Class: X
Subject: Mathematics
Topic: Arithmetic Progression
No. of Questions: 25.

Q1. Let a sequence be defined by $a_1=3$, $a_n=3a_{n-1}+1$ for all n>1. Find the first four terms of the sequence.

Sol. $a_1=3$
$a_n= 3a_{n-1} + 1$ for all n>1
Put n=2, 3 and 4 we get,
$a_2= 3 * 3 + 1=10$
$a_3= 3 * 10 + 1=31$
$a_4= 3 * 31 + 1=94$

Q2. Find the common difference and write the three next terms of the AP: 3, -2, -7, -12…

Sol. Second term – first term= -2 - 3= -5
Third term – second term= -7 – (-2) = -5
Common difference=-5
$a_5= -12 + (-5)= -17$
$a_6= -17 + (-5)= -22$
$a_7= -22 + (-5)= -27$

Q3. Find the 6th term from the end of the AP: 17, 14, 11,…., -40. (CBSE-2005)

Sol. l= -40, d=-3
6th term from the end= l – (6 – 1)d= -40 -5 *-3= -25

Q4. For what value of n is the nth term of the following two AP's the same?
i) 1,7,13.19….. ii) 69, 69, 67,….. (CBSE-2006)

Sol. First AP: a= 1, d=6
$a_n= 1 + (n – 1)*6= 6n – 5$
Second AP: a=69, d=-1
a_n'= 69 + (n – 1) * -1= -n + 70
The 2 AP's have identical nth term
$a_n= a_n$'
6n – 5= -n + 70
n= 75/7, which is not a natural number
There is no value for n for which the 2 AP's will have identical terms.

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Q5. If the 8^{th} term of an AP is 31 and the 15^{th} term is 16 more than the 11^{th} term, find the AP. (CBSE-2006)

Sol. $a_8 = 31$ and $a_{15} = 16 + a_{11}$
$a + 7d = 31$ and $a + 14d = 16 + a + 10d$
$a + 7d = 31$ and $4d = 16$
$a + 7d = 31$ and $d = 4$
$a + 7 * 4 = 31$
$a = 3$
AP: 3,7,11,15,19……

Q6. Which term of the AP: 5,15,25,…. will be 130 more than its 31^{st} term? (CBSE-2006)

Sol. $a = 5$, $d = 10$
$a_{31} = a + 30d = 5 + 30 * 10 = 305$
$a_n = 130 + a_{31}$
$a + (n - 1)d = 130 + 305$
$5 + (n - 1)10 = 435$
$10(n - 1) = 430$
$n - 1 = 43$
$n = 44$

Q7. If the 10^{th} term of an AP is 52 and 17^{th} term is 20 more than the 13^{th} term, find the AP. (CBSE-2006)

Sol. $a_{10} = 52$ and $a_{17} = a_{13} + 20$
$a + 9d = 52$ and $1 + 16d = a + 12d + 20$
$a + 9d = 52$ and $4d = 20$
$a + 9d = 52$ and $d = 5$
$a + 45 = 52$ and $d = 5$
$a = 7$ and $d = 5$
AP: 7, 12, 17, 22…..

Q8. The sum of 5^{th} and 9^{th} terms of an AP is 72 and the sum of 7^{th} term and 12^{th} term is 97. Find the AP. (CBSE-2009)

Sol. $a_5 + a_9 = 72$ and $a_7 + a_{12} = 97$
$(a + 4d) + (a + 8d) = 72$ and $(a + 6d) + (a + 11d) = 97$
$2a + 12d = 72$……….(1)
$2a + 17d = 97$……….(2)
Subtract (1) from (2)
$5d = 25$

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D=5
Put d=5 in (1)
A=6
AP: 6, 11, 16, 21, 26….

Q9. If the p^{th} term of an AP is q and the q^{th} term is p, prove that its n^{th} term is (p + q – n) (CBSE-2008)

Sol. pth term= q
a + (p – 1)d=q……(1)
qth term= p
a + (q – 1)=p……(2)
Subtract (2) from (1)
(p – q)d= (q – p)
d=-1
Put d in (1)
a + (p – 1)d=q
a=(p + q – 1)
nth term= a + (n – 1)d= (p + q – 1) + (n – 1)*(-1)= (p + q – n)

Q10. If m^{th} term of an AP ne 1/n and n^{th} term be 1/m, then show that its $(mn)^{th}$ term is 1.

Sol. 1/n= mth term
1/n= a + (m – 1)d…..(1)
1/m= nth term
1/m= a+(n – 1)d……(2)
Subtract (2) from (1)
1/n – 1/m= (m – n)d
d= 1/mn
Put d= 1/mn in (1)
1/n= a +(m – 1)/mn
1/n= a + 1/n – 1/mn
a= 1/mn
(mn)th term= a+ (mn – 1)d= 1/mn + (mn – 1)d1/mn=1

Q11. If m times the m^{th} term of an AP is equal to n times its n^{th} term, show that the $(m + n)^{th}$ term of the AP is zero. (CBSE-2008)

Sol. (m times mth term)= (n times nth term)
m a_m= n a_n
m{a + (m – 1)d}= n{a + (n – 1)d}
m{a + (m – 1)d}- n{a + (n – 1)d}=0
a(m – n) + {m(m – 1) – n(n – 1)}d=0

$a(m-n) + \{(m^2 - n^2) - (m-n)\}d=0$
$a(m-n) + (m-n)(m+n-1)d=0$
$(m-n)\{a + (m+n-1)d\}=0$
$\{a + (m+n-1)d\}=0$
$a_{m+n}=0$
Hence, $(m+n)^{th}$ term is zero.

Q12. How many numbers of two digits are divisible by 7?

Sol. AP: 14, 21, 28, 35….., 98
nth term= 98
$14 + (n-1)7=98$
$14 + 7n - 7=98$
$7n=91$, $n=13$

Q13. If p^{th}, q^{th} and r^{th} term of an AP are a, b, c respectively, then show that $a(q-r) + b(r-p) + c(p-q)=0$.

Sol. a= pth term=
$a=A + (p-1)D$….(1)
$b=A + (q-1)D$….(2)
$c= A + (r-1)D$….(3)
$a(q-r) + b(r-p) + c(p-q)= \{A + (p-1)D\}(q-r) + \{A + (q-1)D\}(r-p) + \{A + (r-1)D\}(p-q)$

$=A\{(q-r)+(r-p)+(p-q)\} + D\{(p-1)(q-r) + (q-1)(r-p)+(r-1)(p-q)\}$
$=A * 0 + D*0=0$

Q14. Jasleen saved Rs. 5 in the first week of the year and then increased her weekly savings by Rs.1.75 each week. In what week will her weekly savings be Rs.20.75?

Sol. a=5 and d=1.75
nth term= 20.75
$a + (n-1)d=20.75$
$5 + (n-1)1.75=20.75$
$(n-1) * 1.75=20.75$
$n-1= 15.75/1.75$
$n-1=9$
$n=10$

Q15. Find 4 numbers in AP whose sum is 20 and the sum of whose squares is 120.

Sol. Let the numbers be (a – 3d), (a – d), (a + d) and (a + 3d)
Sum of numbers=20
(a – 3d) + (a – d) + (a + d) + (a + 3d)=20
4a=20
a=5
Sum of square120
$(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$
$4a^2 + 20d^2 = 120$
$a^2 + 5d^2 = 30$
$25 + 5d^2 = 30$
$5d^2 = 5$
d=±1
If d =1 then numbers are: 2,4,6,8 and if d=-1 then numbers are: 8,6,4,2

Q16. If the numbers a,b,c,d,e form an AP then find the value of a – 4b + 6c – 4d + e.

Sol. b=a + D, c=a + 2D, d=a + 3D and e=a + 4D
a – 4b + 6c – 4d + e= a – 4(a + d) + 6(a + 2D) – 4(a + 3D) + (a + 4D)
= a – 4a - 4D + 6a + 12D – 4a - 12D + a + 4D=0

Q17. The sum of 3 numbers in AP is -3, and their product is 8. Find the numbers.

Sol. Let the numbers be (a – d), a, (a+d)
Sum of numbers=-3
(a – d) + a + (a + d)=-3
3a=-3
a=-1
Product of numbers=8
(a – d) * a * (a+d)=8
$a(a^2 - d^2) = 8$
$(-1)(1 - d^2) = 8$
$d^2 = 9$, d=±3
If d=3 the numbers are -4,-1,2. If d=-3 then numbers are 2.-1,-4.

Q18. The sum of first six terms of an AP is 42. The ratio of its 10^{th} term to its 30^{th} term is 1:3. Calculate the first and the 13^{th} term of an AP. (CBSE-2009)

Sol. $S_6 = 42$
6/2{2a + (6 – 1)d{=42
2a + 5d=14……(1)

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$a_{10}:a_{30}= 1:3$
$(a + 9d)/(a + 29d)= 1/3$
$3a + 27d=a + 29d$
$2a – 2d=0$
$a=d$………(2)
On solving (1) and (2), we get $a=d=2$
$a_{13}= a + 12d= 2 + 2 *12=26$

Q19. If nth term of an AP is $(2n + 1)$, find the sum of first n terms of the AP. (CBSE-2005)

Sol. $a_n=(2n + 1)$
$a_1= 2 * 1 + 1=3$
$a=a_1=3$ and $l=a_n=2n + 1$
$S_n= n/2(a + l)= n/2\{3 + (2n + 1)\}$
$=n/2(2n + 4)= n(n+2)$

Q20. If S_n, the sum of first n terms of an AP is given by $S_n= 5n^2 + 3n$, then find the term. (CBSE-2009)

Sol. $a_n= S_n – S_{n-1}$
$a_n=(5n^2 + 3n) – \{5(n – 1)^2 + 3(n – 1)\}$
$a_n= (5n^2 + 3n) – (5n^2 – 7n + 2)$
$a_n=10n – 2$

Q21. In an AP, the sum of first n terms is $3n^2/2 + 5n/2$. Find its 25^{th} term. (CBSE-2006)

Sol. $S_n= 3n^2/2 + 5n/2$
$S_{n-1}= 3/2(n – 1)^2 + 5/2(n – 1)$
$a_n= S_n – S_{n-1}= 3n^2/2 + 5n/2 – \{3/2(n – 1)^2 + 5/2(n – 1)\}$
$a_n= 3/2\{n^2 – (n – 1)^2\} + 5/2\{ n – (n – 1)\}$
$a_n= 3/2(2n – 1) + 5/2$
$a_{25}= 3/2(2 * 25 – 1) + 5/2= 76$

Q22. How many terms of the series 54, 51, 48,…. Be taken so that their sum is 513? Explain the double answer. (CBSE-2005)

Sol. $a=54$ and $d= -3$
$S_n= 513$
$n/2\{2a + (n – 1)d\}=513$
$n/2[108 + (n – 1) * -3]=513$
$n^2 – 37n + 342=0$
$(n – 18)(n – 19)=0$

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n=18 or 19
Here the common difference is negative so 19^{th} term is given by
a_{19}= 54 + (19 – 1) * -3=0
Thus, the sum of 18 terms as well as of 19 terms is 513.

Q23. The sum of n, 2n, 3n terms of an AP are S_1, S_2, S_3 respectively. Prove that $S_3=3(S_2 – S_1)$.

Sol. S_1= Sum of n terms= n/2{2a + (n – 1)d}
S_2= Sum of 2n terms= 2n/2{2a + (n – 1)d}
S_3= Sum of 3n terms= 3n/2{2a + (n – 1)d}
$S_2 – S_1$= 2n/2{2a + (n – 1)d} – n/2 {2a + (n – 1)d}= n/2{2a + (3n – 1)d}
$3(S_2 – S_1)$= 3n/2{2a + (3n – 1)d}= S_3
S_3= $S_2 – S_1$.

Q24. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

Sol. $S_m=S_n$
m/2{2a + (m – 1)d}= n/2{2a + (n – 1)d}
2a(m – n) + {m(m – 1) – n(n – 1)}d=0
$2a(m – n) + \{(m^2 – n^2) – (m – n)\}d=0$
(m – n){2a + (m + n – 1)d}=0
2a + (m + n – 1)d=0
S_{m+n}=(m+n)/2{2a + (m + n – 1)d}
S_{m+n}= (m + n)/2 * 0=0

Q25. The ratio of the sum of n terms of two AP's is (7n+1) : (4n+27). Find the ratio of their m^{th} terms.

Sol. S_n= n/2{$2a_1 + (n – 1)d_1$} and S_n'=n/2{$2a_2 + (n – 1)d_2$}
S_n/S_n'= n/2{$2a_1 + (n – 1)d_1$}/ n/2{$2a_2 + (n – 1)d_2$}= {$2a_1 + (n – 1)d_1$}/{$2a_2 + (n – 1)d_2$}
It is given that
S_n/S_n'=(7n + 1)/(4n + 27)
{$2a_1 + (n – 1)d_1$}/{$2a_2 + (n – 1)d_2$}= (7n + 1)/(4n + 27)
We replace n by (2m – 1)
{$2a_1 + (2m - 1 – 1)d_1$}/{$2a_2 + (2m - 1 – 1)d_2$}= (14m - 6)/(8m + 23)
Required ratio is (14m - 6)/(8m + 23)