34587645138205409280x + 345878030101659200022 \\
 \underline{-(34587645138205409280x^2 + 34587645138205409280x - 172938225691027046400)} \\
 34587645138205409280x^2 - 69175290276410818560x + 691756060203318400022 \\
 \underline{-(34587645138205409280x^2 + 34587645138205409280x - 172938225691027046400)} \\
 -69175290276410818560x + 691757789494409600022 \\
 \underline{-(69175290276410818560x^2 + 69175290276410818560x - 345876451382054092800)} \\
 69175290276410818560x^2 - 138350580552821637120x + 1383514718988819200022 \\
 \underline{-(69175290276410818560x^2 + 69175290276410818560x - 345876451382054092800)} \\
 -138350580552821637120x + 1383518177170000000022 \\
 \underline{-(138350580552821637120x^2 + 138350580552821637120x - 691752902764108185600)} \\
 138350580552821637120x^2 - 276701161105643274240x + 2767036354340000000022 \\
 \underline{-(138350580552821637120x^2 + 138350580552821637120x - 691752902764108185600)} \\
 -276701161105643274240x + 2767043273360000000022 \\
 \underline{-(276701161105643274240x^2 + 276701161105643274240x - 1383505805528216371200)} \\
 276701161105643274240x^2 - 553402322211286548480x + 5534056546720000000022 \\
 \underline{-(276701161105643274240x^2 + 276701161105643274240x - 1383505805528216371200)} \\
 -553402322211286548480x + 5534068382348800000022 \\
 \underline{-(553402322211286548480x^2 + 553402322211286548480x - 2767011611056432742400)} \\
 553402322211286548480x^2 - 1106804644422573096960x + 11068096764697600000022 \\
 \underline{-(553402322211286548480x^2 + 553402322211286548480x - 2767011611056432742400)} \\
 -1106804644422573096960x + 11068124435808000000022 \\
 \underline{-(1106804644422573096960x^2 + 1106804644422573096960x - 5534023222112865484800)} \\
 1106804644422573096960x^2 - 2213609288845146193920x + 22136150871616000000022 \\
 \underline{-(1106804644422573096960x^2 + 1106804644422573096960x - 5534023222112865484800)} \\
 -2213609288845146193920x + 22136206212800000000022 \\
 \underline{-(2213609288845146193920x^2 + 2213609288845146193920x - 11068046444225730969600)} \\
 221$$

## Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

$$\begin{array}{r} 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ t^2 - 3 \end{array}$$

Answer:

$$\begin{array}{r} 2t^2 + 3t + 4 \\ t^2 - 3 \end{array}$$

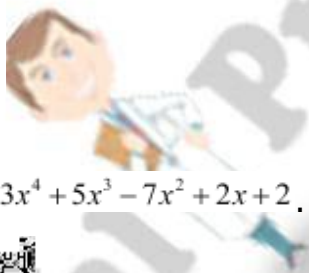
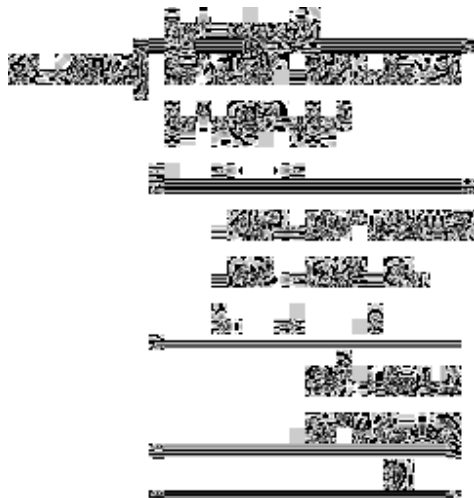
$$t^2 - 3 = t^2 + 0t - 3$$

$$\begin{array}{r} 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ -(2t^4 + 0t^3 - 6t^2) \hline \phantom{2t^4 + } 3t^3 + 4t^2 - 9t - 12 \\ \phantom{2t^4 + } -(3t^3 + 0t^2 - 9t) \hline \phantom{2t^4 + } \phantom{3t^3 + } 4t^2 + 9t - 12 \\ \phantom{2t^4 + } \phantom{3t^3 + } -(4t^2 + 0t - 12) \hline \phantom{2t^4 + } \phantom{3t^3 + } \phantom{4t^2 + } 9t + 0 \end{array}$$

Since the remainder is 0,

Hence,  $t^2 - 3$  is a factor of  $2t^4 + 3t^3 - 2t^2 - 9t - 12$ .

$$\begin{array}{r} 2t^2 + 3t + 4 \\ t^2 - 3 \end{array}$$



Since the remainder is 0,

Hence,  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ .



$$\begin{array}{r}
 x^2 - 1 \\
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \phantom{+ 3x + 1} \\
 -x^3 \phantom{+ 3x + 1} \\
 \underline{-x^3 \phantom{+ 3x - 1}} \phantom{+ 3x + 1} \\
 + \phantom{+ 3x - 1} \phantom{+ 3x + 1} \\
 \underline{\phantom{+ 3x - 1} \phantom{+ 3x + 1}} \\
 2
 \end{array}$$

Since the remainder  $\neq 0$ ,

Hence,  $x^3 - 3x + 1$  is not a factor of  $x^5 - 4x^3 + x^2 + 3x + 1$ .

### Question 3:

Obtain all other zeroes of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ , if two of its zeroes are

$$-1 \pm 2i$$

Answer:

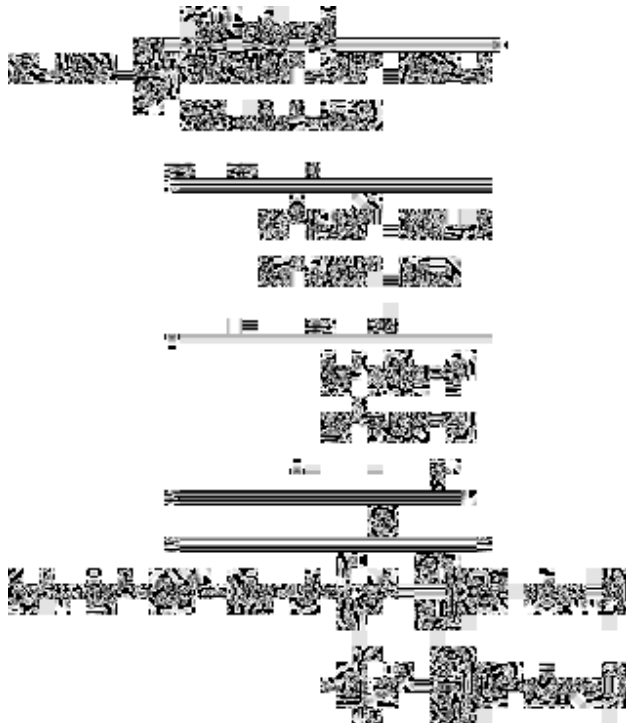
$$-1 \pm 2i, -1 \pm 2i$$

Since the two zeroes are  $-1 \pm 2i$ ,

$$(x + 1 - 2i)(x + 1 + 2i)$$

is a factor of  $3x^4 + 6x^3 - 2x^2 - 10x - 5$ .

Therefore, we divide the given polynomial by  $x^2 - \frac{5}{3}$ .



We factorize  $x^2 + 2x + 1$

$$= (x+1)^2$$

Therefore, its zero is given by  $x + 1 = 0$

$$x = -1$$

As it has the term  $(x+1)^2$ , therefore, there will be 2 zeroes at  $x = -1$ .

Hence, the zeroes of the given polynomial are  $-1$  and  $-1$ .

**Question 4:**

On dividing  $x^3 - 3x^2 + x + 2$  by a polynomial  $g(x)$ , the quotient and remainder were  $x - 2$  and  $-2x + 4$ , respectively. Find  $g(x)$ .

Answer:



$g(x) = ?$  (Divisor)

Quotient =  $(x - 2)$

Remainder =  $(-2x + 4)$

Dividend = Divisor  $\times$  Quotient + Remainder



$g(x)$  is the quotient when we divide  $(x^3 - 3x^2 + 3x - 2)$  by  $(x - 2)$



Question 5:

Give examples of polynomial  $p(x)$ ,  $g(x)$ ,  $q(x)$  and  $r(x)$ , which satisfy

the division algorithm and

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(i)  $\deg p(x) = \deg q(x)$

(ii)  $\deg q(x) = \deg r(x)$

(iii)  $\deg r(x) = 0$

Answer:

According to the division algorithm, if  $p(x)$  and  $g(x)$  are two polynomials with

$g(x) \neq 0$ , then we can find polynomials  $q(x)$  and  $r(x)$  such that

$$p(x) = g(x) \times q(x) + r(x),$$

where  $r(x) = 0$  or degree of  $r(x) < \text{degree of } g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i)  $\deg p(x) = \deg q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant ( i.e., when any polynomial is divided by a constant).

Let us assume the division of  $6x^2 + 2x + 2$  by 2.

Here,  $p(x) = 6x^2 + 2x + 2$

$$g(x) = 2$$

$$q(x) = 3x^2 + x + 1 \text{ and } r(x) = 0$$

Degree of  $p(x)$  and  $q(x)$  is the same i.e., 2.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$$

$$= 6x^2 + 2x + 2$$

Thus, the division algorithm is satisfied.



(ii)  $\deg q(x) = \deg r(x)$

Let us assume the division of  $x^3 + x$  by  $x^2$ ,

Here,  $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$  and  $r(x) = x$

Clearly, the degree of  $q(x)$  and  $r(x)$  is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)  $\deg r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of  $x^3 + 1$  by  $x^2$ .

Here,  $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$  and  $r(x) = 1$

Clearly, the degree of  $r(x)$  is 0.

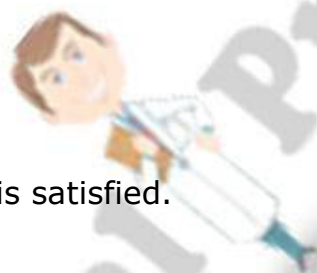
Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.



## Exercise 2.4

### Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

$$x^3 - 2x^2 - 13x + 20$$

Answer:

(i)

$$x^3 - 2x^2 - 13x + 20$$

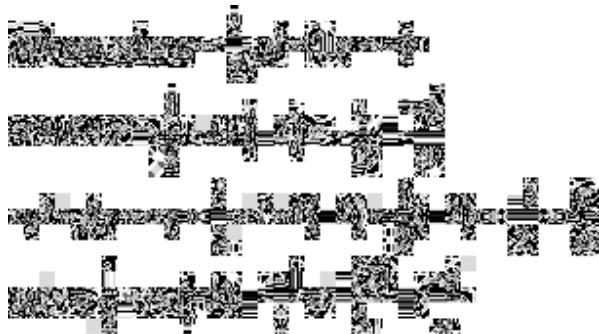
$$= (x - \frac{1}{2})(x - 1)(x + 2)$$

$$= (x - \frac{1}{2})(x^2 + x - 2)$$

$$= (x - \frac{1}{2})(x - 1)(x + 2)$$

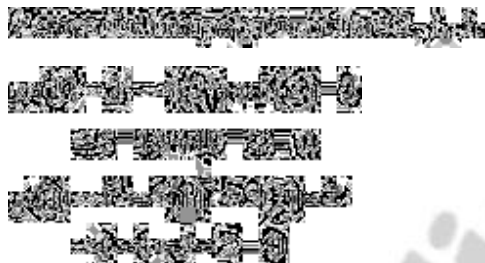
Therefore,  $\frac{1}{2}$ , 1, and  $-2$  are the zeroes of the given polynomial.

Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we obtain  $a = 2$ ,  
 $b = 1$ ,  $c = -5$ ,  $d = 2$



Therefore, the relationship between the zeroes and the coefficients is verified.

(ii)  $p(x) = x^3 - 4x^2 + 5x - 2$



Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with  $ax^3 + bx^2 + cx + d$ , we obtain  $a = 1$ ,  $b = -4$ ,  $c = 5$ ,  $d = -2$ .

Verification of the relationship between zeroes and coefficient of the given polynomial



Multiplication of zeroes taking two at a time =  $(2)(1) + (1)(1) + (2)(1)$

$$= 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$$

$$\text{Multiplication of zeroes} = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$$

Hence, the relationship between the zeroes and the coefficients is verified.

### Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2,  $-7$ ,  $-14$  respectively.

Answer:

Let the polynomial be  $ax^3 + bx^2 + cx + d$  and the zeroes be  $\alpha, \beta$ , and  $\gamma$ .

It is given that

$$\begin{aligned} \alpha + \beta + \gamma &= 2 \\ \alpha\beta + \beta\gamma + \alpha\gamma &= -7 \\ \alpha\beta\gamma &= -14 \end{aligned}$$

If  $a = 1$ , then  $b = -2$ ,  $c = -7$ ,  $d = 14$

Hence, the polynomial is  $x^3 - 2x^2 - 7x + 14$ .

### Question 3:

If the zeroes of polynomial  $x^3 - 3x^2 + x + 1$  are  $a-b, a, a+b$ , find  $a$  and  $b$ .

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are  $a - b, a, a + b$

Comparing the given polynomial with  $x^3 + px^2 + qx + r$ , we obtain

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$$p = 1, q = -3, r = 1, t = 1$$

The zeroes are  $1-b, 1, 1+b$ .

Hence,  $a = 1$  and  $b = \sqrt{2}$  or  $-\sqrt{2}$ .

**Question 4:**

If two zeroes of the polynomial are  $2 \pm \sqrt{3}$ , find other zeroes.

**Answer:**

Given that  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  are zeroes of the given polynomial.

Therefore,  $x^2 + 4 - 4x - 3$   
 $= x^2 - 4x + 1$  is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing  $x^3 - 3x^2 - 28x + 28$  by  $x^2 - 4x + 1$ .

$$\begin{array}{r}
 x^3 - 3x^2 - 28x + 28 \\
 \underline{-(x^2 - 4x + 1)} \phantom{+ 1} \\
 2x^2 - 24x + 27 \phantom{+ 1} \\
 \underline{-(2x^2 - 8x + 2)} \phantom{+ 1} \\
 -16x + 25 \phantom{+ 1} \\
 \underline{-(-16x + 64 - 16)} \phantom{+ 1} \\
 39 - 64 \phantom{+ 1} \\
 \underline{-(39 - 156 + 39)} \phantom{+ 1} \\
 121 - 39 \phantom{+ 1} \\
 \underline{-(121 - 484 + 121)} \phantom{+ 1} \\
 603 - 121 \phantom{+ 1} \\
 \underline{-(603 - 2412 + 603)} \phantom{+ 1} \\
 1810 - 603 \phantom{+ 1} \\
 \underline{-(1810 - 7240 + 1810)} \phantom{+ 1} \\
 5430 - 1810 \phantom{+ 1} \\
 \underline{-(5430 - 21720 + 5430)} \phantom{+ 1} \\
 11320 - 5430 \phantom{+ 1} \\
 \underline{-(11320 - 45280 + 11320)} \phantom{+ 1} \\
 45280 - 11320 \phantom{+ 1} \\
 \underline{-(45280 - 181120 + 45280)} \phantom{+ 1} \\
 181120 - 45280 \phantom{+ 1} \\
 \underline{-(181120 - 724480 + 181120)} \phantom{+ 1} \\
 724480 - 181120 \phantom{+ 1} \\
 \underline{-(724480 - 2907920 + 724480)} \phantom{+ 1} \\
 2907920 - 724480 \phantom{+ 1} \\
 \underline{-(2907920 - 11631680 + 2907920)} \phantom{+ 1} \\
 11631680 - 2907920 \phantom{+ 1} \\
 \underline{-(11631680 - 46526720 + 11631680)} \phantom{+ 1} \\
 46526720 - 11631680 \phantom{+ 1} \\
 \underline{-(46526720 - 186106880 + 46526720)} \phantom{+ 1} \\
 186106880 - 46526720 \phantom{+ 1} \\
 \underline{-(186106880 - 745670720 + 186106880)} \phantom{+ 1} \\
 745670720 - 186106880 \phantom{+ 1} \\
 \underline{-(745670720 - 2982682880 + 745670720)} \phantom{+ 1} \\
 2982682880 - 745670720 \phantom{+ 1} \\
 \underline{-(2982682880 - 11922723520 + 2982682880)} \phantom{+ 1} \\
 11922723520 - 2982682880 \phantom{+ 1} \\
 \underline{-(11922723520 - 47782717440 + 11922723520)} \phantom{+ 1} \\
 47782717440 - 11922723520 \phantom{+ 1} \\
 \underline{-(47782717440 - 191130873600 + 47782717440)} \phantom{+ 1} \\
 191130873600 - 47782717440 \phantom{+ 1} \\
 \underline{-(191130873600 - 766683461120 + 191130873600)} \phantom{+ 1} \\
 766683461120 - 191130873600 \phantom{+ 1} \\
 \underline{-(766683461120 - 1238693537760 + 766683461120)} \phantom{+ 1} \\
 1238693537760 - 766683461120 \phantom{+ 1} \\
 \underline{-(1238693537760 - 1957387075520 + 1238693537760)} \phantom{+ 1} \\
 1957387075520 - 1238693537760 \phantom{+ 1} \\
 \underline{-(1957387075520 - 3134565718400 + 1957387075520)} \phantom{+ 1} \\
 3134565718400 - 1957387075520 \phantom{+ 1} \\
 \underline{-(3134565718400 - 6211702877440 + 3134565718400)} \phantom{+ 1} \\
 6211702877440 - 3134565718400 \phantom{+ 1} \\
 \underline{-(6211702877440 - 12386935377600 + 6211702877440)} \phantom{+ 1} \\
 12386935377600 - 6211702877440 \phantom{+ 1} \\
 \underline{-(12386935377600 - 24773670755200 + 12386935377600)} \phantom{+ 1} \\
 24773670755200 - 12386935377600 \phantom{+ 1} \\
 \underline{-(24773670755200 - 49547341510400 + 24773670755200)} \phantom{+ 1} \\
 49547341510400 - 24773670755200 \phantom{+ 1} \\
 \underline{-(49547341510400 - 99094683020800 + 49547341510400)} \phantom{+ 1} \\
 99094683020800 - 49547341510400 \phantom{+ 1} \\
 \underline{-(99094683020800 - 198189366041600 + 99094683020800)} \phantom{+ 1} \\
 198189366041600 - 99094683020800 \phantom{+ 1} \\
 \underline{-(198189366041600 - 396378732083200 + 198189366041600)} \phantom{+ 1} \\
 396378732083200 - 198189366041600 \phantom{+ 1} \\
 \underline{-(396378732083200 - 792757464166400 + 396378732083200)} \phantom{+ 1} \\
 792757464166400 - 396378732083200 \phantom{+ 1} \\
 \underline{-(792757464166400 - 1585514928332800 + 792757464166400)} \phantom{+ 1} \\
 1585514928332800 - 792757464166400 \phantom{+ 1} \\
 \underline{-(1585514928332800 - 3171029856665600 + 1585514928332800)} \phantom{+ 1} \\
 3171029856665600 - 1585514928332800 \phantom{+ 1} \\
 \underline{-(3171029856665600 - 6342059713331200 + 3171029856665600)} \phantom{+ 1} \\
 6342059713331200 - 3171029856665600 \phantom{+ 1} \\
 \underline{-(6342059713331200 - 12684119426662400 + 6342059713331200)} \phantom{+ 1} \\
 12684119426662400 - 6342059713331200 \phantom{+ 1} \\
 \underline{-(12684119426662400 - 25368238853324800 + 12684119426662400)} \phantom{+ 1} \\
 25368238853324800 - 12684119426662400 \phantom{+ 1} \\
 \underline{-(25368238853324800 - 50736477706649600 + 25368238853324800)} \phantom{+ 1} \\
 50736477706649600 - 25368238853324800 \phantom{+ 1} \\
 \underline{-(50736477706649600 - 101472955413299200 + 50736477706649600)} \phantom{+ 1} \\
 101472955413299200 - 50736477706649600 \phantom{+ 1} \\
 \underline{-(101472955413299200 - 202945910826598400 + 101472955413299200)} \phantom{+ 1} \\
 202945910826598400 - 101472955413299200 \phantom{+ 1} \\
 \underline{-(202945910826598400 - 405891821653196800 + 202945910826598400)} \phantom{+ 1} \\
 405891821653196800 - 202945910826598400 \phantom{+ 1} \\
 \underline{-(405891821653196800 - 811783643306393600 + 405891821653196800)} \phantom{+ 1} \\
 811783643306393600 - 405891821653196800 \phantom{+ 1} \\
 \underline{-(811783643306393600 - 1623567286612787200 + 811783643306393600)} \phantom{+ 1} \\
 1623567286612787200 - 811783643306393600 \phantom{+ 1} \\
 \underline{-(1623567286612787200 - 3247134573225574400 + 1623567286612787200)} \phantom{+ 1} \\
 3247134573225574400 - 1623567286612787200 \phantom{+ 1} \\
 \underline{-(3247134573225574400 - 6494269146451148800 + 3247134573225574400)} \phantom{+ 1} \\
 6494269146451148800 - 3247134573225574400 \phantom{+ 1} \\
 \underline{-(6494269146451148800 - 12988538292902297600 + 6494269146451148800)} \phantom{+ 1} \\
 12988538292902297600 - 6494269146451148800 \phantom{+ 1} \\
 \underline{-(12988538292902297600 - 25977076585804595200 + 12988538292902297600)} \phantom{+ 1} \\
 25977076585804595200 - 12988538292902297600 \phantom{+ 1} \\
 \underline{-(25977076585804595200 - 51954153171609190400 + 25977076585804595200)} \phantom{+ 1} \\
 51954153171609190400 - 25977076585804595200 \phantom{+ 1} \\
 \underline{-(51954153171609190400 - 103908306343218380800 + 51954153171609190400)} \phantom{+ 1} \\
 103908306343218380800 - 51954153171609190400 \phantom{+ 1} \\
 \underline{-(103908306343218380800 - 207816612686436761600 + 103908306343218380800)} \phantom{+ 1} \\
 207816612686436761600 - 103908306343218380800 \phantom{+ 1} \\
 \underline{-(207816612686436761600 - 415633225372873523200 + 207816612686436761600)} \phantom{+ 1} \\
 415633225372873523200 - 207816612686436761600 \phantom{+ 1} \\
 \underline{-(415633225372873523200 - 831266450745747046400 + 415633225372873523200)} \phantom{+ 1} \\
 831266450745747046400 - 415633225372873523200 \phantom{+ 1} \\
 \underline{-(831266450745747046400 - 1662532901491494092800 + 831266450745747046400)} \phantom{+ 1} \\
 1662532901491494092800 - 831266450745747046400 \phantom{+ 1} \\
 \underline{-(1662532901491494092800 - 3325065802982988185600 + 1662532901491494092800)} \phantom{+ 1} \\
 3325065802982988185600 - 1662532901491494092800 \phantom{+ 1} \\
 \underline{-(3325065802982988185600 - 6650131605965976371200 + 3325065802982988185600)} \phantom{+ 1} \\
 6650131605965976371200 - 3325065802982988185600 \phantom{+ 1} \\
 \underline{-(6650131605965976371200 - 13300263211931952742400 + 6650131605965976371200)} \phantom{+ 1} \\
 13300263211931952742400 - 6650131605965976371200 \phantom{+ 1} \\
 \underline{-(13300263211931952742400 - 26600526423863905484800 + 13300263211931952742400)} \phantom{+ 1} \\
 26600526423863905484800 - 13300263211931952742400 \phantom{+ 1} \\
 \underline{-(26600526423863905484800 - 53201052847727810969600 + 26600526423863905484800)} \phantom{+ 1} \\
 53201052847727810969600 - 26600526423863905484800 \phantom{+ 1} \\
 \underline{-(53201052847727810969600 - 106402105695455621939200 + 53201052847727810969600)} \phantom{+ 1} \\
 106402105695455621939200 - 53201052847727810969600 \phantom{+ 1} \\
 \underline{-(106402105695455621939200 - 212804211390911243878400 + 106402105695455621939200)} \phantom{+ 1} \\
 212804211390911243878400 - 106402105695455621939200 \phantom{+ 1} \\
 \underline{-(212804211390911243878400 - 425608422781822487756800 + 212804211390911243878400)} \phantom{+ 1} \\
 425608422781822487756800 - 212804211390911243878400 \phantom{+ 1} \\
 \underline{-(425608422781822487756800 - 851216845563644975513600 + 425608422781822487756800)} \phantom{+ 1} \\
 851216845563644975513600 - 425608422781822487756800 \phantom{+ 1} \\
 \underline{-(851216845563644975513600 - 1702433691127289951027200 + 851216845563644975513600)} \phantom{+ 1} \\
 1702433691127289951027200 - 851216845563644975513600 \phantom{+ 1} \\
 \underline{-(1702433691127289951027200 - 3404867382254579902054400 + 1702433691127289951027200)} \phantom{+ 1} \\
 3404867382254579902054400 - 1702433691127289951027200 \phantom{+ 1} \\
 \underline{-(3404867382254579902054400 - 6809734764509159804108800 + 3404867382254579902054400)} \phantom{+ 1} \\
 6809734764509159804108800 - 3404867382254579902054400 \phantom{+ 1} \\
 \underline{-(6809734764509159804108800 - 13619469529018319608217600 + 6809734764509159804108800)} \phantom{+ 1} \\
 13619469529018319608217600 - 6809734764509159804108800 \phantom{+ 1} \\
 \underline{-(13619469529018319608217600 - 27238939058036639216435200 + 13619469529018319608217600)} \phantom{+ 1} \\
 27238939058036639216435200 - 13619469529018319608217600 \phantom{+ 1} \\
 \underline{-(27238939058036639216435200 - 54477878116073278432870400 + 27238939058036639216435200)} \phantom{+ 1} \\
 54477878116073278432870400 - 27238939058036639216435200 \phantom{+ 1} \\
 \underline{-(54477878116073278432870400 - 108955756232146556865740800 + 54477878116073278432870400)} \phantom{+ 1} \\
 108955756232146556865740800 - 54477878116073278432870400 \phantom{+ 1} \\
 \underline{-(108955756232146556865740800 - 217911512464293113731481600 + 108955756232146556865740800)} \phantom{+ 1} \\
 217911512464293113731481600 - 108955756232146556865740800 \phantom{+ 1} \\
 \underline{-(217911512464293113731481600 - 435823024928586227462963200 + 217911512464293113731481600)} \phantom{+ 1} \\
 435823024928586227462963200 - 217911512464293113731481600 \phantom{+ 1} \\
 \underline{-(435823024928586227462963200 - 871646049857172454925926400 + 435823024928586227462963200)} \phantom{+ 1} \\
 871646049857172454925926400 - 435823024928586227462963200 \phantom{+ 1} \\
 \underline{-(871646049857172454925926400 - 1743292099714344909851852800 + 871646049857172454925926400)} \phantom{+ 1} \\
 1743292099714344909851852800 - 871646049857172454925926400 \phantom{+ 1} \\
 \underline{-(1743292099714344909851852800 - 3486584199428689819703705600 + 1743292099714344909851852800)} \phantom{+ 1} \\
 3486584199428689819703705600 - 1743292099714344909851852800 \phantom{+ 1} \\
 \underline{-(3486584199428689819703705600 - 6973168398857379639407411200 + 3486584199428689819703705600)} \phantom{+ 1} \\
 6973168398857379639407411200 - 3486584199428689819703705600 \phantom{+ 1} \\
 \underline{-(6973168398857379639407411200 - 13946336797714759278814822400 + 6973168398857379639407411200)} \phantom{+ 1} \\
 13946336797714759278814822400 - 6973168398857379639407411200 \phantom{+ 1} \\
 \underline{-(13946336797714759278814822400 - 27892673595429518557629644800 + 13946336797714759278814822400)} \phantom{+ 1} \\
 27892673595429518557629644800 - 13946336797714759278814822400 \phantom{+ 1} \\
 \underline{-(27892673595429518557629644800 - 55785347190859037115259289600 + 27892673595429518557629644800)} \phantom{+ 1} \\
 55785347190859037115259289600 - 27892673595429518557629644800 \phantom{+ 1} \\
 \underline{-(55785347190859037115259289600 - 111570694381718074230518579200 + 55785347190859037115259289600)} \phantom{+ 1} \\
 111570694381718074230518579200 - 55785347190859037115259289600 \phantom{+ 1} \\
 \underline{-(111570694381718074230518579200 - 223141388763436148461037158400 + 111570694381718074230518579200)} \phantom{+ 1} \\
 223141388763436148461037158400 - 111570694381718074230518579200 \phantom{+ 1} \\
 \underline{-(223141388763436148461037158400 - 446282777526872296922074316800 + 223141388763436148461037158400)} \phantom{+ 1} \\
 446282777526872296922074316800 - 223141388763436148461037158400 \phantom{+ 1} \\
 \underline{-(446282777526872296922074316800 - 892565555053744593844148633600 + 446282777526872296922074316800)} \phantom{+ 1} \\
 892565555053744593844148633600 - 446282777526872296922074316800 \phantom{+ 1} \\
 \underline{-(892565555053744593844148633600 - 1785131110107489187688297267200 + 892565555053744593844148633600)} \phantom{+ 1} \\
 1785131110107489187688297267200 - 892565555053744593844148633600 \phantom{+ 1} \\
 \underline{-(1785131110107489187688297267200 - 3570262220214978375376594534400 + 1785131110107489187688297267200)} \phantom{+ 1} \\
 3570262220214978375376594534400 - 1785131110107489187688297267200 \phantom{+ 1} \\
 \underline{-(3570262220214978375376594534400 - 7140524440429956750753189068800 + 3570262220214978375376594534400)} \phantom{+ 1} \\
 7140524440429956750753189068800 - 3570262220214978375376594534400 \phantom{+ 1} \\
 \underline{-(7140524440429956750753189068800 - 14281048880859913501506378137600 + 7140524440429956750753189068800)} \phantom{+ 1} \\
 14281048880859913501506378137600 - 7140524440429956750753189068800 \phantom{+ 1} \\
 \underline{-(14281048880859913501506378137600 - 28562097761719827003012756275200 + 14281048880859913501506378137600)} \phantom{+ 1} \\
 28562097761719827003012756275200 - 14281048880859913501506378137600 \phantom{+ 1} \\
 \underline{-(28562097761719827003012756275200 - 57124195523439654006025512550400 + 28562097761719827003012756275200)} \phantom{+ 1} \\
 57124195523439654006025512550400 - 28562097761719827003012756275200 \phantom{+ 1} \\
 \underline{-(57124195523439654006025512550400 - 114248391046879308012051025100800 + 57124195523439654006025512550400)} \phantom{+ 1} \\
 114248391046879308012051025100800 - 57124195523439654006025512550400 \phantom{+ 1} \\
 \underline{-(114248391046879308012051025100800 - 228496782093758616024102050201600 + 114248391046879308012051025100800)} \phantom{+ 1} \\
 228496782093758616024102050201600 - 114248391046879308012051025100800 \phantom{+ 1} \\
 \underline{-(228496782093758616024102050201600 - 456993564187517232048204100403200 + 228496782093758616024102050201600)} \phantom{+ 1} \\
 456993564187517232048204100403200 - 228496782093758616024102050201600 \phantom{+ 1} \\
 \underline{-(456993564187517232048204100403200 - 913987128375034464096408200806400 + 456993564187517232048204100403200)} \phantom{+ 1} \\
 913987128375034464096408200806400 - 456993564187517232048204100403200 \phantom{+ 1} \\
 \underline{-(913987128375034464096408200806400 - 1827974256750068928192816401612800 + 913987128375034464096408200806400)} \phantom{+ 1} \\
 1827974256750068928192816401612800 - 913987128375034464096408200806400 \phantom{+ 1} \\
 \underline{-(1827974256750068928192816401612800 - 3655948513500137856385632803225600 + 1827974256750068928192816401612800)} \phantom{+ 1} \\
 3655948513500137856385632803225600 - 1827974256750068928192816401612800 \phantom{+ 1} \\
 \underline{-(3655948513500137856385632803225600 - 7311897027000275712771265606451200 + 3655948513500137856385632803225600)} \phantom{+ 1} \\
 73118970$$

### Question 5:

If the polynomial  $x^3 - 2x^2 + kx - 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .

Answer:

By division algorithm,

Dividend = Divisor  $\times$  Quotient + Remainder

Dividend – Remainder = Divisor  $\times$  Quotient

$x^3 - 2x^2 + kx - 10$  will be perfectly divisible by  $x^2 - 2x + k$ .

Let us divide  $x^3 - 2x^2 + kx - 10$  by  $x^2 - 2x + k$

$$\begin{array}{r} x + 0 \\ x^3 - 2x^2 + kx - 10 \\ \underline{-(x^2 - 2x + k)} \phantom{- 10} \\ 0x^3 - x^2 + (k+2)x - 10 - k \\ \underline{-(x^2 - 2x + k)} \phantom{- 10} \\ 0x^3 + x^2 + (k+4)x - 10 - k \\ \underline{-(x^2 - 2x + k)} \phantom{- 10} \\ 0x^3 + 0x^2 + (k+6)x - 10 - 2k \\ \underline{-(kx - 2kx + k^2)} \phantom{- 10} \\ 0x^3 + 0x^2 + 0x - 10 - 2k + k^2 \end{array}$$

It can be observed that  $-10 - 2k + k^2$  will be 0.

Therefore,  $(-10+2k) = 0$  and  $(10-a-8k+k^2) = 0$

For  $(-10+2k) = 0$ ,

$$2k = 10$$

And thus,  $k = 5$

For  $(10-a-8k+k^2) = 0$

$$10 - a - 8 \times 5 + 25 = 0$$

$$10 - a - 40 + 25 = 0$$

$$-5 - a = 0$$

Therefore,  $a = -5$

Hence,  $k = 5$  and  $a = -5$

