

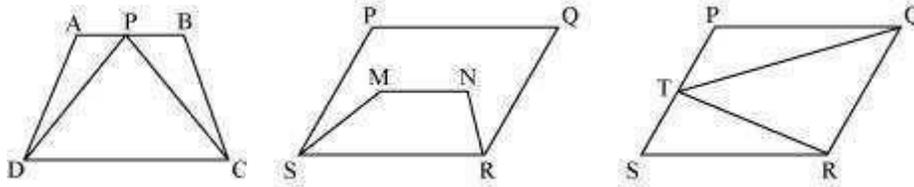
# Class IX Chapter 9 – Areas of Parallelograms and Triangles Maths

## Exercise 9.1 Question

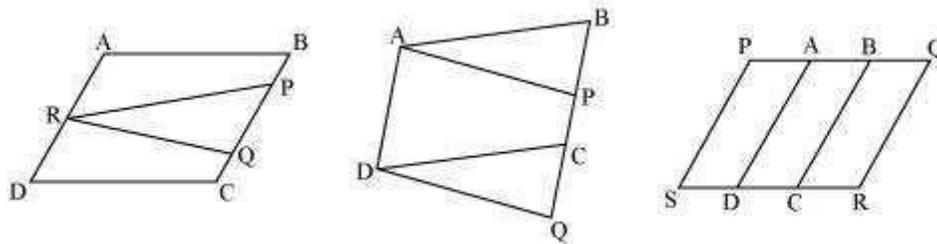
1:

Which of the following figures lie on the same base and between the same parallels.

In such a case, write the common base and the two parallels.



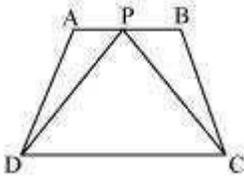
(i)    (ii)    (iii)



(iv)    (v)    (vi)

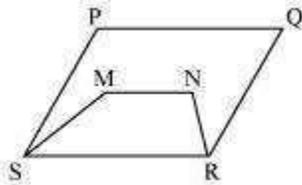
Answer:

(i)



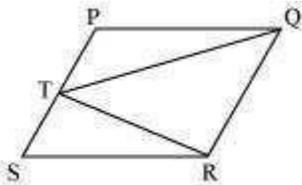
Yes. It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.

(ii)



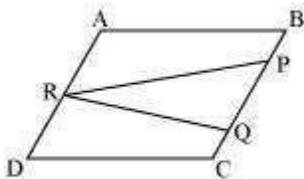
No. It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.

(iii)



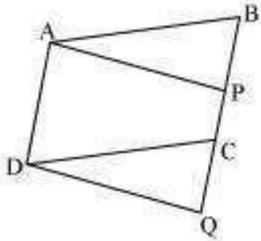
Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)



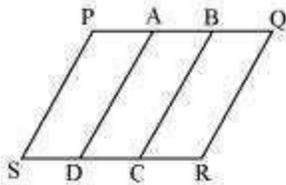
No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and BC. However, these do not have any common base.

(v)



Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and BQ.

(vi)

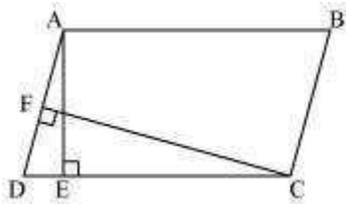


No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.

Exercise 9.2 Question

1:

In the given figure, ABCD is parallelogram,  $AE \perp DC$  and  $CF \perp AD$ . If  $AB = 16$  cm,  $AE = 8$  cm and  $CF = 10$  cm, find  $AD$ .



Answer:

In parallelogram ABCD,  $CD = AB = 16$  cm

[Opposite sides of a parallelogram are equal]

We know that

Area of a parallelogram = Base  $\times$  Corresponding altitude

Area of parallelogram ABCD =  $CD \times AE = AD \times CF$

$16 \text{ cm} \times 8 \text{ cm} = AD \times 10 \text{ cm}$

$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{ cm}$$

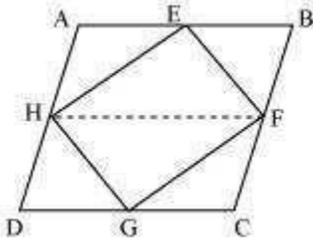
Thus, the length of AD is 12.8 cm.

Question 2:

If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that

$$\text{ar (EFGH)} = \frac{1}{2} \text{ ar (ABCD)}$$

Answer:



Let us join HF.

In parallelogram ABCD,

$AD = BC$  and  $AD \parallel BC$  (Opposite sides of a parallelogram are equal and parallel)

$AB = CD$  (Opposite sides of a parallelogram are equal)

$$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC \quad \text{and } AH \parallel BF$$

$\Rightarrow AH = BF$  and  $AH \parallel BF$  ( $\because$  H and F are the mid-points of AD and BC)

Therefore, ABFH is a parallelogram.

Since  $\triangle HEF$  and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

$$\therefore \text{Area} (\triangle HEF) = \frac{1}{2} \text{Area} (ABFH) \dots (1)$$

Similarly, it can be proved that

$$\text{Area} (\triangle HGF) = \frac{1}{2} \text{Area} (HDCF) \dots (2)$$

On adding equations (1) and (2), we obtain

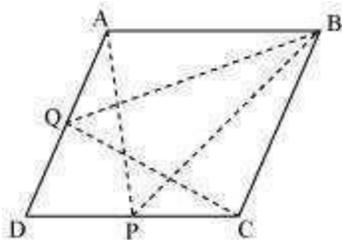
$$\begin{aligned} \text{Area} (\triangle HEF) + \text{Area} (\triangle HGF) &= \frac{1}{2} \text{Area} (ABFH) + \frac{1}{2} \text{Area} (HDCF) \\ &= \frac{1}{2} [\text{Area} (ABFH) + \text{Area} (HDCF)] \end{aligned}$$

$$\Rightarrow \text{Area} (EFGH) = \frac{1}{2} \text{Area} (ABCD)$$

Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that  $\text{ar} (APB) = \text{ar} (BQC)$ .

Answer:



It can be observed that  $\triangle BQC$  and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

$$\therefore \text{Area}(\triangle BQC) = \frac{1}{2} \text{Area}(ABCD) \dots (1)$$

Similarly,  $\triangle APB$  and parallelogram  $ABCD$  lie on the same base  $AB$  and between the same parallel lines  $AB$  and  $DC$ .

$$\therefore \text{Area}(\triangle APB) = \frac{1}{2} \text{Area}(ABCD) \dots (2)$$

From equation (1) and (2), we obtain

$$\text{Area}(\triangle BQC) = \text{Area}(\triangle APB) \quad \text{Question}$$

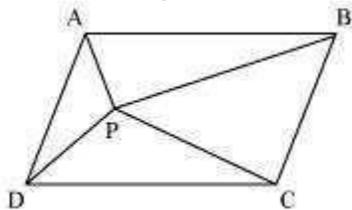
4:

In the given figure,  $P$  is a point in the interior of a parallelogram  $ABCD$ . Show that

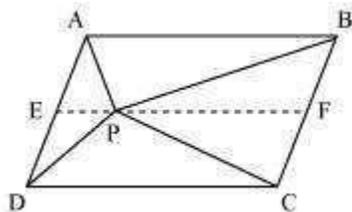
$$(i) \text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(ABCD)$$

$$(ii) \text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

[Hint: Through  $P$ , draw a line parallel to  $AB$ ]



Answer:



(i) Let us draw a line segment  $EF$ , passing through point  $P$  and parallel to line segment  $AB$ .

In parallelogram  $ABCD$ ,

AB || EF (By construction) ... (1) ABCD is a parallelogram.

∴ AD || BC (Opposite sides of a parallelogram)

⇒ AE || BF ... (2)

From equations (1) and (2), we obtain

AB || EF and AE || BF

Therefore, quadrilateral ABFE is a parallelogram.

It can be observed that  $\Delta APB$  and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

$$\therefore \text{Area} (\Delta APB) = \frac{1}{2} \text{Area} (ABFE) \dots (3)$$

Similarly, for  $\Delta PCD$  and parallelogram EFCD,

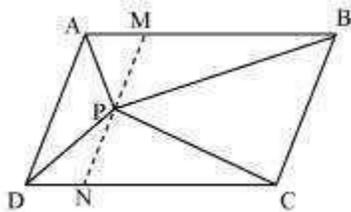
$$\text{Area} (\Delta PCD) = \frac{1}{2} \text{Area} (EFCD) \dots (4)$$

Adding equations (3) and (4), we obtain

$$\text{Area} (\Delta APB) + \text{Area} (\Delta PCD) = \frac{1}{2} [\text{Area} (ABFE) + \text{Area} (EFCD)]$$

$$\text{Area} (\Delta APB) + \text{Area} (\Delta PCD) = \frac{1}{2} \text{Area} (ABCD) \dots (5)$$

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

MN || AD (By construction) ... (6) ABCD is a parallelogram.

∴ AB || DC (Opposite sides of a parallelogram)

⇒ AM || DN ... (7)

From equations (6) and (7), we obtain

MN || AD and AM || DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that  $\Delta APD$  and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$$\therefore \text{Area} (\Delta APD) = \frac{1}{2} \text{Area} (AMND) \dots (8)$$

Similarly, for  $\Delta PCB$  and parallelogram MNCB,

$$\text{Area} (\Delta PCB) = \frac{1}{2} \text{Area} (MNCB) \dots (9)$$

Adding equations (8) and (9), we obtain

$$\begin{aligned} \text{Area} (\Delta APD) + \text{Area} (\Delta PCB) &= \frac{1}{2} [\text{Area} (AMND) + \text{Area} (MNCB)] \\ \text{Area} (\Delta APD) + \text{Area} (\Delta PCB) &= \frac{1}{2} \text{Area} (ABCD) \dots (10) \end{aligned}$$

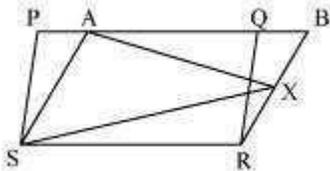
On comparing equations (5) and (10), we obtain Area

$$(\Delta APD) + \text{Area} (\Delta PBC) = \text{Area} (\Delta APB) + \text{Area} (\Delta PCD) \quad \text{Question 5:}$$

In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i)  $\text{ar}(\text{PQRS}) = \text{ar}(\text{ABRS})$

(ii)  $\text{ar}(\triangle \text{PXS}) = \frac{1}{2} \text{ar}(\text{PQRS})$



Answer:

(i) It can be observed that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

$\therefore \text{Area}(\text{PQRS}) = \text{Area}(\text{ABRS}) \dots (1)$

(ii) Consider  $\triangle \text{AXS}$  and parallelogram ABRS.

As these lie on the same base and are between the same parallel lines AS and BR,

$\therefore \text{Area}(\triangle \text{AXS}) = \frac{1}{2} \text{Area}(\text{ABRS}) \dots (2)$

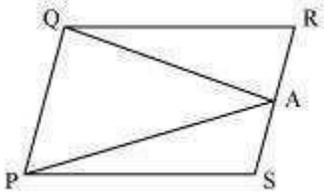
From equations (1) and (2), we obtain

$\text{Area}(\triangle \text{AXS}) = \frac{1}{2} \text{Area}(\text{PQRS})$

Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer:



From the figure, it can be observed that point A divides the field into three parts.

These parts are triangular in shape –  $\Delta PSA$ ,  $\Delta PAQ$ , and  $\Delta QRA$

Area of  $\Delta PSA$  + Area of  $\Delta PAQ$  + Area of  $\Delta QRA$  = Area of  $\parallel gm$  PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

$$\therefore \text{Area} (\Delta PAQ) = \frac{1}{2} \text{Area} (PQRS) \dots (2)$$

From equations (1) and (2), we obtain

$$\text{Area} (\Delta PSA) + \text{Area} (\Delta QRA) = \frac{1}{2} \text{Area} (PQRS) \dots (3)$$

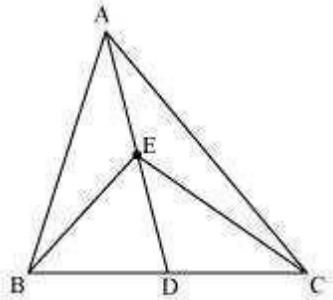
Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

### Exercise 9.3 Question

1:

In the given figure, E is any point on median AD of a  $\Delta ABC$ . Show that ar

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$$



Answer:

AD is the median of  $\triangle ABC$ . Therefore, it will divide  $\triangle ABC$  into two triangles of equal areas.

$\therefore$  Area ( $\triangle ABD$ ) = Area ( $\triangle ACD$ ) ... (1) ED is the median of  $\triangle EBC$ .

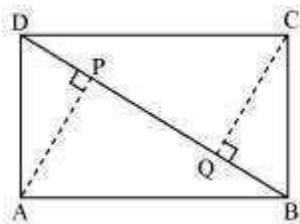
$\therefore$  Area ( $\triangle EBD$ ) = Area ( $\triangle ECD$ ) ... (2)

On subtracting equation (2) from equation (1), we obtain

$$\text{Area}(\triangle ABD) - \text{Area}(\triangle EBD) = \text{Area}(\triangle ACD) - \text{Area}(\triangle ECD)$$

$$\text{Area}(\triangle ABE) = \text{Area}(\triangle ACE) \quad \text{Question 10:}$$

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



- (i)  $\triangle APB \cong \triangle CQD$
- (ii)  $AP = CQ$

(i) In  $\triangle APB$  and  $\triangle CQD$ ,

$$\angle APB = \angle CQD \text{ (Each } 90^\circ\text{)}$$

$$AB = CD \text{ (Opposite sides of parallelogram } ABCD\text{)}$$

$$\angle ABP = \angle CDQ \text{ (Alternate interior angles for } AB \parallel CD\text{)}$$

$$\therefore \triangle APB \cong \triangle CQD \text{ (By AAS congruency)}$$

(ii) By using the above result

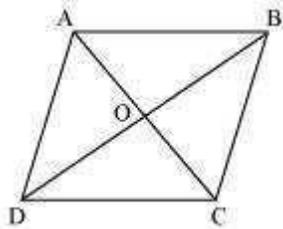
$\triangle APB \cong \triangle CQD$ , we obtain

$$AP = CQ \text{ (By CPCT) Question}$$

3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer:



We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in  $\triangle ABC$ . Therefore, it will divide it into two triangles of equal areas.

$$\therefore \text{Area } (\triangle AOB) = \text{Area } (\triangle BOC) \dots (1) \text{ In}$$

$\triangle BCD$ , CO is the median.

$$\therefore \text{Area } (\triangle BOC) = \text{Area } (\triangle COD) \dots (2)$$

$$\text{Similarly, Area } (\triangle COD) = \text{Area } (\triangle AOD) \dots (3)$$

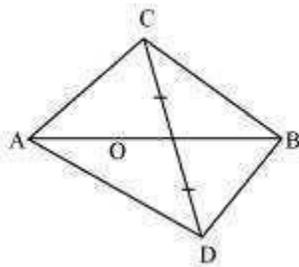
From equations (1), (2), and (3), we obtain

$$\text{Area } (\triangle AOB) = \text{Area } (\triangle BOC) = \text{Area } (\triangle COD) = \text{Area } (\triangle AOD)$$

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that  $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$ .



Answer:

Consider  $\triangle ACD$ .

Line-segment CD is bisected by AB at O. Therefore, AO is the median of  $\triangle ACD$ .

$$\therefore \text{Area}(\triangle ACO) = \text{Area}(\triangle ADO) \dots (1)$$

Considering  $\triangle BCD$ , BO is the median.

$$\therefore \text{Area}(\triangle BCO) = \text{Area}(\triangle BDO) \dots (2)$$

Adding equations (1) and (2), we obtain

$$\text{Area}(\triangle ACO) + \text{Area}(\triangle BCO) = \text{Area}(\triangle ADO) + \text{Area}(\triangle BDO)$$

$$\Rightarrow \text{Area}(\triangle ABC) = \text{Area}(\triangle ABD)$$

Question 6:

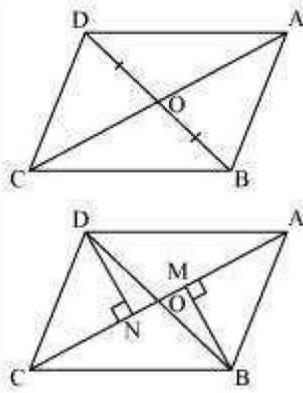
In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that  $OB = OD$ . If  $AB = CD$ , then show that:

(i)  $\text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$

(ii)  $\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$

(iii)  $DA \parallel CB$  or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]



Let us draw  $DN \perp AC$  and  $BM \perp AC$ .

(i) In  $\triangle DON$  and  $\triangle BOM$ ,

$\angle DNO = \angle BMO$  (By construction)

$\angle DON = \angle BOM$  (Vertically opposite angles)

$OD = OB$  (Given)

By AAS congruence rule,

$\triangle DON \cong \triangle BOM$

$\therefore DN = BM \dots (1)$

We know that congruent triangles have equal areas.

$\therefore \text{Area}(\triangle DON) = \text{Area}(\triangle BOM) \dots (2)$

In  $\triangle DNC$  and  $\triangle BMA$ ,

$\angle DNC = \angle BMA$  (By construction)

$CD = AB$  (Given)

$DN = BM$  [Using equation (1)]

$\therefore \triangle DNC \cong \triangle BMA$  (RHS congruence rule)

$\therefore \text{Area}(\triangle DNC) = \text{Area}(\triangle BMA) \dots (3)$

On adding equations (2) and (3), we obtain

$\text{Area}(\triangle DON) + \text{Area}(\triangle DNC) = \text{Area}(\triangle BOM) + \text{Area}(\triangle BMA)$

Therefore,  $\text{Area}(\triangle DOC) = \text{Area}(\triangle AOB)$

(ii) We obtained,

$$\text{Area } (\triangle DOC) = \text{Area } (\triangle AOB)$$

$$\perp \text{ Area } (\triangle DOC) + \text{Area } (\triangle OCB) = \text{Area } (\triangle AOB) + \text{Area } (\triangle OCB)$$

(Adding Area  $(\triangle OCB)$  to both sides)

$$\perp \text{ Area } (\triangle DCB) = \text{Area } (\triangle ACB)$$

(iii) We obtained,

$$\text{Area } (\triangle DCB) = \text{Area } (\triangle ACB)$$

If two triangles have the same base and equal areas, then these will lie between the same parallels.

$$\perp DA \parallel CB \dots (4)$$

In quadrilateral ABCD, one pair of opposite sides is equal ( $AB = CD$ ) and the other pair of opposite sides is parallel ( $DA \parallel CB$ ).

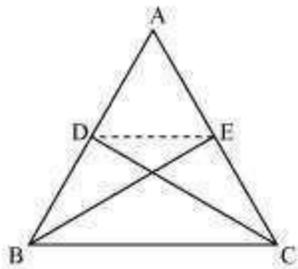
Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of  $\triangle ABC$  such that  $\text{ar}(\triangle DBC) = \text{ar}(\triangle ECB)$ . Prove that  $DE \parallel BC$ .

Answer:

Answer:

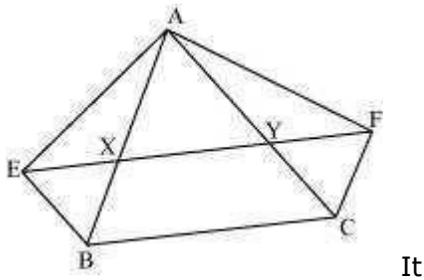


Since  $\triangle BCE$  and  $\triangle BCD$  are lying on a common base BC and also have equal areas,  $\triangle BCE$  and  $\triangle BCD$  will lie between the same parallel lines.

$$\perp DE \parallel BC$$

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF)



It is given that

$XY \parallel BC$  and  $EY \parallel BC$

$BE \parallel AC$  and  $BE \parallel CY$

Therefore, EBCY is a parallelogram.

It is given that

$XY \parallel BC$  and  $XF \parallel BC$

$AB \parallel FC$  and  $AB \parallel XB$

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

$$\therefore \text{Area (EBCY)} = \text{Area (BCFX)} \dots (1)$$

Consider parallelogram EBCY and  $\triangle AEB$

These lie on the same base BE and are between the same parallels BE and AC.

$$\perp \text{ Area } (\triangle ABE) = \frac{1}{2} \text{ Area } (EBCY) \dots (2)$$

Also, parallelogram BCFX and  $\triangle ACF$  are on the same base CF and between the same parallels CF and AB.

$$\perp \text{ Area } (\triangle ACF) = \frac{1}{2} \text{ Area } (BCFX) \dots (3)$$

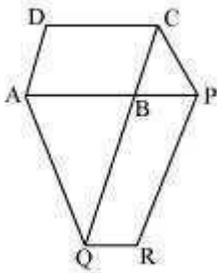
From equations (1), (2), and (3), we obtain

$$\text{Area } (\triangle ABE) = \text{Area } (\triangle ACF)$$

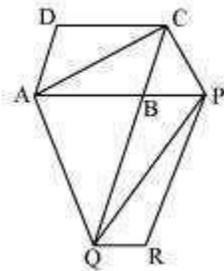
Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that  $\text{ar } (ABCD) = \text{ar } (PBQR)$ .

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]



Answer:



Let us join AC and PQ.

$\triangle ACQ$  and  $\triangle AQP$  are on the same base AQ and between the same parallels AQ and CP.

$$\perp \text{Area} (\triangle ACQ) = \text{Area} (\triangle APQ)$$

$$\perp \text{Area} (\triangle ACQ) - \text{Area} (\triangle ABQ) = \text{Area} (\triangle APQ) - \text{Area} (\triangle ABQ)$$

$$\perp \text{Area} (\triangle ABC) = \text{Area} (\triangle QBP) \dots (1)$$

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

$$\perp \text{Area} (\triangle ABC) = \frac{1}{2} \text{Area} (\text{ABCD}) \dots (2)$$

$$\text{Area } (\triangle QBP) = \frac{1}{2} \text{Area } (PBQR) \dots (3)$$

From equations (1), (2), and (3), we obtain

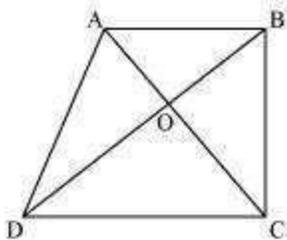
$$\frac{1}{2} \text{Area } (ABCD) = \frac{1}{2} \text{Area } (PBQR)$$

Area (ABCD) = Area (PBQR) Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

Prove that ar (AOD) = ar (BOC).

Answer:



It can be observed that  $\triangle DAC$  and  $\triangle DBC$  lie on the same base DC and between the same parallels AB and CD.

$$\therefore \text{Area } (\triangle DAC) = \text{Area } (\triangle DBC)$$

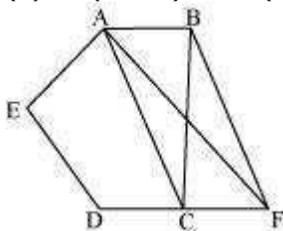
$$\therefore \text{Area } (\triangle DAC) - \text{Area } (\triangle DOC) = \text{Area } (\triangle DBC) - \text{Area } (\triangle DOC)$$

$$\therefore \text{Area } (\triangle AOD) = \text{Area } (\triangle BOC)$$

Question 11:

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that (i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Answer:

(i)  $\triangle ACB$  and  $\triangle ACF$  lie on the same base AC and are between

The same parallels AC and BF.  $\therefore$

$$\text{Area } (\triangle ACB) = \text{Area } (\triangle ACF)$$

(ii) It can be observed that

$$\text{Area } (\triangle ACB) = \text{Area } (\triangle ACF)$$

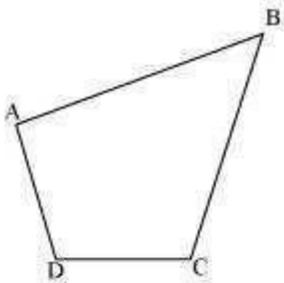
$$\perp \text{ Area } (\triangle ACB) + \text{Area } (ACDE) = \text{Area } (ACF) + \text{Area } (ACDE)$$

$$\perp \text{ Area } (ABCDE) = \text{Area } (AEDF)$$

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer:

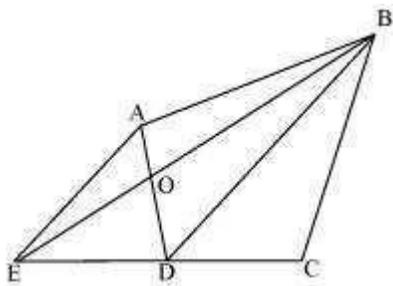


Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion  $\triangle AOB$  can be cut from the original field so that the new shape of the field will be  $\triangle BCE$ . (See figure)

We have to prove that the area of  $\triangle AOB$  (portion that was cut so as to construct Health Centre) is equal to the area of  $\triangle DEO$  (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)



It can be observed that  $\triangle DEB$  and  $\triangle DAB$  lie on the same base  $BD$  and are between the same parallels  $BD$  and  $AE$ .  $\therefore \text{Area}(\triangle DEB) = \text{Area}(\triangle DAB)$

$$\perp \text{Area}(\triangle DEB) - \text{Area}(\triangle DOB) = \text{Area}(\triangle DAB) - \text{Area}(\triangle DOB)$$

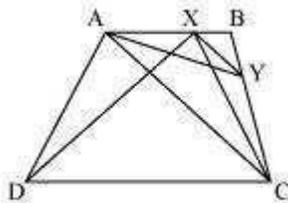
$$\perp \text{Area}(\triangle DEO) = \text{Area}(\triangle AOB)$$

Question 13:

$ABCD$  is a trapezium with  $AB \parallel DC$ . A line parallel to  $AC$  intersects  $AB$  at  $X$  and  $BC$  at  $Y$ .

Prove that  $\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$ .

[Hint: Join  $CX$ .] Answer:



It can be observed that  $\triangle ADX$  and  $\triangle ACX$  lie on the same base  $AX$  and are between the same parallels  $AB$  and  $DC$ .

$$\perp \text{Area}(\triangle ADX) = \text{Area}(\triangle ACX) \dots (1)$$

$\triangle ACY$  and  $\triangle ACX$  lie on the same base  $AC$  and are between the same parallels  $AC$  and  $XY$ .

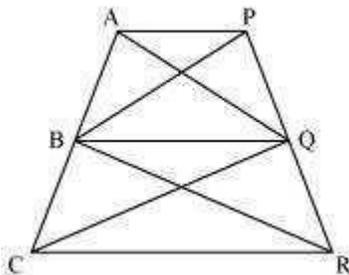
$$\perp \text{Area}(\triangle ACY) = \text{Area}(\triangle ACX) \dots (2)$$

From equations (1) and (2), we obtain

$$\text{Area}(\triangle ADX) = \text{Area}(\triangle ACY) \quad \text{Question 14:}$$

In the given figure,  $AP \parallel BQ \parallel CR$ . Prove that  $\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$ .

Answer:



Since  $\triangle ABQ$  and  $\triangle PBQ$  lie on the same base  $BQ$  and are between the same parallels

AP and BQ,

$$\perp \text{Area} (\triangle ABQ) = \text{Area} (\triangle PBQ) \dots (1)$$

Again,  $\triangle BCQ$  and  $\triangle BRQ$  lie on the same base BQ and are between the same parallels BQ and CR.

$$\perp \text{Area} (\triangle BCQ) = \text{Area} (\triangle BRQ) \dots (2)$$

On adding equations (1) and (2), we obtain

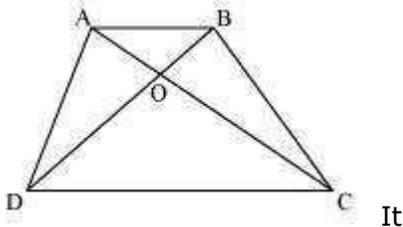
$$\text{Area} (\triangle ABQ) + \text{Area} (\triangle BCQ) = \text{Area} (\triangle PBQ) + \text{Area} (\triangle BRQ) \perp$$

$$\text{Area} (\triangle AQC) = \text{Area} (\triangle PBR)$$

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that  $\text{ar} (\triangle AOD) = \text{ar} (\triangle BOC)$ . Prove that ABCD is a trapezium.

Answer:



is given that

$$\text{Area} (\triangle AOD) = \text{Area} (\triangle BOC)$$

$$\text{Area} (\triangle AOD) + \text{Area} (\triangle AOB) = \text{Area} (\triangle BOC) + \text{Area} (\triangle AOB)$$

$$\text{Area} (\triangle ADB) = \text{Area} (\triangle ACB)$$

We know that triangles on the same base having areas equal to each other lie between the same parallels.

Therefore, these triangles,  $\triangle ADB$  and  $\triangle ACB$ , are lying between the same parallels. i.e.,

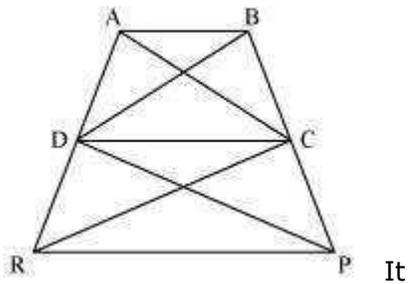
$$AB \parallel CD$$

Therefore, ABCD is a trapezium.

Question 16:

In the given figure,  $\text{ar} (\triangle DRC) = \text{ar} (\triangle DPC)$  and  $\text{ar} (\triangle BDP) = \text{ar} (\triangle ARC)$ . Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Answer:



is given that

$$\text{Area } (\triangle DRC) = \text{Area } (\triangle DPC)$$

As  $\triangle DRC$  and  $\triangle DPC$  lie on the same base  $DC$  and have equal areas, therefore, they must lie between the same parallel lines.  $\perp DC \parallel RP$

Therefore,  $DCPR$  is a trapezium. It is also given that

$$\text{Area } (\triangle BDP) = \text{Area } (\triangle ARC)$$

$$\perp \text{Area } (\triangle BDP) - \text{Area } (\triangle DPC) = \text{Area } (\triangle ARC) - \text{Area } (\triangle DRC)$$

$$\perp \text{Area } (\triangle BDC) = \text{Area } (\triangle ADC)$$

Since  $\triangle BDC$  and  $\triangle ADC$  are on the same base  $CD$  and have equal areas, they must lie between the same parallel lines.  $\perp AB \parallel CD$

Therefore,  $ABCD$  is a trapezium.

### Exercise 9.4 Question

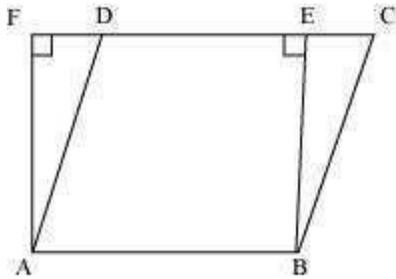
1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer:

As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths.

Therefore,

$$AB = EF \text{ (For rectangle)}$$

$$AB = CD \text{ (For parallelogram)} \quad \perp$$

$$CD = EF$$

$$\perp AB + CD = AB + EF \dots (1)$$

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.  $\perp AF < AD$

And similarly,  $BE < BC$

$$\perp AF + BE < AD + BC \dots (2)$$

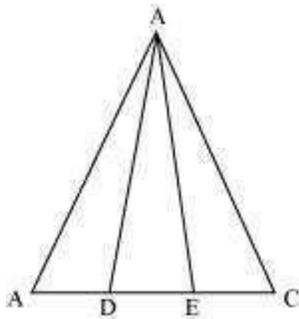
From equations (1) and (2), we obtain

$$AB + EF + AF + BE < AD + BC + AB + CD$$

Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD **Question 2:**

In the following figure, D and E are two points on BC such that  $BD = DE = EC$ . Show that  $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$ .

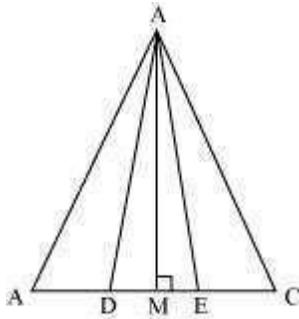
Can you answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



[Remark: Note that by taking  $BD = DE = EC$ , the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide  $\triangle ABC$  into n triangles of equal areas.]

Answer:

Let us draw a line segment  $AM \perp BC$ .



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$\text{Area} (\triangle ADE) = \frac{1}{2} \times DE \times AM$$

$$\text{Area} (\triangle ABD) = \frac{1}{2} \times BD \times AM$$

$$\text{Area} (\triangle AEC) = \frac{1}{2} \times EC \times AM$$

It is given that  $DE = BD = EC$

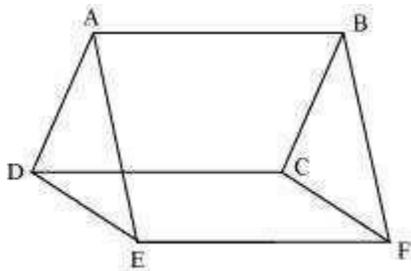
$$\perp \frac{1}{2} \times DE \times AM = \frac{1}{2} \times BD \times AM = \frac{1}{2} \times EC \times AM$$

$$\perp \text{Area} (\triangle ADE) = \text{Area} (\triangle ABD) = \text{Area} (\triangle AEC)$$

It can be observed that Budhia has divided her field into 3 equal parts.

Question 3:

In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that  $\text{ar} (\triangle ADE) = \text{ar} (\triangle BCF)$ .



Answer:

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.  $\perp AD = BC \dots (1)$

Similarly, for parallelograms DCFE and ABFE, it can be proved that

$$DE = CF \dots (2)$$

$$\text{And, } EA = FB \dots (3)$$

In  $\triangle ADE$  and  $\triangle BCF$ ,

$$AD = BC \text{ [Using equation (1)]}$$

$$DE = CF \text{ [Using equation (2)]}$$

$$EA = FB \text{ [Using equation (3)]}$$

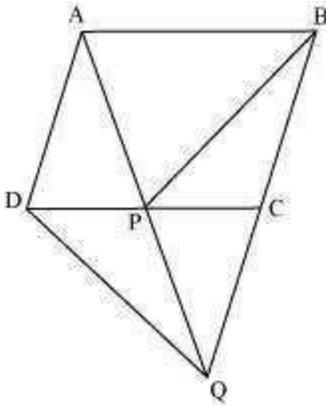
$\therefore \triangle ADE \cong \triangle BCF$  (SSS congruence rule)

$$\therefore \text{Area}(\triangle ADE) = \text{Area}(\triangle BCF)$$

Question 4:

In the following figure, ABCD is parallelogram and BC is produced to a point Q such that  $AD = CQ$ . If AQ intersect DC at P, show that  $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$ .

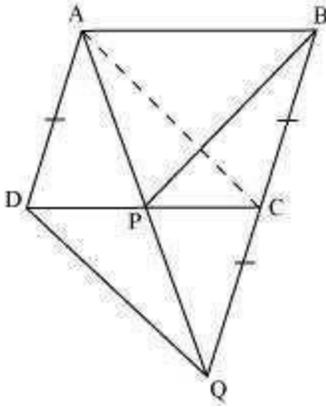
[Hint: Join AC.]



Answer:

It is given that ABCD is a parallelogram.

$AD \parallel BC$  and  $AB \parallel DC$  (Opposite sides of a parallelogram are parallel to each other) Join point A to point C.



Consider  $\triangle APC$  and  $\triangle BPC$

$\triangle APC$  and  $\triangle BPC$  are lying on the same base  $PC$  and between the same parallels  $PC$  and  $AB$ . Therefore,

$$\text{Area}(\triangle APC) = \text{Area}(\triangle BPC) \dots (1)$$

In quadrilateral  $ACDQ$ , it is given that

$$AD = CQ$$

Since  $ABCD$  is a parallelogram,

$AD \parallel BC$  (Opposite sides of a parallelogram are parallel)

$CQ$  is a line segment which is obtained when line segment  $BC$  is produced.

$$\perp AD \parallel CQ$$

We have,

$$AC = DQ \text{ and } AC \parallel DQ$$

Hence,  $ACQD$  is a parallelogram.

Consider  $\triangle DCQ$  and  $\triangle ACQ$

These are on the same base  $CQ$  and between the same parallels  $CQ$  and  $AD$ .

Therefore,

$$\text{Area}(\triangle DCQ) = \text{Area}(\triangle ACQ)$$

$$\perp \text{Area}(\triangle DCQ) - \text{Area}(\triangle PQC) = \text{Area}(\triangle ACQ) - \text{Area}(\triangle PQC)$$

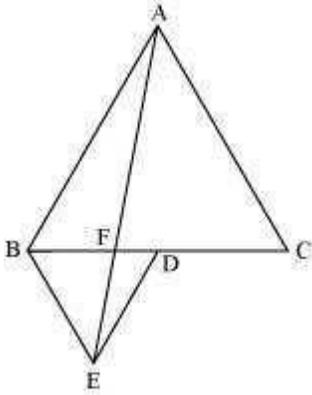
$$\perp \text{Area}(\triangle DPQ) = \text{Area}(\triangle APC) \dots (2)$$

From equations (1) and (2), we obtain

$$\text{Area}(\triangle BPC) = \text{Area}(\triangle DPQ) \quad \text{Question}$$

5:

In the following figure,  $ABC$  and  $BDE$  are two equilateral triangles such that  $D$  is the midpoint of  $BC$ . If  $AE$  intersects  $BC$  at  $F$ , show that



$$\text{ar}(\text{BDE}) = \frac{1}{4} \text{ar}(\text{ABC})$$

(i)

$$\text{ar}(\text{BDE}) = \frac{1}{2} \text{ar}(\text{BAE})$$

(ii)

$$\text{ar}(\text{ABC}) = 2 \text{ar}(\text{BEC})$$

(iii)

$$\text{ar}(\text{BFE}) = \text{ar}(\text{AFD})$$

(iv)

$$\text{ar}(\text{BFE}) = 2 \text{ar}(\text{FED})$$

(v)

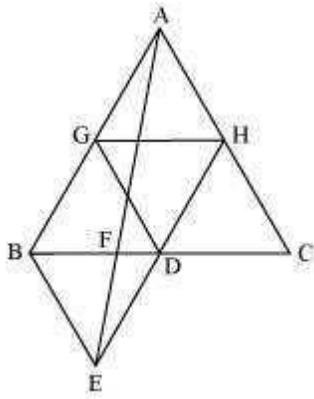
$$\text{ar}(\text{FED}) = \frac{1}{8} \text{ar}(\text{AFC})$$

(vi)

[Hint: Join EC and AD. Show that  $BE \parallel AC$  and  $DE \parallel AB$ , etc.] Answer:

(i) Let G and H be the mid-points of side AB and AC respectively.

Line segment GH is joining the mid-points. Therefore, it will be parallel to third side BC and also its length will be half of the length of BC (mid-point theorem).



$$\perp GH = \frac{1}{2} BC \text{ and } GH \parallel BD$$

$\perp GH = BD = DC$  and  $GH \parallel BD$  (D is the mid-point of BC)

Consider quadrilateral GHDB.

$GH \parallel BD$  and  $GH = BD$

Two line segments joining two parallel line segments of equal length will also be equal and parallel to each other.

Therefore,  $BG = DH$  and  $BG \parallel DH$

Hence, quadrilateral GHDB is a parallelogram.

We know that in a parallelogram, the diagonal bisects it into two triangles of equal area.

Hence,  $\text{Area}(\triangle BDG) = \text{Area}(\triangle HGD)$

Similarly, it can be proved that quadrilaterals DCHG, GDHA, and BEDG are

parallelograms and their respective diagonals are dividing them into two triangles of equal area.

$\text{ar}(\triangle GDH) = \text{ar}(\triangle CHD)$  (For parallelogram DCHG)  $\text{ar}(\triangle GDH)$

$= \text{ar}(\triangle HAG)$  (For parallelogram GDHA)  $\text{ar}(\triangle BDE) = \text{ar}$

$(\triangle BDG)$  (For parallelogram BEDG)  $\text{ar}(\triangle ABC) = \text{ar}(\triangle BDG) +$

$\text{ar}(\triangle GDH) + \text{ar}(\triangle CHD) + \text{ar}(\triangle HAG)$   $\text{ar}(\triangle ABC) = 4 \times$

$\text{ar}(\triangle BDE)$

$$\text{Hence, } \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

(ii)  $\text{Area}(\triangle BDE) = \text{Area}(\triangle AED)$  (Common base DE and  $DE \parallel AB$ )

$$\text{Area } (\triangle BDE) - \text{Area } (\triangle FED) = \text{Area } (\triangle AED) - \text{Area } (\triangle FED)$$

$$\text{Area } (\triangle BEF) = \text{Area } (\triangle AFD) \quad (1)$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABF) + \text{Area } (\triangle AFD)$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABF) + \text{Area } (\triangle BEF) \quad [\text{From equation (1)}]$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABE) \quad (2) \quad AD$$

is the median in  $\triangle ABC$ .

$$\begin{aligned} \text{ar } (\triangle ABD) &= \frac{1}{2} \text{ar } (\triangle ABC) \\ &= \frac{4}{2} \text{ar } (\triangle BDE) && \text{(As proved earlier)} \\ \text{ar } (\triangle ABD) &= 2 \text{ar } (\triangle BDE) && (3) \end{aligned}$$

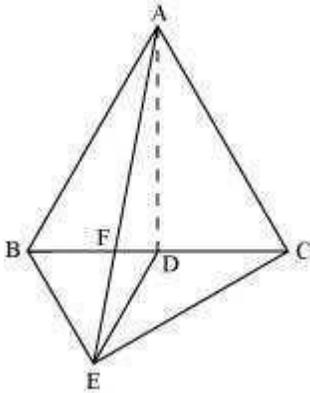
From (2) and (3), we obtain

$$2 \text{ar } (\triangle BDE) = \text{ar } (\triangle ABE)$$

$$\text{ar } (\triangle BDE) = \frac{1}{2} \text{ar } (\triangle ABE)$$

Or,

(iii)



$$\text{ar } (\triangle ABE) = \text{ar } (\triangle BEC) \quad (\text{Common base BE and } BE \parallel AC) \text{ ar}$$

$$(\triangle ABF) + \text{ar } (\triangle BEF) = \text{ar } (\triangle BEC)$$

Using equation (1), we obtain ar

$$(\triangle ABF) + \text{ar } (\triangle AFD) = \text{ar } (\triangle BEC) \text{ ar}$$

$$(\triangle ABD) = \text{ar } (\triangle BEC)$$

$$\frac{1}{2} \text{ar } (\triangle ABC) = \text{ar } (\triangle BEC)$$

$$\text{ar } (\triangle ABC) = 2 \text{ar } (\triangle BEC)$$

(iv) It is seen that  $\triangle BDE$  and  $\triangle AED$  lie on the same base (DE) and between the parallels DE and AB.

$$\begin{aligned} \perp \text{ar} (\triangle BDE) &= \text{ar} (\triangle AED) - \perp \text{ar} (\triangle FED) \\ &= \text{ar} (\triangle AED) - \text{ar} (\triangle FED) \quad \perp \text{ar} (\triangle BFE) = \text{ar} (\triangle AFD) \end{aligned}$$

(v) Let  $h$  be the height of vertex E, corresponding to the side BD in  $\triangle BDE$ .

Let  $H$  be the height of vertex A, corresponding to the side BC in  $\triangle ABC$ .

$$\text{ar}(\text{BDE}) = \frac{1}{4} \text{ar}(\text{ABC}).$$

In (i), it was shown that

$$\therefore \frac{1}{2} \times \text{BD} \times h = \frac{1}{4} \left( \frac{1}{2} \times \text{BC} \times H \right)$$

$$\Rightarrow \text{BD} \times h = \frac{1}{4} (2\text{BD} \times H)$$

$$\Rightarrow h = \frac{1}{2} H$$

In (iv), it was shown that  $\text{ar}(\triangle \text{BFE}) = \text{ar}(\triangle \text{AFD})$ .

$$\perp \text{ar}(\triangle \text{BFE}) = \text{ar}(\triangle \text{AFD})$$

$$\frac{1}{2} \times \text{FD} \times H = \frac{1}{2} \times \text{FD} \times 2h = 2 \left( \frac{1}{2} \times \text{FD} \times h \right)$$

=

$$= 2 \text{ar}(\triangle \text{FED})$$

$$\text{ar}(\text{BFE}) = 2 \text{ar}(\text{FED}).$$

Hence,

$$(ii) \text{Area}(\triangle \text{AFC}) = \text{area}(\triangle \text{AFD}) + \text{area}(\triangle \text{ADC})$$

$$= \text{ar}(\text{BFE}) + \frac{1}{2} \text{ar}(\text{ABC}) \quad \left[ \text{In (iv), ar}(\text{BFE}) = \text{ar}(\text{AFD}) ; \text{AD is median of } \triangle \text{ABC} \right]$$

$$= \text{ar}(\text{BFE}) + \frac{1}{2} \times 4 \text{ar}(\text{BDE}) \quad \left[ \text{In (i), ar}(\text{BDE}) = \frac{1}{4} \text{ar}(\text{ABC}) \right]$$

$$= \text{ar}(\text{BFE}) + 2 \text{ar}(\text{BDE}) \quad \dots(5)$$

$$\text{ar}(\text{BFE}) = 2 \text{ar}(\text{FED}).$$

Now, by (v),  $\dots(6)$

$$\text{ar}(\text{BDE}) = \text{ar}(\text{BFE}) + \text{ar}(\text{FED}) = 2 \text{ar}(\text{FED}) + \text{ar}(\text{FED}) = 3 \text{ar}(\text{FED}) \quad \dots(7)$$

Therefore, from equations (5), (6), and (7), we get:

$$\text{ar}(\text{AFC}) = 2 \text{ar}(\text{FED}) + 2 \times 3 \text{ar}(\text{FED}) = 8 \text{ar}(\text{FED})$$

$$\therefore \text{ar}(\text{AFC}) = 8 \text{ar}(\text{FED})$$

$$\text{Hence, ar}(\text{FED}) = \frac{1}{8} \text{ar}(\text{AFC})$$

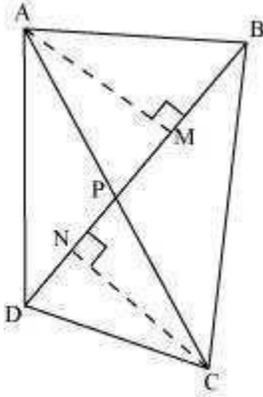
Question 6:

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) = \text{ar}(\text{APD}) \times \text{ar}(\text{BPC})$$

[Hint: From A and C, draw perpendiculars to BD] Answer:

Let us draw  $AM \perp BD$  and  $CN \perp BD$



$$= \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

Area of a triangle

$$\text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) = \left[ \frac{1}{2} \times \text{BP} \times \text{AM} \right] \times \left[ \frac{1}{2} \times \text{PD} \times \text{CN} \right]$$

$$= \frac{1}{4} \times \text{BP} \times \text{AM} \times \text{PD} \times \text{CN}$$

$$\text{ar}(\text{APD}) \times \text{ar}(\text{BPC}) = \left[ \frac{1}{2} \times \text{PD} \times \text{AM} \right] \times \left[ \frac{1}{2} \times \text{CN} \times \text{BP} \right]$$

$$= \frac{1}{4} \times \text{PD} \times \text{AM} \times \text{CN} \times \text{BP}$$

$$= \frac{1}{4} \times \text{BP} \times \text{AM} \times \text{PD} \times \text{CN}$$

$$\therefore \text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) = \text{ar}(\text{APD}) \times \text{ar}(\text{BPC})$$

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

$$\text{ar}(\text{PRQ}) = \frac{1}{2} \text{ar}(\text{ARC}) \quad \text{ar}(\text{RQC}) = \frac{3}{8} \text{ar}(\text{ABC})$$

(i)

(ii)

$$\text{ar}(\text{PBQ}) = \text{ar}(\text{ARC})$$

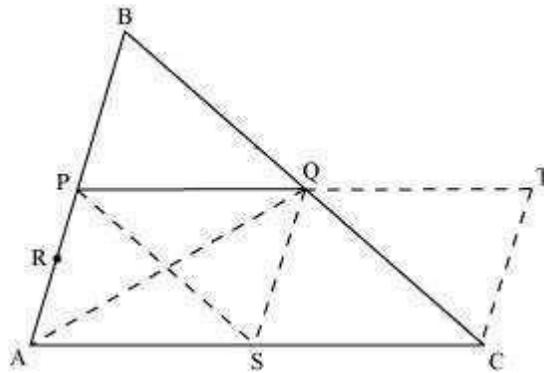
(iii)

Answer:

Take a point S on AC such that S is the mid-point of AC.

Extend PQ to T such that  $PQ = QT$ .

Join TC, QS, PS, and AQ.



In  $\triangle ABC$ , P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC$$

$\perp$   $PQ \parallel AS$  and  $PQ = AS$  (As S is the mid-point of AC)

$\perp$  PQSA is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.

$$\perp \text{ ar } (\triangle PAS) = \text{ ar } (\triangle SQP) = \text{ ar } (\triangle PAQ) = \text{ ar } (\triangle SQA)$$

Similarly, it can also be proved that quadrilaterals PSCQ, QSCT, and PSQB are also parallelograms and therefore,

$$\text{ ar } (\triangle PSQ) = \text{ ar } (\triangle CQS) \text{ (For parallelogram PSCQ) ar}$$

$$(\triangle QSC) = \text{ ar } (\triangle CTQ) \text{ (For parallelogram QSCT) ar}$$

$$(\triangle PSQ) = \text{ ar } (\triangle QBP) \text{ (For parallelogram PSQB) Thus, ar } (\triangle PAS) = \text{ ar } (\triangle SQP) = \text{ ar}$$

$$(\triangle PAQ) = \text{ ar } (\triangle SQA) = \text{ ar } (\triangle QSC) = \text{ ar } (\triangle CTQ) = \text{ ar}$$

$$(\triangle QBP) \dots (1)$$

$$\text{ Also, ar } (\triangle ABC) = \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PAS) + \text{ ar } (\triangle PQS) + \text{ ar } (\triangle QSC) \text{ ar}$$

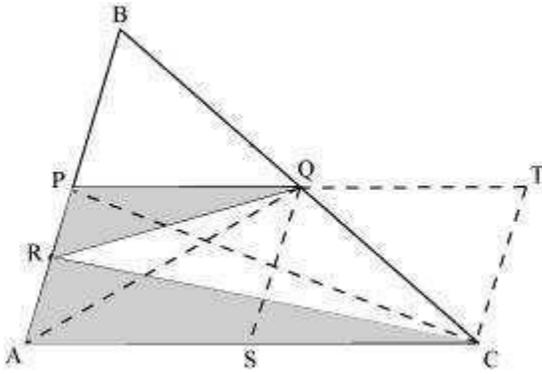
$$(\triangle ABC) = \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ)$$

$$= \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ) + \text{ ar } (\triangle PBQ)$$

$$= 4 \text{ ar } (\triangle PBQ)$$

$$\perp \text{ ar } (\triangle PBQ) = \frac{1}{4} \text{ ar } (\triangle ABC) \dots (2)$$

(i) Join point P to C.



In  $\Delta PAQ$ , QR is the median.

$$\therefore \text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta PAQ) = \frac{1}{2} \times \frac{1}{4} \text{ar}(\Delta ABC) = \frac{1}{8} \text{ar}(\Delta ABC) \quad \dots (3)$$

In  $\Delta ABC$ , P and Q are the mid -points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$$PQ = \frac{1}{2} AC$$

$$AC = 2PQ \Rightarrow AC = PT$$

Also,  $PQ \parallel AC \Rightarrow PT \parallel AC$

Hence, PACT is a parallelogram.

$$\text{ar}(\text{PACT}) = \text{ar}(\text{PACQ}) + \text{ar}(\Delta QTC)$$

$$= \text{ar}(\text{PACQ}) + \text{ar}(\Delta PBQ) \text{ [Using equation (1)]} \quad \perp$$

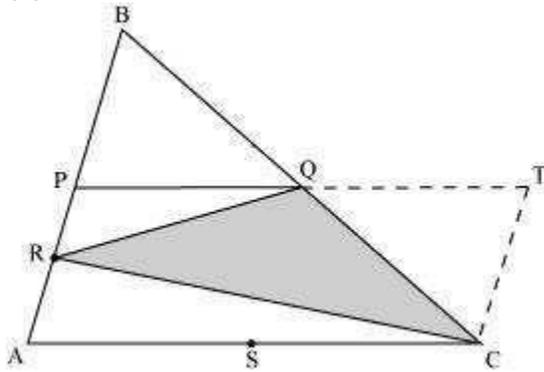
$$\text{ar}(\text{PACT}) = \text{ar}(\Delta ABC) \quad \dots (4)$$

$$\begin{aligned} \text{ar}(\Delta ARC) &= \frac{1}{2} \text{ar}(\Delta PAC) \quad (\text{CR is the median of } \Delta PAC) \\ &= \frac{1}{2} \times \frac{1}{2} \text{ar}(\text{PACT}) \quad (\text{PC is the diagonal of parallelogram PACT}) \\ &= \frac{1}{4} \text{ar}(\Delta PACT) = \frac{1}{4} \text{ar}(\Delta ABC) \end{aligned}$$

$$\Rightarrow \frac{1}{2} \text{ar}(\Delta ARC) = \frac{1}{8} \text{ar}(\Delta ABC)$$

$$\Rightarrow \frac{1}{2} \text{ar}(\Delta ARC) = \text{ar}(\Delta PRQ) \quad [\text{Using equation (3)}] \quad \dots (5)$$

(ii)



$$\text{ar}(\text{PACT}) = \text{ar}(\Delta PRQ) + \text{ar}(\Delta ARC) + \text{ar}(\Delta QTC) + \text{ar}(\Delta RQC)$$

Putting the values from equations (1), (2), (3), (4), and (5), we obtain

$$\text{ar}(\Delta ABC) = \frac{1}{8} \text{ar}(\Delta ABC) + \frac{1}{4} \text{ar}(\Delta ABC) + \frac{1}{4} \text{ar}(\Delta ABC) + \text{ar}(\Delta RQC)$$

$$\text{ar}(\Delta ABC) = \frac{5}{8} \text{ar}(\Delta ABC) + \text{ar}(\Delta RQC)$$

$$\text{ar}(\Delta RQC) = \left(1 - \frac{5}{8}\right) \text{ar}(\Delta ABC)$$

$$\text{ar}(\Delta RQC) = \frac{3}{8} \text{ar}(\Delta ABC)$$

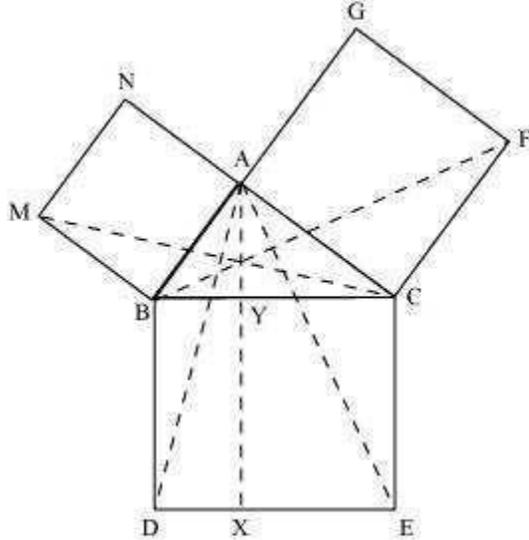
(iii) In parallelogram PACT,

$$\begin{aligned}
 \text{ar}(\triangle ARC) &= \frac{1}{2} \text{ar}(\triangle PAC) \quad (\text{CR is the median of } \triangle PAC) \\
 &= \frac{1}{2} \times \frac{1}{2} \text{ar}(\text{PACT}) \quad (\text{PC is the diagonal of parallelogram PACT}) \\
 &= \frac{1}{4} \text{ar}(\triangle PACT) \\
 &= \frac{1}{4} \text{ar}(\triangle ABC) \\
 &= \text{ar}(\triangle PBQ)
 \end{aligned}$$

Question 8:

In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX  $\perp$  DE

meets BC at Y. Show that:



i)  $\triangle MBC \perp \triangle ABD$

$$\text{ar}(\text{BYXD}) = 2\text{ar}(\text{MBC})$$

(ii)

$$\text{ar}(\text{BYXD}) = 2\text{ar}(\text{ABMN})$$

(iii)

iv)  $\triangle FCB \perp \triangle ACE$

$$\text{ar}(\text{CYXE}) = 2\text{ar}(\text{FCB})$$

(v)

$$\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$$

(vi)

$$\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$$

(vii)

Note: Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in class X.

Answer:

(i) We know that each angle of a square is  $90^\circ$ . Hence,  $\angle ABM = \angle DBC = 90^\circ$

$$\angle ABM + \angle ABC = \angle DBC + \angle ABC$$

$$\angle MBC = \angle ABD$$

In  $\triangle MBC$  and  $\triangle ABD$ ,

$$\angle MBC = \angle ABD \text{ (Proved above)}$$

$$MB = AB \text{ (Sides of square ABMN)}$$

$$BC = BD \text{ (Sides of square BCED)}$$

$$\therefore \triangle MBC \cong \triangle ABD \text{ (SAS congruence rule)}$$

(ii) We have

$$\triangle MBC \cong \triangle ABD$$

$$\therefore \text{ar}(\triangle MBC) = \text{ar}(\triangle ABD) \dots (1)$$

It is given that  $AX \perp DE$  and  $BD \perp DE$  (Adjacent sides of square BDEC)

$$\therefore BD \parallel AX \text{ (Two lines perpendicular to same line are parallel to each other)}$$

$\triangle ABD$  and parallelogram  $BYXD$  are on the same base  $BD$  and between the same parallels  $BD$  and  $AX$ .

$$\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(BYXD)$$

$$\text{ar}(BYXD) = 2 \text{ar}(\triangle ABD)$$

$$\text{Area}(BYXD) = 2 \text{area}(\triangle MBC) \text{ [Using equation (1)]} \dots (2)$$

(iii)  $\triangle MBC$  and parallelogram  $ABMN$  are lying on the same base  $MB$  and between same parallels  $MB$  and  $NC$ .

$$\therefore \text{ar}(\triangle MBC) = \frac{1}{2} \text{ar}(ABMN)$$

$$2 \text{ar}(\triangle MBC) = \text{ar}(ABMN) \quad \text{ar}(BYXD) = \text{ar}(ABMN)$$

$$\text{[Using equation (2)]} \dots (3)$$

(iv) We know that each angle of a square is  $90^\circ$ .

$$\therefore \angle FCA = \angle BCE = 90^\circ$$

$$\angle FCA + \angle ACB = \angle BCE + \angle ACB$$

$$\angle FCB = \angle ACE$$

In  $\triangle FCB$  and  $\triangle ACE$ ,  $\angle FCB$

$$= \angle ACE$$

$FC = AC$  (Sides of square ACFG)

$CB = CE$  (Sides of square BCED)  $\triangle FCB$

$\triangle ACE$  (SAS congruence rule)

v) It is given that  $AX \perp DE$  and  $CE \perp DE$  (Adjacent ( sides of square BDEC)

Hence,  $CE \parallel AX$  (Two lines perpendicular to the same line are parallel to each other)

Consider  $\triangle ACE$  and parallelogram CYXE

$\triangle ACE$  and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.

$$\therefore \text{ar}(\triangle ACE) = \frac{1}{2} \text{ar}(\text{CYXE})$$

$$\angle \text{ar}(\text{CYXE}) = 2 \text{ar}(\triangle ACE) \dots (4)$$

We had proved that  $\angle \triangle FCB$   
 $\angle \triangle ACE$

$$\text{ar}(\triangle FCB) = \text{ar}(\triangle ACE) \dots (5)$$

On comparing equations (4) and (5), we obtain ar

$$(\text{CYXE}) = 2 \text{ar}(\triangle FCB) \dots (6)$$

(vi) Consider  $\triangle FCB$  and parallelogram  
 ACFG

$\triangle FCB$  and parallelogram ACFG are lying on the same base CF and between the same  
 parallels CF and BG.

$$\therefore \text{ar}(\triangle FCB) = \frac{1}{2} \text{ar}(\text{ACFG})$$

$$\angle \text{ar}(\text{ACFG}) = 2 \text{ar}(\triangle FCB) \quad \angle \text{ar}(\text{ACFG}) = \text{ar}(\text{CYXE})$$

[Using equation (6)] ... (7)

(vii) From the figure, it is evident that ar

$$(BCED) = ar (BYXD) + ar (CYXE) \quad \perp \quad ar (BCED) = ar (ABMN) + ar$$

(ACFG) [Using equations (3) and (7)]