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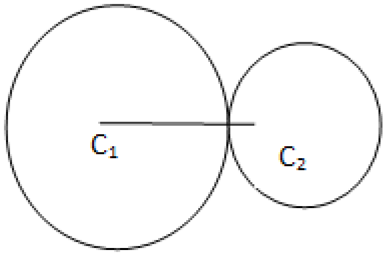
Class: 10
Subject: Math's
Topic: Areas related to circles
No. of Questions: 20

Q.1 The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

Sol: Let the radius of the circle be r cm. Then,
Diameter = 2r cm and Circumference = $2\pi r$ cm
It is given that the circumference exceeds the diameter by 16.8 cm
Circumference = Diameter + 16.8
$2\pi r = 2r + 16.8$
$2 \times 22/7 \times r = 2r + 16.8$
$44r = 14r + 16.8 \times 7$
$44r - 14r = 117.6$
$30r = 117.6$
$r = 117.6/30 = 3.92$
Hence, radius = 3.92 cm

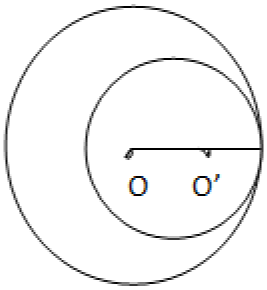
Q.2 Two circles touch externally. The sum of their areas is 130 π sq. cm. and the distance between their centers is 14 cm. Find the radii of the circles.

Sol: If two circles touch externally, then the distances between their centers is the sum of their radii.
Let the radii of the two circles be r_1 cm and r_2 cm respectively.
Let C_1 and C_2 be the centers of the given circles. Then,
$C_1C_2 = r_1 + r_2$
$r_1 + r_2 = 14$
It is given that the sum of the areas of two circles is equal to 130 π cm^2.
$\pi r_1^2 + \pi r_2^2 = 130\pi$
$r_1^2 + r_2^2 = 130$
Now,
$(\pi r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$
$14^2 = 130 + 2r_1r_2$
$196 - 130 = 2r_1r_2$
$r_1r_2 = 33$
Now,
$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$
$(r_1 - r_2)^2 = 130 - 2 \times 33$
$(r_1 - r_2)^2 = 64$
$r_1 - r_2 = 8$
On solving the equations, we get, r_1=11 cm and r_2= 3 cm



Q.3 Two circles touch internally. The sum of their areas is 116 π cm^2 and distance between their centers is 6 cm. Find the radii of the circles.

Sol: Let R and r be the radii of the circles having centers at O and O' respectively.
Then,
Sum of areas = 116π cm^2
$\pi R^2 + \pi r^2 = 116\pi$
$R^2 + r^2 = 116$...... (i)
Distance between the centers = 6 cm
OO' = 6cm
R – r = 6.........(ii)
Now,
$(R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$
$(R + r)^2 + 36 = 2 \times 116$
$(R + r)^2 = (2 \times 116 - 36) = 196$
R + r = 14.......(iii)
Solving (ii) and (iii), we get R = 10 and r = 4.
Hence, radii of the given circles are 10 cm and 4 cm respectively.



Q.4 A copper wire, when bent in the form of a square, encloses an area of 484 cm^2. If the same wire is bent in the form of a circle, find the area enclosed by it. (Use π = 22/7).

Sol: We have,
Area of the square = 484 cm^2
Side of the square 484cm = 22 cm
So, Perimeter of the square = 4 (side) = (4 x 22) cm = 88 cm
Let r be the radius of the circle. Then,
Circumference of the circle = Perimeter of the square.
$2\pi r = 88$

2 x 22/7 x r = 88
r = 14 cm
Area of the circles = πr^2 = {22/7 x $(14)^2$} cm^2 = 616 cm^2

Q.5 A wheel has diameter 84 cm. Find how many complete revolutions must it take to cover 792 meters.

Sol: Let r be the radius of the wheel. Then,
Diameter = 84 cm
2r = 84
r = 42 cm
Circumference of the wheel = 2 π r cm = 2 x 22/7 x 42 cm = 264 cm = 2.64m
So, the wheel covers 2.64 meters in one complete revolution.
Total number of revolutions in covering 792 meters = 792/2.64 = 300.
Hence, the wheel takes 300 revolutions in covering 792 meters.

Q.6 A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheels is 60 cm, calculate the speed per hour with which the boy is cycling.

Sol: We have,
Radius of the wheel = r = 60/2 cm = 30 cm.
Circumference of the wheel = 2 π r = 2 x 22/7 x 30 cm = 1320/7 cm
Distance covered in one revolution = Circumference = 1320/7 cm
Distance covered in 140 revolutions = 1320/7 x 140 cm
= (130 x 20) cm = 26400 cm = 26400/100 m = 264 m = 264/1000 km
It is given that the wheels are making 140 revolutions per minute. So, Distance covered in one minute = Distance covered in 140 revolutions
Distance covered in one minute = 264/1000 km
Distance covered in one hour= 24/1000 x 60 Km= 15.84 Km

Q.7 The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute. (Use π = 22/7)

Sol: Clearly, minute hand of a clock describes a circle of radius equal to its length i.e. 14 cm.
Since the minute hand rotates through 6^o in one minute. Therefore, area swept by the minute hand in one minute is the area of a sector of angle 6^o in a circle of radius 14 cm,
Hence, required area A is given by
A = θ/360 x πr^2
A = {6/360 x 22/7 x $(14)^2$} cm^2
A = {1/60 x 22/7 x 14 x 14} cm^2 = 154/15 cm^2 = 10.26 cm^2

Q.8 The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 2 days. (Take π = 22/7)

Sol: In 2 days, the short hand will complete 4 rounds.
Distance moved by its tip = 4(Circumference of a circle of radius 4 cm)

$= 4 \times (2 \times 22/7 \times 4)$ cm $= 704/7$ cm

In 2 days, the long hand will complete 48 rounds.

Distance moved by its tip = 48(Circumference of a circle of radius 6 cm)

$= 48 \times (2 \times 22/7 \times 6)$ cm $= 12672/7$ cm

Hence, sum of the distances moved by the tips of two hands of the clock = $(704/7 + 12672/7)$ cm $= 1910.85$ cm

Q.9 An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. Find the radius of the circle.

Sol: Let the radius of the circle be r cm and the arc AB of length 5π cm subtends angle θ at the centre O of the circle. Then,

Arc AB = 5π cm and Area of sector OAB = 20π cm^2

$\theta/360 \times 2\pi r = 5\pi$ and $\theta/360 \times \pi r^2 = 20\pi$

$(\theta/360 \times \pi r^2) / (\theta/360 \times 2\pi r) = 20\pi / 5\pi$

$r / 2 = 4$

$r = 8$ cm

Q.10 A chord AB of a circle of radius 15 cm makes an angle of 60^o at the centre of the circle. Find the area of the major and minor segment. (Take $\pi = 3.14$, $\sqrt{3} = 1.73$)

Sol: we know that the area of the minor segment of angle θ^o in a circle of radius r is given by

$A = \{\pi \theta/360 - \sin \theta/2 \cos \theta/2\} r^2$

$A = \{3.14 \times 60 / 360 - \sin 30^o\} (15)^2$ cm^2

$A = \{3.14/6 - \sqrt{3}/4\} \times 225$ cm^2

$A = (0.5233 - 0.4330)\ 225$ cm^2 $= 225 \times 0.902$ cm^2 $= 20.295$ cm^2

Area of the major segment = Area of the circle – Area of the minor segment

$= \{3.14 \times (15)^2 - 20.295\}$ cm^2

$= \{706.5 - 20.295\}$ cm^2 $= 686.205$ cm^2

Q.11 A horse is placed for grazing inside a rectangular field 70m by 52m and is tethered to one corner by a rope 21m long. On how much area can it graze?

Sol: Shaded portion indicates the area which the horse can graze. Clearly, shaded area is the area of a quadrant of a circle of radius r = 21m

Required area = $1/4\pi r^2$

Required area = $\{1/4 \times 22/7 \times (21)^2\}$ cm^2

Required area = $693/2$ cm^2 $= 346.5$ cm^2

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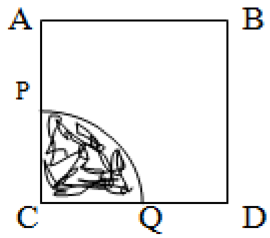
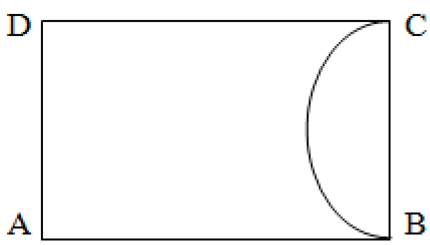


Q.12 A paper is in the form of a rectangle ABCD in which AB = 20cm and BC = 14cm. A semi-circular portion with BC as diameter is cut off. Find the area of a remaining part.

Sol: We have,

Length of the rectangle ABCD = AB = 20cm
Breadth of the rectangle ABCD = BC = 14cm
Area of the rectangle ABCD = (20 x 14)cm^2 = 280 cm^2
Diameter of the semi-circle = BC = 14 cm
Radius of the semi-circle = 7 cm
Area of the semi-circular portion cut off from the rectangle *ABCD*
= 1/2 $(\pi r)^2$ = {1/2 x 22/7 x 7^2} cm^2 = 77 cm^2
Area of the remaining part= Area of rectangle ABCD – Area of semi-circle
= (280 – 77) cm^2 = 203 cm^2



Q.13 A square park has each side of 100m. At each corner of the park, there is a flower bed in the form of a quadrant of radius 14m. Find the area of the remaining part of the park (Use π = 22/7).

Sol: Let A be the area of each quadrant of a circle of radius 14m. Then,
A = 1/4 (πr^2) = 1/4 x 22/7 x 14 x 14 = 154m^2
Area of 4 quadrants = (4 x 154) m^2 = 616 m^2
Area of square park having side 100m long = (100 x 100) m^2 = 10,000 m^2
Hence, area of the remaining part of the park = 10,000 – 616 = 9384 m^2

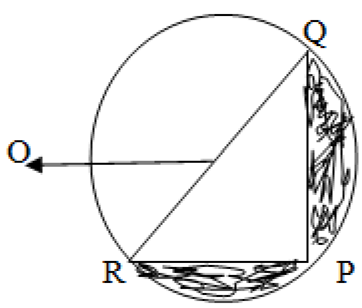
Q.14 Find upto the three places of decimals the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65, measured in centimeters.

Sol: For the first triangle,
a=35, b=53, c=66
s=(a+b+c)/2=(35 + 53 + 66)/2= 77 cm

Now,
Δ_1= Area of the first triangle
Δ_1= $\sqrt{a(s-a)(s-b)(s-c)}$
Δ_1= $\sqrt{77(77-35)(77-53)(77-66)}$ = 924 cm^2

For the second triangle,
a=33, b=56, c=65
s=(33 + 56 + 65)/2= 77 cm
Δ_2= Area of second triangle
Δ_2= $\sqrt{77(77-33)(77-56)(77-65)}$ = 924 cm^2
Let r be the radius of the circle.
Area of the circle= Sum of the areas of two triangles
$\pi r^2 = \Delta_1 + \Delta_2$
πr^2= 924 + 924
$22/7 \times r^2$= 1848
r= $14\sqrt{3}$ cm

Q.15 Find the area of the shaded region from the given figure, if PQ=24 cm, PR=7 cm and O is the centre of the circle.

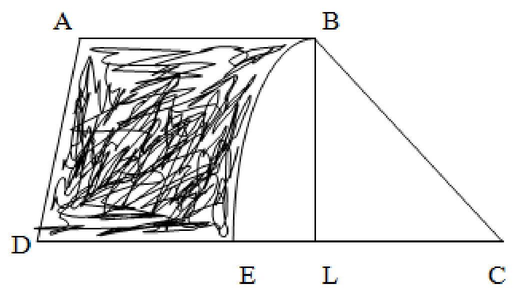


Sol: ∠RPQ is the angle in the semi circle. Therefore, it is a right angle.
Using Pythagoras theorem,
$RQ^2 = RP^2 + PQ^2$
$RQ^2 = 7^2 + 24^2$
RQ^2= 625
RQ= 25 cm
Radius of the circle= 1/2 RQ= 25/2 cm
Area of the shaded region= Area of the semi circle – area of ΔRPQ
= $1/2\ \pi r^2 - 1/2 \times PR \times PQ$
=$1/2 \times 22/7 \times (25/2)^2 - 1/2 \times 7 \times 24$= 4523/28 cm^2

Q.16 ABCD is a trapezium with AB||DC and ∠BCD=60^0, If BEC is a sector of a circle with centre C and AB=BC=7 cm and DE=4 cm, then find the area of the shaded region.

(Use $\sqrt{3} = 1.732$)

(CBSE-2010)



Sol: CE=CB= 7 cm
CD=CE + ED= (7 + 4) cm= 11 cm
In ΔCLB,
sin 60^0= BL/BC
$\sqrt{3}$ /2= BL/7
BL= 7 $\sqrt{3}$ /2 cm
Area of trapezium= 1/2(AB + CD) x BL= 1/2(7 + 11) x 7 $\sqrt{3}$ /2= 63 $\sqrt{3}$ /2 cm^2
Area of sector BEC= $60^0/360^0$ x 22/7 x 7^2= 77/3 cm^2
Required Area= 63 $\sqrt{3}$ /2 – 77/3= 54558 – 25.666= 28.89 cm^2

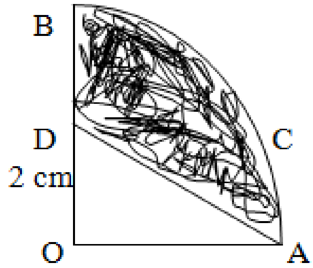
Q.17 The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9.35 A.M.

Sol: Angle described by the minute hand in one minute=6^0
Angle described by the minute hand in 35 minutes= $(6 \times 35)^0$= 210^0
Area swept by the minute hand in 35 minutes= area of a sector of angle 210^0 in a circle of radius 10 cm.
= 210/360 x 22/7 x $(10)^2$= 183.3 cm^2

Q.18 A pendulum swings through an angle of 30^0 and describes an arc 8.8 cm in length. Find the length of the pendulum.

Sol: θ= 30^0, l=arc=8.8 cm
l= θ/360 x 2π r
8.8= 30/360 x 2 x 22/7 x r
r= 8.8 x 6 x 7/ 22= 16.8 cm

Q.19 AOBCA represents a quadrant of a circle of radius 3.5 cm with centre O. Calculate the area of the shaded portion.



Sol: Area of the quadrant AOBCA= 1/4 π r^2= 1/4 x 22/7 x $(3.5)^2$
= 1/4 x 22/7 x 7/2 x 7/2= 27/8 cm^2= 9.625 cm^2
Area of ΔAOD= 1/2 x base x height= 1/2 x (OA x OB)= 1/2[3.5 x 2]= 3.5 cm^2
Area of shaded region= Area of quadrant – area of ΔAOD
= (9.625 – 3.5) cm^2= 6.125 cm^2

Q.20 In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. Find the area of the remaining portion of the triangle. (Take $\sqrt{3}$ = 1.732)

Sol: Let ABC be an equilateral triangle of side 24 cm, and let AD be the perpendicular from A to BC. Since the triangle is equilateral, so D bisects BC.
BD=CD=12 cm
The centre of the inscribed circle will coincide with the centroid of ΔABC.
OD= AD/3
In ΔABD,
AB^2= AD^2 + BD^2
24^2= AD^2 + 12^2
AD= 12 cm
OD= 1/3 AD= 4 $\sqrt{3}$ cm
Area of incircle= $\pi(OD)^2$= 22/7 x $(4\sqrt{3})^2$= 150.85 cm^2
Area of ΔABC= $\sqrt{3}$ /4 x $(24)^2$= 249.4 cm^2
Area of the remaining portion of the triangle= 249.4 – 150.85= 98.55 cm^2